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Research Paper

TWO-PARTITION DISCRETE GLOBAL GRID SYSTEMS – A COMPARISON OF APPROACHES BASED ON ADJUSTED SPHERICAL CUBE AND CYLINDRICAL EQUIDISTANT PROJECTION

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Abstract. *The enormous volumes of geospatial data and the need to process and distribute them cry out for a unified framework that enables their efficient storage, analysis, and a high degree of interoperability. Discrete global grid systems provide such a framework by hierarchically tessellating cells to seamlessly partition and address the globe. Since they are usually based on a regular polyhedron, they partition the entire world into as many discrete data sets as the given polyhedron has sides. In this paper, we try to reduce the number of partitions to two, which is a minimum if we want to obtain spatially convex partitions without interruptions. Two approaches are presented, based on an adjusted spherical cube and an equidistant cylindrical projection. The distortions resulting from the application of these projections are compared and guidelines are presented to improve the quality of their implementation by reducing the distortion of the continental plates and making a better mapping to the WGS84 ellipsoid.*

Key words: *Geospatial reference, discrete global grid system, map projection*

1. INTRODUCTION

The amount of geospatial data is extremely large, growing by several terabytes every day. The sources of this data are diverse, and the largest amount comes in the form of satellite and aerial imagery. The collected data is available in various projections and formats. In order for them to be shared, analyzed, and commonly used, they must be consistent and organized, and

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there must be a unique way to reference them. This requires an appropriate global spatial reference frame.

Discrete Global Grid Systems (DGGS) are a class of spatial reference systems that use a hierarchical tessellation of cells to partition and address the entire planet without gaps or overlaps. The formal development of these systems begins in the middle of the 20th century [1], but they experienced a real expansion only at the transition from the 20th to the 21st century [2]-[4]. The importance of DGGS is also reflected in the fact that the Open Geospatial Consortium (OGC) established the DGGS Standard and Domain Working Groups to promote geospatial data interoperability and published the Discrete Global Grid Systems Abstract Specification [5] in 2017. The standardization process continued and resulted in a formal specification defined by the ISO 19170-1:2021 standard [6].

The most important subclass of DGGSs are systems based on regular, multi-resolution partitions of polyhedra, called Geodesic DGGSs (GDGGS) [4]. The five Platonic solids (tetrahedron, hexahedron, octahedron, icosahedron, and dodecahedron) circumscribed by the sphere are most commonly used as regular base polyhedra. As the number of faces increases, the polyhedron better approximates the spherical surface, reducing the effects of the distortion caused by the projection of the planet onto the polyhedron. However, each face represents a separate partition with its own local coordinate system and separate hierarchical data set, and is adjacent to neighboring independent partitions, which complicates the overall processing of the data. Apart from the problem of merging segments that lie on the boundary of multiple partitions, a larger number of partitions in systems for streaming geospatial data, such as planetary-scale terrain data visualization systems [7], means larger memory requirements and less efficient resource consumption. Therefore, the goal is to reduce the number of partitions to a minimum while keeping the distortion under control.

This paper compares two approaches for designing DGGSs with two partitions based on orthogonal grids. The paper is divided into 6 sections. After a brief introduction to DGGSs in Section 1, Section 2 presents the basic design decisions that fully specify DGGS. Section 3 proposes two partitioning schemes based on an adjusted spherical cube and a cylindrical equidistant projection. Section 4 compares these approaches in terms of the distortion, while Section 5 recommends further improvements by selecting an appropriate ellipsoid to sphere mapping and base cube orientation. The conclusion and directions for further development are given in Section 6.

2. DGGS IMPLEMENTATION

There are many different approaches to the design of GDGGS. However, five basic elements [4] completely determine GDDGS, namely:

1. Regular base polyhedron,
2. Orientation of the base polyhedron with respect to the planet,
3. Hierarchical spatial partitioning of the sides of the polyhedron into cells,
4. Transform the planar faces of the polyhedron into the corresponding spherical or ellipsoidal surface of the planet and vice versa, and
5. Methods for indexing and addressing cells.

Usually, one of the five Platonic solids [8] is chosen as the base polyhedron because they are regular and consist of sides of the same shape (triangles, squares, or pentagons), the same area, and the same number of neighbors. However, in some cases semiregular polyhedra are also used, in particular the truncated icosahedron, one of the 14 Archimedean solids. The truncated icosahedron is suitable for a hexagonal cell structure, but the cells are not uniform. Each of the 12 vertices of the icosahedron is truncated to form a pentagon, resulting in 20 hexagons corresponding to the original triangular faces [8]. The 12 pentagonal cells exist at all levels of tessellation. A considerable number of the proposed GDGGSs are based on the icosahedron and the truncated icosahedron using triangular or hexagonal cells [2]. Despite their good properties in approximating a spheroidal surface, the lack of orthogonal axes and cell congruence, as well as the complicated implementation seem to prevent their wide acceptance. On the other hand, hexahedral GDGGSs lead to larger distortions due to the smaller number of primary partitions. However, their ease of implementation and superior data organization and retrieval properties make them more attractive for use in various applications.

The first hexahedral projection can be attributed to C. G. Reichard, who in 1803 produced a six-sheet atlas of the entire world [5] using a gnomonic projection. The interest in the cube as a base polyhedron returned in the early 1970s, first through the class of conservative finite difference approximations of the primitive equations for quasi-uniform spherical grids derived from regular polyhedra in 1972 [9], and then through the feasibility study of a quadrilateralized spherical cube Earth database system [10] for the US Navy Department in 1975. The proposed quadrilateralized spherical cube projection was used extensively with some modifications more than a decade later for the Cosmic Background Explorer (COBE) project at NASA. The extended studies of a quadrilateralized spherical cube Earth database led to an exact equal-area sphere-to-cube mapping in 1976 [11]. However, the proposed formulation was computationally difficult to solve at that time [12].

In the last two decades, hexahedral projections have come back into focus as a simple method to extend existing 2D spatial databases and information browsing systems to the 3D spherical data model [3] and to accurately render a planetary scale terrain [13]. Some modern open-source libraries, such as S2 [14], use spherical cubes to enable seamless geographic databases with low distortion. Numerous hexahedral projections have been proposed in the last decades, highlighting the relevance and popularity of this approach, especially in geodesy, but also in computer graphics [15]-[16].

Aligning the base polyhedron is the next step in designing the GDGGS. Usually this is done so that the vertices or the centers of the faces coincide with the poles or achieve symmetry with respect to the equator and the prime meridian. Considering that it is often important to represent the continental plates without interruptions and with as little distortion as possible, the alignment can support some of the desired properties, such as unfolding the entire world into the Dymaxion map [17] without landmass interruptions, completely covering the area of interest with only one face of the polyhedron [8], or globally minimizing the landmass distortion [18]. Since DGGS should provide an efficient organization of data for the entire world without favoring certain areas or having to consider it as an unfolded 2D map, an approach that minimizes the distortion on a global scale is certainly the most acceptable approach for aligning the base polyhedron.

The third step in the definition of the GDGGS is the spatial partitioning of the sides of the polyhedron into cells. The partitioning must be such that it is possible to form a multi-resolution grid. In general, four cell topologies [4] are used, namely: square, triangle, diamond, and hexagon. There are two basic properties for the relationship of cells in two adjacent hierarchy levels, namely: congruence and alignment. Cells are congruent if the area of the cell on a coarser level represents the union of the areas of the cells on a finer level, and aligned if the centers of the cells on different levels coincide. The ratio of cell areas on two adjacent levels is called the aperture. The aperture depends on the shape of the cell, but also on whether we want to maintain the property of congruence or alignment, and how fine we want the transition between different levels to be. The smaller the aperture, the smaller the difference in resolution between two adjacent levels, and the larger the number of levels in the data set. If the base polyhedron is a cube and the sides are square, the most natural subdivision is into square cells, and the most common aperture is 4 or 9, although other values are used [19]. Aperture 4 allows the formation of congruent and unaligned grids, while aperture 9 forms congruent and aligned grids, but with a larger difference in the resolution of adjacent levels. In the present work, aperture 4 is used because it represents a good balance between a smooth transition and the number of levels with different resolutions, but also because of the possibility of a direct mapping to the technology used for visualization.

The fourth step in defining GDGGS is to decide how to map the surface of the polyhedron onto a sphere or an ellipsoid. Two approaches are possible [2]: direct spherical subdivision and projection. The direct spherical subdivision creates partitions directly on the spherical surface, while the projection uses the inverse transformation of mapping the spheroidal surface onto flat faces of the base polyhedron. Since it is impossible to project a curved surface onto a plane without distortion, we can choose whether to preserve the surface (equal-area projection) or the shape (conformal projection) when choosing the projection. The better one feature is preserved, the more the other is violated. For this reason, projections are used that are neither equal-area nor conformal, but keep both types of distortion under control. Detailed reviews of the properties of projections used to map the sphere onto a cube and vice versa, especially from the point of view of their application in computer graphics and visualization of planetary scale terrain, can be found in [15] and [16]. In the next section we present some of the projections that can be used for bisecting the planet.

The final step in defining GDGGS is to choose a cell indexing method. Dealing with an extremely large number of cells requires an efficient addressing scheme, usually referred to as indexing. Many indexing methods have been proposed for DGGSs, but all of them can be classified into three types [20]: hierarchy-based, space-filling curves, and axis-based indexing. Hierarchy-based indexing follows the refinement process, which is usually given by a space-partitioning data structure in the background. This index is usually represented by a string whose length corresponds to the resolution and where each character or number represents the spatial position of the cell within its parent structure. The space-filling curve indexing is based on recursively defined 1D curves that systematically traverse a space, which in the case of 2D space is a unit square. This indexing scheme is popular because it serializes a multidimensional address into a very compact single number and reduces the memory access time by preserving the relationship between near neighbors. The two most popular space-filling cures used in the cube-based GDGGS indexing are Hilbert and Morton curves. The axis-based indexing is a natural way of cell addressing using an m-dimensional position vector. By unfolding the base cube into a plane, a unique 2D coordinate system can be used for all of its faces, but most implementations use separate coordinate systems for each face. Since the axis-based indexing is suitable for accessing multidimensional data, it is a good choice for implementing a data cube based on DGGS [21] as a Digital Earth platform [22]. Digital Earth is a concept of an interactive digital replica of the entire planet that can facilitate a shared understanding of the multiple relationships between the physical and natural environments and society [23].

3. Two-partition World Division

People have always had to georeference to determine their position in the space that surrounds them. Traditionally, parts of the Earth's surface were converted into planar maps because they were easier to produce and store and proved to be very effective until the need arose to apply them to the entire planet.

Since the Earth has a spherical shape, it is impossible to unfold its surface in a plane without causing a considerable distortion of area and shape. Projecting the entire surface onto only one plane either results in extremely large distortions that grow into singularities (most common at the poles, since these are less important for cartography), or the map is interrupted (discontinuous). Discontinuous maps reduce the distortion, but the partitions are separated from each other and in a mutual position that does not correspond to their relationship on the surface of the planet. Apart from the problem of representing areas that lie on partition boundaries (partition merging problem), such maps have an interrupted visual context of integrity. It is precisely for this reason that the equidistant cylindrical projection (invented by Marinus of Tyre around 100 AD), shown in Fig. 1, is still the most commonly used for organizing global data, despite its extremely large distortion, because the relationship between the position on the map and the corresponding geographic location on the Earth is particularly simple.

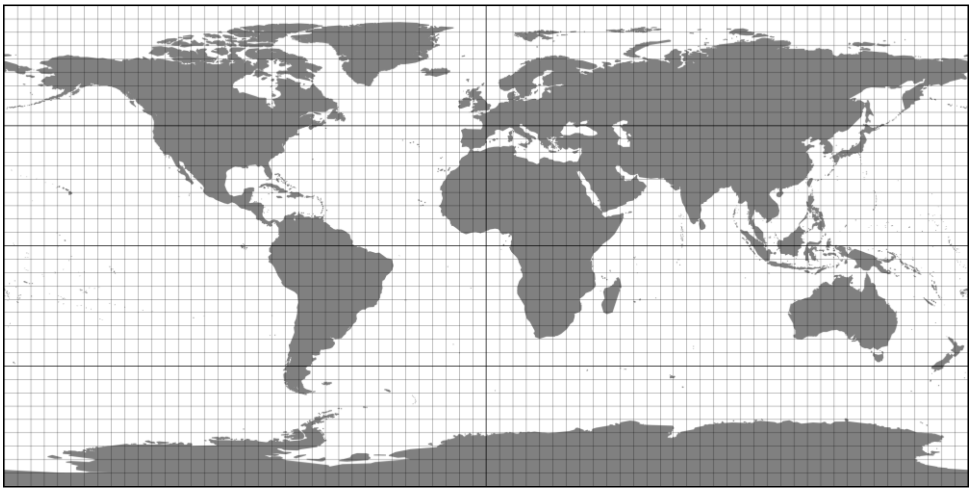


Fig. 1 Equidistant cylindrical projection

Of all the basic polyhedra used in GDGGSs, only the hexahedron, i.e., the cube, allows the merging of partitions into continuous 2D maps. This is a very important feature given the way raster data is stored in modern computer systems. There have been attempts to dynamically create a continuous map composed of the faces of a cube [24] centered on the focal point, but the drawback is the discontinuity in the vertices of the cube (Fig. 2). Another problem is the need to rotate certain sections of the map with the shift of the focal point, which makes it difficult to update the map and disturbs the spatial coherence.

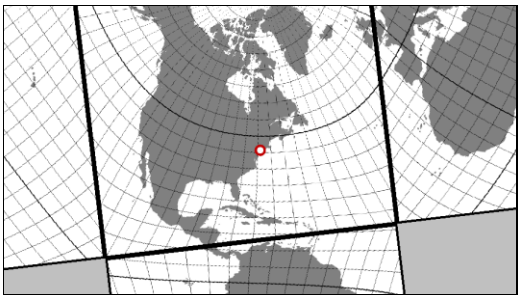


Fig. 2 An example of a continuous 2D map based on a spherical cube [24].

From the point of view of spatial consistency and the formation of a minimum number of partitions, it is optimal to connect three sides of the cube into one partition (Fig. 3). The partitions formed in this way are symmetrical and orthogonal to each other, and their shape is reminiscent of the Yin-Yang symbol, which is why the corresponding grid got its name [25]. An organization with two partitions (Yin-Yang) can also be obtained by two orthogonal cylindrical projections (Fig. 4). In this section, both approaches are presented.

Regardless of the projection method used, the first partition (P0) generally extends horizontally, symmetrically about the equator and the prime meridian (Figs. 3a and 4a), with polar coordinates identical to global geographic coordinates (φ,θ). The second partition (P1) extends along the anti-meridian (180th meridian), is symmetric about it, and contains both poles (Figs. 3b and 4b). The transformation of the local polar coordinates of one partition into another is achieved by equations (1) and (2). Various forms of these transformations can be found in [25].

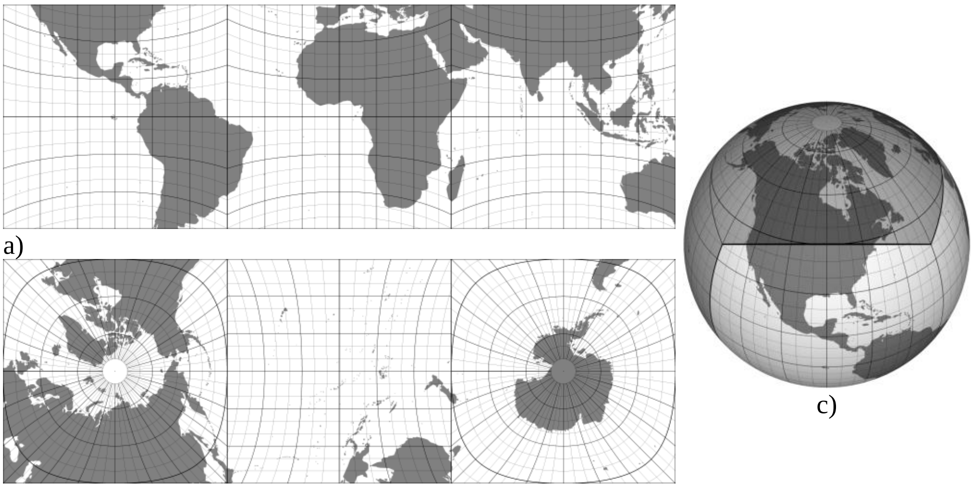
$$\theta_{P1} = arcsin\left(-cos(\theta_{P0}) \cdot sin(\varphi_{P0})\right), \quad (1)$$

$$\varphi_{P1} = -sgn(\theta_{P0}) \cdot arccos\left(-cos(\theta_{P0}) \cdot sin(\varphi_{P0}) / cos(\theta_{P1})\right). \quad (2)$$

A good property of these transformations is that they are the inverse of themselves. This means that two successive applications lead to an identity transformation. So they can be used to transform from any partition to another. The data are defined in the space of the partitions by plane (x,y) coordinates. The function that transforms polar coordinates (φ,θ) into plane coordinates (x,y) is called the *forward* transformation (a projection of a sphere onto a plane) and the function that transforms plane coordinates into polar coordinates (a projection of a plane onto a sphere) is called the *inverse* transformation. The properties of projections are precisely defined by these functions.

Partitioning Based on Spherical Cubes

There are many projections that can be used to project a spherical surface onto a cube, popularly called spherical cubes. In [15], only some of the most commonly used are shown. These projections can be equal-area or conformal, and most have neither of these properties. However, the projections that are neither equal-area nor conformal are commonly used because they balance the effects of two types of distortions with some additional practical properties. In this work, the adjusted spherical cube projection [26]-[27] was used, which is very simple to implement and yet very effective. The forward transformation is defined by equations (3) and (4), while equations (5) and (6) define the inverse transformation [15].

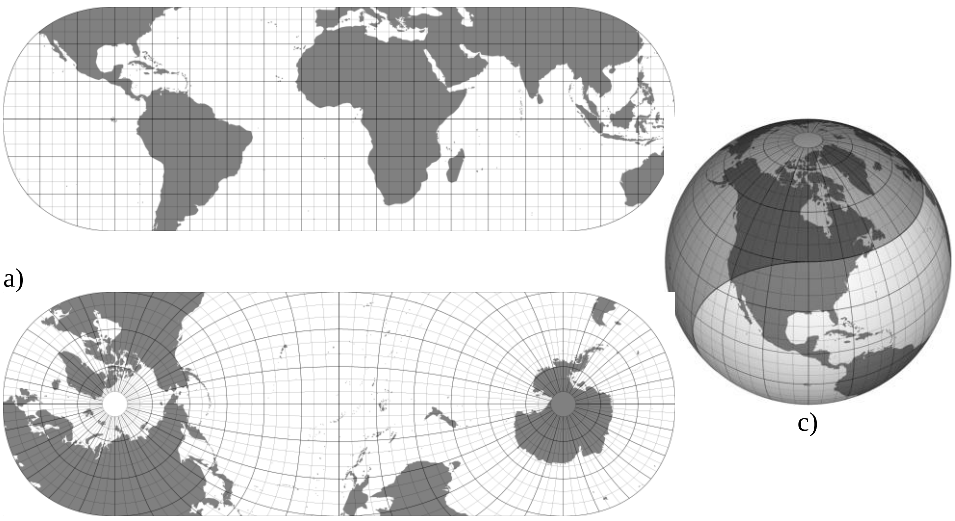


a)

b)

c)

Fig. 3 Partitioning based on an adjusted spherical cube: a) partition P0, b) partition P1, and c) partitions applied to the sphere, with partition P1 shown in a darker shade.



a)

b)

c)

Fig. 4 Partitioning based on cylindrical projection: a) partition P0, b) partition P1, and c) partitions applied to the sphere, with partition P1 shown in a darker shade.

$$x = \varphi \cdot 4 / \pi \tag{3}$$

$$y = \arctan\left(\tan(\theta) / \cos(\varphi)\right) \cdot 4 / \pi \tag{4}$$

$$\varphi = x \cdot \pi / 4 \tag{5}$$

$$\theta = \arctan\left(\tan(y \cdot \pi / 4) * \cos(\varphi)\right) \tag{6}$$

In contrast to the classical spherical cubes, where the plane coordinates (*x*,*y*) lie in the interval [-1,1], in the case of the two-partition projection x $\in$ [-3,3], so that equation (3) is changed to (7).

$$x = \begin{cases} (\varphi + \pi / 2) \cdot 4 / \pi - 2, & \varphi < -\pi / 4 \\ (\varphi - \pi / 2) \cdot 4 / \pi + 2, & \varphi > \pi / 4 \\ \varphi \cdot 4 / \pi, & |\varphi| < \pi / 4 \end{cases} \tag{7}$$

Another problem is to determine to which partition a point with given polar coordinates (φ,θ) belongs. If $|\varphi|<3\pi/4$ and $|\theta|<3\pi/4$ holds, then the point certainly belongs to partition P1. However, if this condition is not satisfied, it does not mean that it belongs to partition P0, because the angles of the polar faces of the cube are below the 45th parallel. The problem can be solved by assuming that the point belongs to the partition P0 and checking whether the y-coordinate obtained by the forward transformation has an absolute value smaller than 1. If this is the case, the assumption that the point belongs to the partition P0 is also correct.

Partitioning Based on Cylindrical Projection

Since the partitions enclose the sphere cylindrically, the use of two orthogonal cylindrical projections can reduce the distortion by avoiding the bends at the edges of the cube. However, this approach also has a drawback, since it leads to a partial overlap of the square cells at the edge of the partition [25]. When rectangular partitions are used, 6.4% of their areas overlap. The challenge in using this projection is to efficiently remove the overlapping areas and precisely cut and fit the partitions. In Fig. 4, the overlapping areas have been removed, resulting in the oval shape of the partitions.

Using the equidistant cylindrical projection simplifies the forward and inverse transformations as much as possible. For the partition P0, the plane coordinates (x,y) are obtained by dividing the polar coordinates (φ,θ) by the constant PI /4 so that the range of coordinates is identical to that of the projection based on the spherical cube. In the case of partition P1, the global polar coordinates must first be transformed to the polar coordinates of the respective partition using equations (1) and (2) before dividing by the constant $\pi/4$. The inverse transformation for the partition P0 multiplies the plane coordinates by the constant $\pi/4$, while for P1 it additionally transforms the polar coordinates of the partition into global polar coordinates using equations (1) and (2).

Since the overlapping regions are in the leftmost and rightmost sixths of the partition, the point (x,y) belongs exclusively to the current partition if x $\in$ [-2,2]. If the previous condition is not satisfied, the point does not belong to the overlapping region if $|\theta_{P1}| < \pi/4$. θ_{P1} is obtained by applying the inverse transformation and converting the polar coordinates to the space of another partition by applying equations (1) and (2).

4. Comparison of the Proposed Projections by Distortion

The two most commonly used metrics for representing shape deformation in projection are *angular* and *areal distortions*. Ideally, two lines should intersect at the same angle on both the surface of the globe and the projected map. If the projection is conformal, the angles and therefore the shape of the features are preserved. For the non-conformal projections, the angular distortion represents the maximum deviation from the correct angle at a given location. The projection can also change the scale of features. The ratio between the projected area and the original area is called areal distortion. Equal-area projections preserve the area.

Distortion parameters are often represented with Tissot’s indicatrices [28]. An infinitesimal circle on the Earth projects as an infinitesimal ellipse on a map projection, with major and minor axes directly related to areal and angular distortions. This ellipse of distortion is called an indicatrix. Table 1 compares the two proposed projections using Tissot's formulas. In addition to the areal and angular distortion, the aspect distortion was added as the ratio of the major and minor semiaxis of Tissot’s indicatrices.

Tab. 1 Comparison of distortion parameters for two proposed projections.

Projection	Distortion	Min.	Max.	Ave.
Adjusted spherical cube	Areal	1.621	2.292	1.925
	Angular [°]	0.000	31.07	11.57
	Aspect	1.000	1.732	1.234
Equidistant cylindrical	Areal	1.621	2.292	1.805
	Angular [°]	0.000	19.75	5.864
	Aspect	1.000	1.414	1.113

Table 1 shows that both projections have the same ranges of areal distortion, but that the mean value is significantly lower for the cylindrical equidistant projection. This is due to the fact that the areal distortion is greatest at the edges, where the projection plane ends. For an equidistant cylindrical projection it is at $\pm\pi/4$, while for a spherical cube it occurs at all edges of the cube. Since a partition of a spherical cube consists of three connected sides of a cube, a much larger area is affected by the distortion. Fig. 5 shows graphically the effects of distortion on the partition P0.

When considering angular distortion, the equidistant cylindrical projection is significantly better for both maximum and mean values. This indicates that the shapes are better preserved with this projection. The angular distortion is greatest at the vertices of the spherical cube, where it reaches 31°. Only the angular distortion is given in degrees. There are no units of measurement for the other types of distortions. The cylindrical equidistance projection is also better when considering the aspect distortion. The largest aspect distortion in the adjusted spherical cube is also at the vertices.

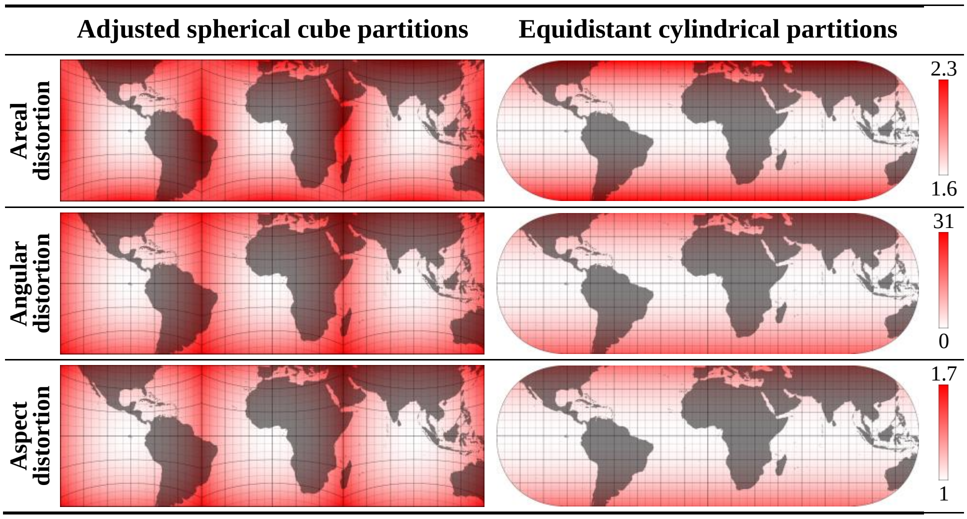


Fig. 5 Graphical comparison of the distortions of the two proposed projections.

A statistical distribution of the distortion values can be seen in the histograms in Fig. 6. The distributions were estimated based on over 50 million samples per partition. It can be seen that the cylindrical equidistant projection not only has lower values for all three types of distortion, but also a better distribution of these values.

From a visualization point of view, the problem with the two-partition cylindrical equidistant projection is the overlap of the partitions. Namely, the algorithms for representing the terrain on a planetary scale must very precisely truncate the vertices of one partition that merge into another, requiring additional techniques to close pinholes that occur at the boundary of the junction. Second, precise merging of partitions requires that there be overlapping cells, which requires capturing data that should not be displayed. And third, addressing partitions must be done at the level of the entire bounding rectangle, including overlapping areas, even if the cells within them do not contain data. All this complicates the application compared to spherical cubes.

5. Further Improvements

In all the projections defined before, the sphere was mapped onto the plane and vice versa. However, the planet Earth is not an ideal sphere. Currently, the most commonly used model is the World Geodetic System 1984 (WGS84) reference ellipsoid [29]. All global navigation and most global data sets are based on this model. Therefore, it is necessary to include the mapping of the ellipsoid to the sphere in the forward and inverse transformations of the previously defined projections (Fig. 7) to allow access to this data.

The transformation from ellipsoid to sphere is not a unique process and depends on the property we want to preserve. The most common method is to convert the geodetic latitude to one of the "auxiliary" latitudes. Auxiliary latitudes were systematically described and all formulas derived by O. Adams [30], but they gained greater popularity much later through the working manual of Snyder [28]. In [18], it was shown that in the adjusted spherical cube projection the smallest angular distortion is obtained by applying the geocentric latitude and the smallest areal distortion is obtained by applying the approximated authalic latitude. The properties are 0.8% to 1.3% better than when the geodesic latitude is used, i.e., when the ellipsoid is not converted into a sphere. Considering the minimal deviation of the WGS84 reference ellipsoid from the ideal sphere, the advantages of the carefully selected ellipsoid to sphere transformation in mapping are not negligible.

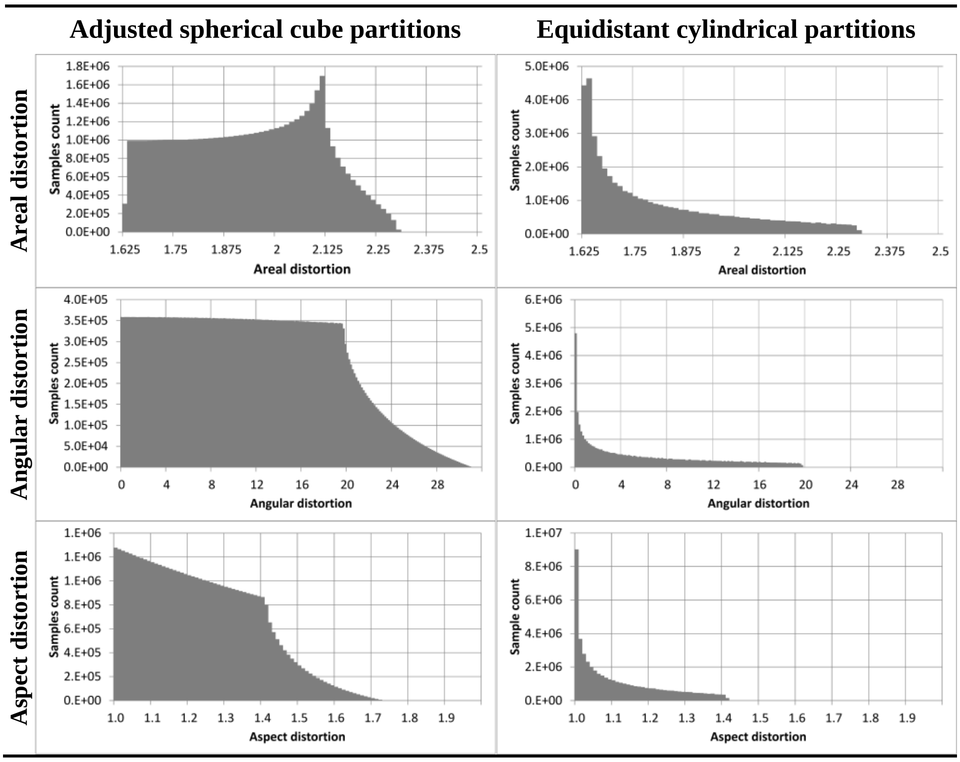


Fig. 6 The distribution of distortion values for the two proposed projections.

Another improvement is to reduce the distortion of the landmass. This can be done by orienting the base cube or the orthogonal cylinders, depending on which projection is considered. The basic orientation shown in Section 3 is defined to maximally simplify the transformations and to make the partitions symmetric with respect to the equator, the prime meridian, and the antimeridian. By introducing rotations, the mapping of the ellipsoid to the plane and vice versa becomes more complex, so that we can now speak of transformation pipelines instead of simple forward and inverse transformations (Fig. 7).

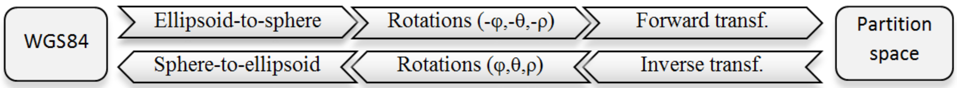


Fig. 7 Transformation pipelines for converting WGS84 coordinates into a partition space (forward transformation) and back to WGS84 (inverse transformation).

The rotation angles depend on the projection chosen, but also on the type of the distortion we want to minimize. For the case of an adjusted spherical cube, with the constraint that the angles are integer and we want to minimize the angular distortion, the optimal rotation angles of the base cube are: $\varphi = 17°$ (in the longitudinal direction), $\theta = -10°$ (in the latitudinal direction), and $\rho = 32°$ (about the axis perpendicular to the axes of the two previous rotations), as shown in [18]. The choice to optimize the angular distortion also results in a more faithful representation of the shape of the continents. Moreover, the arrangement of the continents after the rotations is such that their overlap with the edges of the cube is reduced. Fig. 8 shows the appearance of the partitions after the orientation of the base cube according to the previously set parameters.

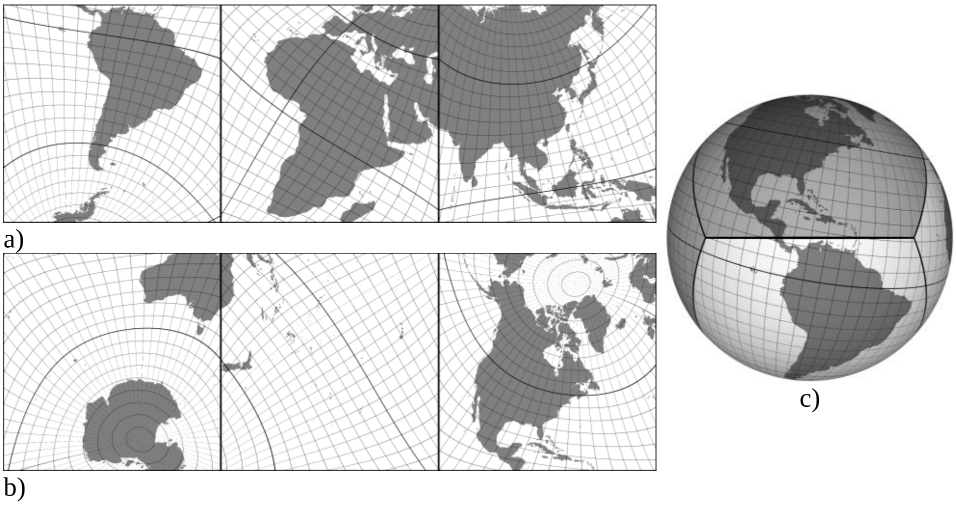


Fig. 8 Partitioning based on an adjusted spherical cube, optimally rotated to minimize angular distortion of the continental plates: a) partition P0, b) partition P1, and c) partitions applied to the sphere, with partition P1 shown in a darker shade.

6. Conclusion

The problem of organizing, storing, and efficiently sharing enormous amounts of geospatial data has not yet been satisfactorily solved. DGGSs provide a framework for unified access to global data, but typically divide the data into several separate partitions according to the number of sides of the polyhedron on which they are based. In this paper, the two methods to divide the entire world into only two disjoint, convex and orthogonal partitions are proposed. This is the minimum number if we want to avoid singularities or interrupted partitions.

The effects of distortion were compared based on their maximum and mean values, spatial distribution, and statistical distribution of values. It is found that the partitioning based on two orthogonal cylindrical equidistant projections has lower distortions than the adjusted spherical cube. However, the problem with this approach is the presence of overlapping areas that can complicate the combination and visualization of the data.

Distortions can be further reduced by rotating the projection planes, that is, rotating the sides of the cube or cylinder so that the regions of interest are as far as possible from the edges of the projection planes. The most common goal is to reduce the distortion of land masses, since they have much greater cartographic significance than large areas of water. Distortion can also be affected by the choice of an ellipsoid-to-sphere mapping.

The results of the analysis of the proposed projections with two partitions show that both approaches are very applicable in the organization of global data. From the point of view of less distortion, the approach based on orthogonal cylindrical projections gives much better results, but the approach via spherical cubes forms completely disjoint and rectangular partitions, which makes the application much easier.

Given the large number of projections based on spherical cubes, further research will be directed toward finding a projection that has better distortion properties while retaining all the advantages of rectangular partitions. The second research direction will focus on solving problems related to three-dimensional visualization of data addressed by the proposed two-partition DGGSs, and adapting existing algorithms for planetary-scale terrain rendering to take advantage of such organization of data.

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