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STATISTICS OF SIGNAL TO INTERFERENCE RATIO PROCESS AT OUTPUT OF MOBILE-TO-MOBILE RAYLEIGH FADING CHANNEL IN THE PRESENCE OF COCHANNEL INTERFERENCE

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Abstract. Dual-hop cooperative communications in interference-limited Rayleigh fading channel are investigated in this paper. The paper considers the first- and second-order statistics of the signal to interference ratio process at the input of the destination mobile station. The exact closed-form expressions for the first-order statistical measures, the probability density function and cumulative distribution function, are derived. We also derive the approximate closed form expressions for the second-order statistics, the level crossing rate and the average fade duration. The obtained theoretical results are verified by the Monte-Carlo simulations.

Key words: mobile-to-mobile channel, Rayleigh fading, probability density function, level crossing rate, average fade duration, cooperative communications

1. INTRODUCTION

An efficient way to improve capacity, reliability and energy efficiency of mobile communications is to use the cooperation between users [1,2]. In cooperative communications, the mobile stations are connected either directly, or via a relay or both.

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The cooperative communications are especially important for mobile-to-mobile (M2M) channels, where both mobile stations are in motion and often there is no direct link between the source (S) and the destination (D). M2M channel will have even higher importance in the future, because of increasing importance of vehicle-to-vehicle (V2V) communications. In order to further increase the mobile system capacity, the frequency reuse is implemented. On the other hand, the frequency reuse causes strong cochannel interference (CCI). CCI is much stronger than the noise, and the noise may be neglected. Such an environment is called interference-limited.

Cooperative systems in Rayleigh fading channel are often analyzed in literature [3–13]. The outage probability, in the presence of Rayleigh interference, is considered in [3,4] when the desired signal is transmitted over Nakagami-m and Weibull fading channel, respectively. The average bit error rate, in the generalized K fading environment, is analyzed in [5]. Best relay selection cooperative communications, with the respect to the outage probability, are investigated in [6] for decode-and-forward (DF) and in [7,8] for amplify-and-forward (AF) relaying strategy. The outage performance of dual-hop cooperative systems in Rayleigh fading channels is studied for AF relays in [9–12] and for the case of DF relaying in [11]. Multihop relay systems with CCI in Rayleigh fading channel are evaluated in [13]. Second-order statistics, level crossing rate (LCR) and average fade duration (AFD), are analyzed in [14,15]. Paper [14] considers two-hop AF relaying in Rayleigh fading channel in the presence of thermal noise and CCI. On the other hand, paper [15] evaluates second-order statistical parameters for multiple hop Rayleigh fading channel, however in the absence of interference.

Having in mind the above analysis, it may be noticed that there is a lack of research of dual-hop cooperative systems in interference-limited Rayleigh fading channel, especially the investigation of the second-order statistics, LCR and AFD. Therefore, in this paper we derive the exact closed-form expressions for the first-order statistics of the signal to interference ratio at the input of the destination mobile station, probability density function (PDF) and cumulative distribution function (CDF) with AF relaying in mobile-to-mobile channel. The Laplace approximation for the LCR and AFD is also derived. The theoretical results are verified with the Monte-Carlo simulation. The analysis is also valid for mobile-to-base station channel, without the direct link between S and D. Besides, first order statistics with static transmitters and receivers may also be determined by the given analysis.

The paper is structured as follows. Section 2 describes the system model. First- and second-order performance measures are derived in Sections 3 and 4, respectively. Some numerical results, which show the influence of the fading channel parameters on the system's performance, are given in Section 5. The concluding remarks are given in Section 6.

2. SYSTEM MODEL

As already mentioned, we consider a dual-hop cooperative relay system, as shown in Fig. 1. Due to obstacles, there is no line-of-sight between the source and destination, and therefore S and D are connected only via a relay (R). The interference is present at both sections, S-R and R-D. Such a scenario is very likely for M2M communications, because

of the movement of both S and D. On the other hand, in cellular mobile-to-base station communications, there is usually also a direct link between S and D, due to high altitude location of the base station.



Fig. 1 System model

Random variables, describing the desired signal x_1 , x_2 and interference y_1 , y_2 envelopes, at the first and second section, are independent non-identically distributed Rayleigh random variables with the following probability density functions:

$$p_{x_{i}}(x_{i}) = \frac{x_{i}}{\sigma_{x_{i}}^{2}} \cdot \exp\left(-\frac{x_{i}^{2}}{2\sigma_{x_{i}}^{2}}\right), \quad x_{i} \ge 0, i = 1, 2,$$

$$p_{y_{i}}(y_{i}) = \frac{y_{i}}{\sigma_{y_{i}}^{2}} \cdot \exp\left(-\frac{y_{i}^{2}}{2\sigma_{y_{i}}^{2}}\right), \quad y_{i} \ge 0, i = 1, 2,$$
(1)

where $\sigma_{x_1}, \sigma_{x_2}, \sigma_{y_1}$ and σ_{y_2} are Rayleigh fading parameters. The signal to interference ratio is z_1, z_2 and z.

3. PDF, CDF AND OUTAGE PROBABILITY

The desired received signal envelope at the input of the destination mobile station is given by $x = x_1 \cdot x_2$. Similarly, the cochannel interference envelope at the input of the destination mobile station is equal to $y = y_1 \cdot y_2$. Since we consider AF relays, the signal to interference ratio at the input of the destination mobile station can be expressed as the ratio [16]

$$z = \frac{x}{y} = \frac{x_1 \cdot x_2}{y_1 \cdot y_2} = z_1 \cdot z_2,$$
 (2)

where $z_1 = x_1 / y_1$ and $z_2 = x_2 / y_2$. We will first derive PDF of z_1 and z_2 , and finally of $z = z_1 \cdot z_2$. Since z_i , i = 1, 2 is the ratio of the random variables x_i and y_i , its PDF is defined as

$$p_{z_{i}}(z_{i}) = \int_{0}^{\infty} dy_{i} \cdot y_{i} \cdot p_{x_{i}}(y_{i} \cdot z_{i}) \cdot p_{y_{i}}(y_{i})$$

$$= \int_{0}^{\infty} dy_{i} \cdot y_{i} \cdot \frac{y_{i}z_{i}}{\sigma_{x_{i}}^{2}} \cdot \exp\left(-\frac{y_{i}^{2}z_{i}^{2}}{2\sigma_{x_{i}}^{2}}\right) \cdot \frac{y_{i}}{\sigma_{y_{i}}^{2}} \cdot \exp\left(-\frac{y_{i}^{2}}{2\sigma_{y_{i}}^{2}}\right)$$

$$= \frac{z_{i}}{\sigma_{x_{i}}^{2}\sigma_{y_{i}}^{2}} \int_{0}^{\infty} dy_{i} \cdot y_{i}^{3} \cdot \exp\left(-y_{i}^{2}\left(\frac{z_{i}^{2}}{2\sigma_{x_{i}}^{2}} + \frac{1}{2\sigma_{y_{i}}^{2}}\right)\right)$$

$$= \frac{z_{i}}{\sigma_{x_{i}}^{2}\sigma_{y_{i}}^{2}} \int_{0}^{\infty} ds \cdot s \cdot \exp\left(-s\left(\frac{z_{i}^{2}}{2\sigma_{x_{i}}^{2}} + \frac{1}{2\sigma_{y_{i}}^{2}}\right)\right).$$
(3)

After integrating by parts, we get

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$$p_{z_i}(z_i) = \frac{8\sigma_{y_i}^2 \sigma_{z_i}^2 \cdot z_i}{\left(2\sigma_{z_i}^2 + 2\sigma_{y_i}^2 \cdot z_i^2\right)^2}.$$
 (4)

Now, we have

$$p_{z}(z) = \int_{0}^{\infty} dz_{2} \frac{1}{z_{2}} p_{z_{1}}(z/z_{2}) p_{z_{2}}(z_{2}).$$
(5)

After substituting (3) in (5), we get

$$p_{z}(z) = \frac{4 \cdot \sigma_{x_{1}}^{2} \cdot \sigma_{y_{2}}^{2}}{\sigma_{y_{1}}^{2} \cdot \sigma_{x_{2}}^{2}} \cdot \frac{1}{z^{3}} \cdot \int_{0}^{\infty} dz_{2} \cdot \frac{z_{2}^{3}}{\left(1 + \frac{\sigma_{x_{1}}^{2}}{\sigma_{y_{1}}^{2} z^{2}} z_{2}^{2}\right)^{2}} \left(1 + \frac{\sigma_{y_{2}}^{2}}{\sigma_{x_{2}}^{2}} z_{2}^{2}\right)^{2}}.$$
 (6)

Using 3.259.3 [17], the final expression for the destination SIR PDF may be obtained as

$$p_{z}(z) = \frac{1}{3} \cdot \frac{\sigma_{y_{1}}^{2} \cdot \sigma_{y_{2}}^{2}}{\sigma_{x_{1}}^{2} \cdot \sigma_{x_{2}}^{2}} \cdot z \cdot {}_{2}F_{1}\left(2, 2; 4; 1 - z^{2} \frac{\sigma_{y_{1}}^{2} \cdot \sigma_{y_{2}}^{2}}{\sigma_{x_{1}}^{2} \cdot \sigma_{x_{2}}^{2}}\right),$$
(7)

where $_{2}F_{1}(a,b;c;d)$ is Hypergeometric function [17].

The cumulative distribution function of z is defined as

$$F_z(z) = \int_0^z dt \cdot p_z(t) .$$
(8)

By inserting (6) in (8), and after some mathematical manipulations we get

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$$F_{z}(z) = 2 \cdot \frac{\sigma_{y_{2}}^{2}}{\sigma_{x_{2}}^{2}} \cdot \int_{0}^{\infty} dz_{2} \cdot \frac{z_{2}}{\left(1 + \frac{\sigma_{x_{1}}^{2}}{\sigma_{y_{1}}^{2} z^{2}} z_{2}^{2}\right)} \left(1 + \frac{\sigma_{y_{2}}^{2}}{\sigma_{x_{2}}^{2}} z_{2}^{2}\right)^{2} .$$
(9)

Again, using 3.259.3 [17], we obtain the final expression

$$F_{z}(z) = \frac{1}{2} \cdot \frac{\sigma_{y_{1}}^{2} \cdot \sigma_{y_{2}}^{2}}{\sigma_{x_{1}}^{2} \cdot \sigma_{x_{2}}^{2}} \cdot z^{2} \cdot {}_{2}F_{1}\left(2, 2; 3; 1 - z^{2} \frac{\sigma_{y_{1}}^{2} \cdot \sigma_{y_{2}}^{2}}{\sigma_{x_{1}}^{2} \cdot \sigma_{x_{2}}^{2}}\right).$$
(10)

The outage probability, defined as the probability that the signal to interference ratio at the destination mobile station is lower than a certain threshold z_{th} is equal to

$$P_{out}\left(z_{th}\right) = \Pr\left[z < z_{th}\right] = F_z\left(z_{th}\right). \tag{11}$$

4. LEVEL CROSSING RATE AND AVERAGE FADE DURATION

The destination mobile station signal to interference ratio LCR is evaluated as the average value of the first derivative of the SIR.

Having in mind (2), the first derivative of z is

$$\dot{z} = \frac{x_2}{y_1 \cdot y_2} \dot{x}_1 + \frac{x_1}{y_1 \cdot y_2} \dot{x}_2 - \frac{x_1 \cdot x_2}{y_1^2 \cdot y_2} \dot{y}_1 - \frac{x_1 \cdot x_2}{y_1 \cdot y_2^2} \dot{y}_2.$$
(12)

Random variables \dot{x}_1 , \dot{x}_2 , \dot{y}_1 , and \dot{y}_2 have zero-mean Gaussian distribution with the following variances [18]:

$$\sigma_{\dot{x}_{i}}^{2} = 2\pi^{2} f_{m}^{2} \sigma_{x_{i}}^{2},$$

$$\sigma_{\dot{y}_{i}}^{2} = 2\pi^{2} f_{m}^{2} \sigma_{y_{i}}^{2}, i = 1, 2.$$
(13)

where f_m is the maximum Doppler frequency.

Since a linear transformation of a Gaussian random variable is also a Gaussian random variable, \dot{z} follows a conditional Gaussian distribution with mean [15]

$$\overline{\dot{z}} = \frac{x_2}{y_1 \cdot y_2} \overline{\dot{x}_1} + \frac{x_1}{y_1 \cdot y_2} \overline{\dot{x}_2} - \frac{x_1 \cdot x_2}{y_1^2 \cdot y_2} \overline{\dot{y}_1} - \frac{x_1 \cdot x_2}{y_1 \cdot y_2^2} \overline{\dot{y}_2} = 0,$$
(14)

and variance

$$\sigma_{z}^{2} = 2\pi^{2} f_{m}^{2} \frac{x_{2}^{2}}{y_{1}^{2} \cdot y_{2}^{2}} \sigma_{x_{1}}^{2} \left(1 + z^{2} \frac{y_{1}^{2} \cdot y_{2}^{2}}{x_{2}^{4}} \frac{\sigma_{x_{2}}^{2}}{\sigma_{x_{1}}^{2}} + z^{2} \frac{y_{2}^{2}}{x_{2}^{2}} \frac{\sigma_{y_{1}}^{2}}{\sigma_{x_{1}}^{2}} + z^{2} \frac{y_{1}^{2}}{x_{2}^{2}} \frac{\sigma_{y_{2}}^{2}}{\sigma_{x_{1}}^{2}} \right).$$
(15)

The joint probability density function of z, \dot{z}, x_2, y_1 , and y_2 is

$$p_{zz_{x_2}y_1y_2}(z, \dot{z}, x_2, y_1, y_2) = p_{\dot{z}}(\dot{z} \mid z, x_2, y_1, y_2) \cdot p_{zx_2y_1y_2}(z, x_2, y_1, y_2).$$
(16)

where

$$p_{zx_2y_1y_2}(z, x_2, y_1, y_2) = p_{x_2}(x_2) \cdot p_{y_1}(y_1) \cdot p_{y_2}(y_2) \cdot p_z(z \mid x_2, y_1, y_2).$$
(17)

The conditional joint probability density function of z is

$$p_{z}(z \mid x_{2}, y_{1}, y_{2}) = \left| \frac{dx_{1}}{dz} \right| p_{x_{1}}\left(\frac{z \cdot y_{1} \cdot y_{2}}{x_{2}} \right).$$
(18)

From (2) we get

$$\left|\frac{dx_1}{dz}\right| = \frac{y_1 \cdot y_2}{x_2} \,. \tag{19}$$

After substituting (17), (18), and (19) into (16), the expression for $p_{z \neq x_2 y_1 y_2}(z, \dot{z}, x_2, y_1, y_2)$ becomes

$$p_{z\dot{z}x_{2}y_{1}y_{2}}(z, \dot{z}, x_{2}, y_{1}, y_{2}) = \frac{y_{1} \cdot y_{2}}{x_{2}} p_{x_{1}}\left(\frac{z \cdot y_{1} \cdot y_{2}}{x_{2}}\right)$$

$$\cdot p_{x_{2}}(x_{2}) \cdot p_{y_{1}}(y_{1}) \cdot p_{y_{2}}(y_{2}) \cdot p_{\dot{z}}(\dot{z} \mid z, x_{2}, y_{1}, y_{2}).$$
(20)

The joint probability density function of z and \dot{z} is

$$p_{z\dot{z}}(z,\dot{z}) = \int_{0}^{\infty} dx_{2} \int_{0}^{\infty} dy_{1} \int_{0}^{\infty} dy_{2} \cdot p_{z\dot{z}x_{2}y_{1}y_{2}}(z,\dot{z},x_{2},y_{1},y_{2}).$$
(21)

From (20) and (21), we get

$$p_{z\dot{z}}(z,\dot{z}) = \int_{0}^{\infty} dx_{2} \int_{0}^{\infty} dy_{1} \int_{0}^{\infty} dy_{2} \cdot \frac{y_{1} \cdot y_{2}}{x_{2}} p_{x_{1}} \left(\frac{z \cdot y_{1} \cdot y_{2}}{x_{2}} \right) \cdot p_{x_{2}}(x_{2}) \cdot p_{y_{1}}(y_{1}) \cdot p_{y_{2}}(y_{2}) \cdot p_{\dot{z}}(\dot{z} \mid z, x_{2}, y_{1}, y_{2}).$$
(22)

Finally, the level crossing rate of signal to interference ratio process at the input of the destination mobile station is [19]

$$N_{z}(z) = \int_{0}^{\infty} d\dot{z} \cdot \dot{z} \cdot p_{z\dot{z}}(z\dot{z})$$

= $\int_{0}^{\infty} dx_{2} \int_{0}^{\infty} dy_{1} \int_{0}^{\infty} dy_{2} \cdot \frac{y_{1} \cdot y_{2}}{x_{2}} p_{x_{1}}\left(\frac{z \cdot y_{1} \cdot y_{2}}{x_{2}}\right) \cdot p_{x_{2}}(x_{2}) \cdot p_{y_{1}}(y_{1}) \cdot p_{y_{2}}(y_{2})$ (23)
 $\cdot \int_{0}^{\infty} d\dot{z} \cdot \dot{z} \cdot p_{z}(\dot{z} \mid z, x_{2}, y_{1}, y_{2}).$

Having in mind that

$$\int_{0}^{\infty} d\dot{z} \cdot \dot{z} \cdot p_{\dot{z}}(\dot{z} \mid z, x_{2}, y_{1}, y_{2}) = \frac{1}{\sqrt{2\pi\sigma_{\dot{z}}}} \int_{0}^{\infty} \dot{z} \cdot e^{-\frac{\dot{z}^{2}}{2\sigma_{z}^{2}}} d\dot{z} = \frac{\sigma_{z}}{\sqrt{2\pi}}, \quad (24)$$

we have

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$$N_{z}(z) = \int_{0}^{\infty} dx_{2} \int_{0}^{\infty} dy_{1} \int_{0}^{\infty} dy_{2} \cdot \frac{y_{1} \cdot y_{2}}{x_{2}} p_{x_{1}}\left(\frac{z \cdot y_{1} \cdot y_{2}}{x_{2}}\right) \cdot p_{x_{2}}(x_{2}) \cdot p_{y_{1}}(y_{1}) \cdot p_{y_{2}}(y_{2}) \cdot \frac{\sigma_{z}}{\sqrt{2\pi}}.$$
 (25)

After substituting the probability density functions (1) in (25), the expression for the level crossing rate becomes

$$N_{z}(z) = \frac{1}{\sigma_{x_{1}} \cdot \sigma_{x_{2}}^{2} \cdot \sigma_{y_{1}}^{2} \cdot \sigma_{x_{2}}^{2}} \cdot z$$

$$\cdot \sqrt{\pi} \cdot f_{m} \cdot \int_{0}^{\infty} dx_{2} \int_{0}^{\infty} dy_{1} \int_{0}^{\infty} dy_{2} \cdot \cdot \int_{0}^{\sqrt{1+z^{2}}} \frac{y_{1}^{2} \cdot y_{2}^{2}}{x_{2}^{4}} \frac{\sigma_{x_{2}}^{2}}{\sigma_{x_{1}}^{2}} + z^{2} \frac{y_{2}^{2}}{x_{2}^{2}} \frac{\sigma_{y_{1}}^{2}}{\sigma_{x_{1}}^{2}} + z^{2} \frac{y_{1}^{2}}{x_{2}^{2}} \frac{\sigma_{y_{2}}^{2}}{\sigma_{x_{1}}^{2}} \cdot \int_{0}^{\sqrt{1+z^{2}}} \frac{y_{1}^{2} \cdot y_{2}^{2}}{x_{2}^{4}} \frac{\sigma_{x_{2}}^{2}}{\sigma_{x_{1}}^{2}} + z^{2} \frac{y_{1}^{2}}{x_{2}^{2}} \frac{\sigma_{y_{2}}^{2}}{\sigma_{x_{1}}^{2}} \cdot \int_{0}^{\sqrt{1+z^{2}}} \frac{y_{1}^{2} \cdot y_{2}^{2}}{x_{2}^{4}} \frac{\sigma_{x_{2}}^{2}}{\sigma_{x_{1}}^{2}} + z^{2} \frac{y_{1}^{2}}{x_{2}^{2}} \frac{\sigma_{y_{2}}^{2}}{\sigma_{x_{1}}^{2}} \cdot \int_{0}^{\sqrt{1+z^{2}}} \frac{y_{1}^{2} \cdot y_{2}^{2}}{\sigma_{x_{1}}^{2}} + z^{2} \frac{y_{1}^{2}}{\sigma_{x_{2}}^{2}} \frac{\sigma_{y_{2}}^{2}}{\sigma_{x_{1}}^{2}} \cdot \int_{0}^{\sqrt{1+z^{2}}} \frac{y_{1}^{2} \cdot y_{2}^{2}}{\sigma_{x_{1}}^{2}} + z^{2} \frac{y_{1}^{2} \sigma_{y_{2}}^{2}}{\sigma_{x_{1}}^{2}} \cdot \int_{0}^{\sqrt{1+z^{2}}} \frac{y_{1}^{2} \cdot y_{1}^{2} \cdot y_{2}^{2}}{\sigma_{x_{1}}^{2}} + z^{2} \frac{y_{1}^{2} \sigma_{x_{2}}^{2}}{\sigma_{x_{1}}^{2}} \cdot \int_{0}^{\sqrt{1+z^{2}}} \frac{y_{1}^{2} \cdot y_{1}^{2} \cdot y_{2}^{2}}{\sigma_{x_{1}}^{2}} \cdot \int_{0}^{\sqrt{1+z^{2}}} \frac{y_{1}^{2} \cdot y_{1}^{2} \cdot y_{2}^{2}}{\sigma_{x_{1}}^{2}} \cdot \int_{0}^{\sqrt{1+z^{2}}} \frac{y_{1}^{2} \cdot y_{1}^{2} \cdot y_{2}^{2}}{\sigma_{x_{1}}^{2}} - \frac{y_{1}^{2} \cdot y_{1}^{2} \cdot y_{2}^{2}}{\sigma_{x_{1}}^{2}} \cdot \int_{0}^{\sqrt{1+z^{2}}} \frac{y_{1}^{2} \cdot y_{2}^{2} \cdot y_{1}^{2}}{\sigma_{x_{1}}^{2}} \cdot \int_{0}^{\sqrt{1+z^{2}}} \frac{y_{1}^{2} \cdot y_{1}^{2} \cdot y_{2}^{2}}{\sigma_{x_{1}}^{2}} \cdot \int_{0}^{\sqrt{1+z^{2}}} \frac{y_{1}^{2} \cdot y_{2}^{2}}{\sigma_{x_{1}}^{2}} \cdot \int_{0}^{\sqrt{1+z^{$$

The three-fold integral in the above expression is solved by using Laplace approximation theorem for the three-fold integral [20]

$$\int_{0}^{\infty} dx \int_{0}^{\infty} dy \int_{0}^{\infty} dz \cdot g(x, y, z) \exp(-\lambda \cdot f(x, y, z))$$

$$= \left(\frac{2\pi}{\lambda}\right)^{3/2} \frac{g(x_0, y_0, z_0)}{\sqrt{B(x_0, y_0, z_0)}} \exp(-\lambda \cdot f(x_0, y_0, z_0)),$$
(27)

where x_0, y_0 , and z_0 are solutions of the following set of equations:

$$\frac{\partial f(x_0, y_0, z_0)}{\partial x_0} = 0,$$

$$\frac{\partial f(x_0, y_0, z_0)}{\partial y_0} = 0,$$

$$\frac{\partial f(x_0, y_0, z_0)}{\partial z_0} = 0,$$
(28)

and

$$B(x_{0}, y_{0}, z_{0}) = \begin{vmatrix} \frac{\partial^{2} f(x_{0}, y_{0}, z_{0})}{\partial x_{0}^{2}} & \frac{\partial^{2} f(x_{0}, y_{0}, z_{0})}{\partial x_{0} \partial y_{0}} & \frac{\partial^{2} f(x_{0}, y_{0}, z_{0})}{\partial y_{0} \partial z_{0}} \\ \frac{\partial^{2} f(x_{0}, y_{0}, z_{0})}{\partial x_{0} \partial y_{0}} & \frac{\partial^{2} f(x_{0}, y_{0}, z_{0})}{\partial y_{0}^{2}} & \frac{\partial^{2} f(x_{0}, y_{0}, z_{0})}{\partial y_{0} \partial z_{0}} \\ \frac{\partial^{2} f(x_{0}, y_{0}, z_{0})}{\partial x_{0} \partial z_{0}} & \frac{\partial^{2} f(x_{0}, y_{0}, z_{0})}{\partial y_{0} \partial z_{0}} & \frac{\partial^{2} f(x_{0}, y_{0}, z_{0})}{\partial z_{0}^{2}} \end{vmatrix} .$$
(29)

For the considered case, the constant $\lambda = 1$ and functions *f* and *g* are

$$g(x_2, y_1, y_2) = \sqrt{1 + z^2 \frac{y_1^2 \cdot y_2^2}{x_2^4} \frac{\sigma_{x_2}^2}{\sigma_{x_1}^2} + z^2 \frac{y_2^2}{x_2^2} \frac{\sigma_{y_1}^2}{\sigma_{x_1}^2} + z^2 \frac{y_1^2}{x_2^2} \frac{\sigma_{y_2}^2}{\sigma_{x_1}^2}},$$
(30)

$$f(x_2, y_1, y_2) = \frac{1}{2\sigma_{x_1}^2} \frac{z^2 \cdot y_1^2 \cdot y_2^2}{x_2^2} + \frac{1}{2\sigma_{x_2}^2} x_2^2 + \frac{1}{2\sigma_{y_1}^2} y_1^2 + \frac{1}{2\sigma_{y_2}^2} y_2^2 - 2\ln y_1 - 2\ln y_2.$$
(31)

Using the expressions for CDF and LCR, the average fade duration may be defined as

$$T_z(z) = \frac{F_z(z)}{N_z(z)}$$
. (32)

5. NUMERICAL RESULTS

This section presents some numerical results that indicate the influence of different fading parameters on the PDF, outage probability, LCR, and AFD. The obtained theoretical results are confirmed by the Monte-Carlo simulation, with one million simulation steps. Without the loss of generality, the following assumptions are made $\sigma_{x_1} = \sigma_{x_2} = \sigma_x$ and $\sigma_{y_1} = \sigma_{y_2} = \sigma_y$.

Fig. 2 shows PDF of the destination node signal to interference ratio, for different values of ratio $R = 20 \log(\sigma_x / \sigma_y)$. The results show an excellent agreement between the theoretical and simulation results.



Fig. 2 Probability density function of the destination signal to interference ratio



Fig. 3 Outage probability as a function of the outage threshold

Outage probability, P_{out} , as a function of the outage threshold z_{th} is shown in Fig. 3, with *R* as a parameter. It may be noticed that P_{out} is lower for higher *R*, due to better channel conditions for higher signal to interference ratio. Again, the theoretical and simulation results are in good agreement.

Fig. 4 illustrates the outage probability as a function of R. This figure confirms conclusions from the Fig. 3.



Fig. 4 Outage probability as a function of average signal to interference power ratio

Fig. 5 depicts the normalized level crossing rate for different values of R. Besides the Laplace approximation results, we show the results obtained by the numerical integration of (26) in the software package Mathematica. Also, these results are compared to the Monte-Carlo simulation results, based on the sum-of-sinusoids Rayleigh channel model [21]. The carrier frequency is chosen to be 1 GHz, mutual terminals speed is 80 km/h, which resulted in the maximum Doppler frequency of 74 Hz. There is a good agreement between the exact (numerical integration), approximation, and simulation results. The same difference between the theoretical and simulation results for LCR of a Rayleigh random variable is observed in [21] (Fig. 9), too.



Fig. 6 Normalized average fade duration

Based on the level crossing rate and the cumulative distribution function, the normalized average fade duration is calculated and shown in Fig. 6. The exact, Laplace approximation and the simulation results are shown. The average fade duration is lower for higher R, again due to better channel conditions.

6. CONCLUSION

This paper consideres the cooperative mobile-to-mobile communications performance in Rayleigh fading channel in the presence of cochannel interference. Two hop communication is assumed, where source mobile station is connected to the destination mobile station via a relay. Both the desired signal and interference are subject to Rayleigh fading. The exact closed-form of the first-order statistical measures, the probability density function and cumulative distribution function are derived. Besides, an approximate closed-form expression for the level crossing rate and average fade duration is given. The exact and approximate results are compared to the Monte-Carlo simulation results. The analysis shows an excellent agreement between the exact, approximate and simulated results.

In the future work, we will analyze more complex system model, where the source and destination are connected both directly and via a relay.

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