FACTA UNIVERSITATIS
Series: **Economics and Organization** Vol. 17, N° 1, 2020, pp. 17 - 26
https://doi.org/10.22190/FUEO191016002S

Original Scientific Paper

EFFECTS OF APPLYING DIFFERENT RISK MEASURES ON THE OPTIMAL PORTFOLIO SELECTION: THE CASE OF THE BELGRADE STOCK EXCHANGE

UDC 336.76(497.11)

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Abstract. *Despite its wide use in practice, Modern Portfolio Theory and Markowitz's approach to optimization, which is based on quadratic programming and the first two moments of the probability distribution of returns as major parameters, was faced with criticism. Therefore, standard Mean-Variance approach had been modified by applying more appropriate risk measures in optimization algorithm. The aim of this paper is to indicate efficiency of these models as well as justification of their usage in managing stocks portfolio on the Belgrade Stock Exchange.*

Key words: *portfolio optimization, alternative risk measures, Belgrade Stock Exchange*

JEL Classification: G32, G11, O16.

1. INTRODUCTION

Portfolio optimization represents one of the most important aspects of making investment decisions, based on the possibility of a successful assessment of the relationship between return and risk. For decades, in the selection of financial assets and the formulation of an optimal portfolio strategy investors have been dominantly using the Mean-Variance (MV) model (Markowitz, 1952), which forms the basis of Modern Portfolio Theory. Simplified starting assumptions regarding investors' preferences and return distribution, make it possible to form an optimization model, which, according to Markowitz's approach, is based only on the expected return and risk. However, none of the conditions of this model are sustainable in reality. Normal probability distribution is not an adequate model of empirical distribution of returns, characterized by asymmetry and heavy tails, so

Received October 16, 2019 / Accepted January 21, 2020
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that in situations where there is a significant deviation from the expected value, the measures of central tendency and dispersion are no longer sufficient parameters to describe the distribution of returns or to optimize the portfolio.

In the mid-1990s, the MV model began to be critically observed and innovated to respond to investors' demands. Most of the criticisms relate to the parameters of the return distribution - the mean, as a measure of expected return, and the standard deviation, as a measure of risk, and their role in portfolio optimization. Due to the fact that the normal distribution does not have empirical significance in modeling financial series from contemporary markets, in many cases it has been observed that the form of distribution significantly influences portfolio performance as well as the criteria for selecting financial assets (Lamm, 2003). Technological development, especially the development of digital technologies, has transformed many industries and enlarged the number of available investment options. In cases where a large number of financial instruments are being considered, high dimensionality can prevent the precise evaluation of a complex correlation structure and risk. Under the radically changed risk-return trade-off, investors' risk aversion and investment conditions, the application of Markowitz's model in the optimization can result in the allocation of financial resources to suboptimal investment alternatives.

One of the possible ways to improve the MV model is to incorporate various risk measures into the portfolio optimization model (Konno et al., 2002; Chang et al., 2009). New technologies allow designing improved computational frameworks and decision algorithms for portfolio optimization. Modern research shows that optimization models that involve extreme risks and encompass the entire return distribution would provide more adequate solutions to the problem, especially in emerging markets (Stevenson, 2001; Gilmore et al., 2005). Therefore, the aim of this paper is to point out the shortcomings of the MV model in the optimization of the securities portfolio of the Belgrade Stock Exchange by comparing the performance of the optimal portfolios obtained by using different risk measures. The measures used to evaluate portfolio risk in this study are the measures most commonly used in research, as described in the second part of this paper. The method of their calculation and implementation in the optimization algorithms is explained in the third part of the paper, while a comparative analysis of the results of portfolio optimization by applying different risk measures is presented in the fourth part. Concluding remarks and further directions for research are presented in the fifth part of the paper.

2. LITERATURE REVIEW

The effects of portfolio diversification on emerging markets can be highlighted as the most significant feature of financial globalization (Mensi et al., 2017). Emerging and frontier markets are usually considered separately from developed ones because of their specificities - depth and width, legal and institutional infrastructure. These new capital markets of transition countries in Europe, South America, Asia, the Middle East and Africa offer investors unusually high returns, comparing to developed markets, but also higher level of volatility (Bekaert & Harvey, 2017). The peculiarities of the functioning of these markets make it impossible to establish a strong correlation with other world markets, preserving them from the impact of global trends (Berger et al., 2011). Therefore, after the financial crisis, the economic importance of emerging markets significantly increased and the share of emerging market companies' stocks in the MSCI All Country World Index reached 14% (Melas, 2019). Nevertheless, investments in the frontier and emerging markets are fraught with numerous risks, which cannot be adequately measured by the application of classic risk assessment models. Moreover, in post-crisis period modeling of market risk is heavily re-examined, due to the fact that existing risk management models and practices have not provided a reliable framework for measuring and managing risk (Ball, 2009; Hansen, 2013).

Despite numerous empirical studies conducted from the 1950s till now, there is no fundamental theory that offers a generally accepted statistical model based on some theoretical return distribution that takes into account all the observed characteristics of financial time series, such as: volatility clustering, autoregression and return asymmetry, and heavy tails and their properties (Stojanov et al., 2011). Therefore, it is necessary that the methodology for quantifying risk is based on a model that captures the stated properties of financial time series and, accordingly, an appropriate measure of risk. Research shows that investors prefer securities whose distribution is positively asymmetric if the expected return and standard deviation are constant for all considered securities (Guidolin & Timmermann, 2008; Xiong & Idzorek, 2011). This characteristic of investors is particularly evident when referring to risk managers in mutual funds and insurance companies (Zuluaga & Cox, 2010). In accordance with investors' preferences, it is necessary to apply an appropriate risk assessment model, since an inadequate model for risk assessment in portfolio analysis can lead to the application of wrong diversification and risk hedging strategies (Lee, 2011). The appearance of new financial instruments, the different types of investors and the circumstances under which investment is made in the capital markets, has conditioned the consideration of an alternative risk measures to the variance (Hoe et al., 2010; Zhang and Guo, 2018). Implementation of these measures enabled simplification of portfolio optimization algorithms, since the optimal portfolio can be determined by using linear programming.

In order to overcome the disadvantages of variance as a measure of risk, Konno and Yamazaki (1991) proposed a new portfolio optimization model that captures risk with the measure of mean absolute deviation. This model can be used to select assets from a large set of available investment alternatives and, unlike the Markowitz's model, it does not require calculation of the covariance matrix. However, despite the fact that variance and mean absolute deviation are adequate risk measures in a large number of cases (Byrne and Lee, 2004), investors' preferences towards positive deviations from expected returns cannot be incorporated in these models (Lamm, 2003).

By applying downside risk measures in the securities selection model, especially when investing in emerging markets, these deficiencies can be eliminated (Stevenson, 2001; Estrada, 2006). One of the commonly used downside risk measures, introduced by Markowitz (1959) into portfolio analysis, is semivariance. The semivariance is often considered as more appropriate investors' risk measure than the variance. However, computational issues have affected academics and practitioners to use preferably MV approach (Estrada, 2008). The other widely used downside risk measure is the semi-absolute deviation proposed by Speranza (1993). Although, it can be shown that, under certain assumptions, the semi-absolute deviation is equal to one half of the absolute deviation, as well as equivalent to the variance (Chiodi et al., 2003), from a computational point of view, implementation of the semi-absolute deviation in the portfolio optimization model makes its evaluation simplified (Liu and Qin, 2012).

In recent years, due to finance and insurance regulation, Value at Risk (VaR) and Conditional Value at Risk (CVaR) have been used in financial and risk management. The problem of the choice between VaR and CVaR in portfolio optimization is affected by the differences in mathematical properties of these risk measures, stability of statistical estimation and complexity of optimization procedures (Sarykalin et al., 2008). Practical applications indicate that the minimization of CVaR usually leads to near optimal solutions in VaR terms, so it can be concluded that portfolios characterized by low CVaR should have low VaR as well. However, CVaR as a coherent measure of risk can be used in solving optimization problems of large portfolios and a large number of scenarios with linear programming (Krokhmal et al., 2002).

The previous researches on the portfolio optimization conducted on the Serbian and neighboring capital markets, in which Markowitz's model in portfolio selection was applied, identified numerous limiting factors, such as market illiquidity, low turnovers, high oscillations of returns, as well as positive correlation among the returns (Zaimović and Delalić, 2010; Kočović et al., 2015; Radović et al., 2018). Therefore, the main contribution of this paper is to investigate the effects of different risk measures implementation on optimal portfolio selection model on the Belgrade Stock Exchange.

3. Data and Methodological Framework

In order to form a portfolio that will provide investors on the Belgrade Stock Exchange a better performance comparing to the market portfolio, optimization algorithms are applied to a group of stocks – constituents of the market indexes BELEX15 and BELEX*line* baskets. The value of BELEX*line* and BELEX15 indexes are determined by the prices of the most liquid stocks which are continuously being traded on the regulated market of the Belgrade Stock Exchange. An adequate evaluation of the basic characteristics of financial time series requires certain duration of the series, which reduces the number of available stocks to 29. Taking into consideration the required conditions which the companies issuing the stocks need to meet in order to be included in these lists, the starting point in this study is the assumption that they are the most liquid securities on the Belgrade Stock Exchange.

The data used in this study were taken from the website of the Belgrade Stock Exchange (www.belex.rs). The series of values of the indexes, as well as of individual stocks, include records from January 1st, 2008 to December 31st, 2018, which in total includes 2775 trading days. Optimization algorithms are executed using 29 assets observation, each of which with the 2524 in-sample trading data and tested in out-sample period of 250 trading data in the last year. In the models we use the logarithmic returns of the selected stocks' values. The distributions of such returns deviate significantly from the normal distribution, while the stocks' returns have been positively correlated (Stanković et al., 2015).

If it is assumed that investor considers investing financial means in i, $i = 1, 2, 3, \dots, N$ different securities, which returns in the time period t, $t = 1, 2, 3, \dots T$ are $R_i(t)$, the expected return of the investment portfolio $E(R_P)$ can be determined using the following formula:

$$E(R_p) = \sum_{i=1}^{N} w_i E(R_i) \tag{1}$$

where $E(R_i)$ represents the expected return of i security measured by the mean value in the observed period ($\bar{R}$), while w_i is a share of the i security in the investment portfolio.

To determine the optimal portfolio, we use different criteria. If we assume that investors are risk-averse, they will select securities in such a manner to provide the maximum return for an acceptable level of risk or to minimize risk for a given level of return. Portfolio risk assessment includes evaluation of correlation between securities in the portfolio, which determine the level of diversification. The maximum reduction of risk is achieved by a combination of securities, whose returns are perfectly negatively correlated. The degree and magnitude of changes in securities' return may be measured using covariance between the securities' returns. The risk of the portfolio, according to MV model, is measured using variance, and can be determined as follows:

$$\sigma_P^2 = \sum_{i=1}^{N} w_i^2 \sigma_i^2 + 2 \sum_{\substack{i,j=1 \\ i \neq j}}^{N} w_i w_j \, cov(R_i, R_j) \tag{2}$$

where σ_i^2 presents expected risk on investment in i security measured as a dispersion of the returns around the expected returns $E(R_i)$, while $cov(R_i, R_j)$ is covariance between the securities' returns.

As an alternative measure to variance, absolute deviation (AD) is used. The main advantage of applying AD in portfolio optimization is simplicity of its computing, because the assessment of risk in this manner can be made without the need to calculate covariance matrix. Moreover, when the securities are highly correlated, such as the securities in the observed sample, it is unnecessary to determine the covariance matrix. Assuming that there are T different scenarios, i.e. T observations, AD can be calculated as follows:

$$AD = \frac{1}{T} \sum_{t=1}^{T} \left| \sum_{i=1}^{N} (R_i - m(R_i)) w_i \right| \tag{3}$$

where $m(R_i)$ represents the measurement of central tendency and in this study, it is the mean of the return of the observed sample.

Since investors are more sensitive to the returns' decline than to the potential returns' increase, they will follow "safety-first" rule, meaning that they will select assets with the lowest probability of loss below a certain or disaster level. Downside risk measures enable investors to assess the risk below average or expected return, thereby not assuming that assets' return distribution follow the normal probability distribution. Lower semivariance (LSV) is a statistical measure, which represents the squared deviation of the values lower than the mean, and can be calculated in the following manner (Boasson et al., 2011):

$$LSV = \frac{1}{T} \sum_{t=1}^{T} \sum_{i=1}^{N} (m(R_i) w_i - R_i w_i)^2, \; m(R_i) > R_i \tag{4}$$

Accordingly, lower semi absolute deviation (LSAD) is a statistical measure, which shows the absolute deviations of the values from the mean, and can be calculated using the following formula:

$$LSAD = \frac{1}{T} \sum_{t=1}^{T} \left| \sum_{i=1}^{N} (m(R_i) - R_i) w_i \right|, \; m(R_i) > R_i \tag{5}$$

Computational setting of Value at Risk (VaR) as a risk measure was presented within Modern Portfolio Theory (Holton, 2002). Today VaR is a standardized risk measure, which is applied in the risk assessment and capital adequacy of both financial and nonfinancial institutions. The VaR model is determined by two main parameters: confidence interval α, $\alpha \in (0,1)$ and holding period h within the VaR is assessed. For the given confidence interval α and holding period h, VaR_α can be defined as the least number l, such that in the future period it will not be greater than expected loss L with probability 1-α. According to the probability theory, VaR is a lower quantile of return distribution (McNeil et al., 2002), that in the case of a portfolio can be summarized as follows:

$$VaR_{P,h} = -Q_{1-\alpha}(R_{P_{h-1}}, R_{P_{h-2}}, \dots, R_{P_1}) \tag{6}$$

However, VaR as a risk measure is not subadditive. Moreover, VaR does not provide information regarding the value of the loss that exceeds the value of VaR. Implementation of this measure in optimization problem and risk management, contrary to the expectations, does not contribute to risk reduction and investors' utility maximization rather recommending the positions with higher risk exposure, due to which investors suffer grater losses under the terms of significant volatility (Basak & Shapiro, 2001, Yamai & Yoshiba, 2002). The drawbacks of VaR can be mitigated by Conditional Value at Risk (CVaR) which is a conditional expectation that gives the expected loss beyond the VaR. Correspondingly to VaR model, computational setting of CVaR model is determined by two main parameters: confidence interval α, $\alpha \in (0,1)$ and holding period h within the risk is assessed. For the given confidence interval α and holding period h, CVaR_α can be defined as a mean value of α-quantile of empirical distribution of L, and, according to the formula (6), it can be quantified in the following manner:

$$CVaR_\alpha(R_P) = E(R_P | R_P \leq VaR_\alpha(R_P)) \tag{7}$$

Portfolio optimization model under the assumption that investors tend to minimize the risk of the portfolio (L_P) is defined using the above described measures of risk and in general can be summarized as follows:

$$minimize\ L_P \tag{8}$$

subject to:

$$\textstyle\sum_{i=1}^{N} w_i E(R_i) = m(R_P) \tag{9}$$

$$\textstyle\sum_{i=1}^{N} w_i = 1, 0 \leq w_i \leq 0.2 \tag{10}$$

The set constraints should enable comparison with market portfolio approximated by the market indexes BELEX15 and BELEX*line*. Detailed explanation of applied optimization algorithms can be found, for example, in the studies of Byrne and Lee (2004), Krokhmal et al. (2002) and Liu and Qin (2012).

4. RESULTS AND DISCUSSION

Optimizing allocation of financial means on the Belgrade Stock Exchange using various measures of risk presented in this study results in five different investment portfolios, which achieve better performance compared to market portfolios in the out-sample period (Table 1). The effects of these investments are measured by total return, total risk and the Sharpe ratio. Although none of the obtained portfolios can be considered effective in terms of Sharpe ratio, all have higher Sharpe ratio than BELEX15 and BELEX*line* index, with significantly higher returns and almost identical risk, but higher risk comparing to market portfolios, at a level of approximately 15%.

Implementation of the standard MV approach in portfolio optimization lead to portfolio that consists of 27 stocks and it enables investors to achieve 3.30% return within holding period of 250 days. Investors should invest in a similar number of stocks according to MSV model (26) and MVaR model (23) in order to realize 2% higher return comparing to market portfolio. However, transaction costs and Belgrade Stock

Market illiquidity may reduce the possibilities to invest in stocks according to these models of optimization. On the other hand, the use of AD and SAD measures of risk in the portfolio optimization model resulted in portfolios, whose structures make smaller number of shares (5) and significantly better performances in comparison to other alternatives. The optimal portfolios determined by using MSAD and MAD models realize identical Sharpe ratio (0.81124) and return of 12.87%.

Table 1 Optimal portfolios performances

Optimizaton method	No. of stocks	Sharpe ratio	Holding period return	Holding period variance
Mean – Variance (MV)	27	0.21258	3.30%	15.54%
Mean – Semi Variance (MSV)	26	0.13973	2.19%	15.64%
Mean – Absolute Deviation (MAD)	5	0.81124	12.87%	15.87%
Mean – Semi Absolute Deviation (MSAD)	5	0.81124	12.87%	15.87%
Mean – Value at Risk (MVaR)	23	0.17024	2.68%	15.71%
BELEX15	11	0.02759	0.25%	9.00%
BELEXline	21	-0.65957	-4.50%	6.83%

Source: Authors' calculation

In order to compare the structure of obtained portfolios we estimate similarity index. Portfolio similarity index, as a measure of similarity of portfolios' structures, includes both similarity in terms of portfolios' compositions, as well as similarity between the weights attached to common assets in two portfolios. The similarity in portfolio composition is measured by the portfolio overlap index that is calculated as the share of common stocks in two portfolios in the average number of stocks in observed portfolios. However, portfolios even of exactly the same composition may differ in the weights attached to common assets. Therefore, portfolio weight index is assessed by summing the minimum weight attached to each asset that overlaps two portfolios. Following Byrne and Lee (2004) the portfolio similarity index is calculated by multiplying the portfolio overlap index by portfolio weight index and the results of portfolio similarity analysis are presented in Table 2.

Table 2 Portfolios similarity indexes

	MV	MSV	MAD	MSAD	MVaR
MV	100.00%				
MSV	92.68%	100.00%			
MAD	17.72%	18.65%	100.00%		
MSAD	17.72%	18.65%	99.22%	100.00%	
MVaR	84.68%	87.81%	20.77%	20.77%	100.00%

Source: Authors' calculation

Comparing the structures of optimal portfolios achieved by using different risk measures in optimization algorithms, it can be concluded that the structures are significantly different. Portfolios obtained by implementing MSV model is the most similar to MV portfolio with 92.68% of similarity. The structure of MVaR optimal portfolio is different in 15.32% from MV portfolio. On the other hand, MAD as well as MSAD optimal portfolios show the least similarity with MV portfolio with a value of similarity index of 17.72%. Considering the fact that mean values of the selected stocks' returns are mostly negative with insignificant deviations from zero, the structures of the MAD and MSAD optimal portfolios are almost identical. However, these portfolios provide the best performances and, despite the investment conditions on the Serbian capital market, are realizable to investors.

5. Conclusion

Modern portfolio theory was the common framework for the development of many theories and concepts in finance that are still widely used, but also the subject of constant criticism. The assumption that most certainly brings into question the appearance of this theory is the probability distribution of returns on financial assets, and thus the basic parameters of the optimization model. Financial time series show numerous anomalies in comparison with the normal distribution, which require consideration of additional parameters necessary for the adequate assessment of return and risk. The emergence of new financial instruments, as well as different types of investors, have made it necessary to consider the effects of extreme risks in the portfolio optimization models. Considering the statistical parameters of return distributions, investors will prefer to allocate their funds in the financial assets, whose return distribution is positively asymmetrical, while they will be averse toward the financial assets, whose return distribution is long tailed. In such terms, the variance as a commonly used measure of risk in portfolio optimization models should be replaced by the downside risk measures. The need to incorporate alternative risk measures in portfolio optimization algorithms is even more pronounced on the emerging capital markets, such as the Belgrade Stock Exchange, considering the statistical characteristics of the financial time series.

The contribution of this research results is substantial, taking into consideration that portfolio specialists working at the Serbian capital market, as an emerging market, have not widely incorporated optimization models into their common practice. One possible explanation of such a situation is that extreme events and economic disturbances, such as global financial crisis from 2008, change financial environment so that past data have low importance when predicting future. However, global changes in living and business environment are posing a complex set of emerging risks, so the future research will be aimed at sustainable portfolio management.

Acknowledgement: *The paper is a part of the research done with the support of the Erasmus+ Programme of the European Union within the project no. 611831-EPP-1-2019-1-RS-EPPJMO-MODULE.*

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EFEKTI PRIMENE RAZLIČITIH MERA RIZIKA NA IZBOR OPTIMALNOG PORTFOLIJA: SLUČAJ BEOGRADSKE BERZE

Savremena portfolio teorija i Markovicev pristup optimizaciji, koji se zasniva na kvadratnom programiranju i čiji su osnovni parametri prva dva momenta raspodele verovatnoće prinosa, uprkos širokoj primeni u praksi, suočila se sa brojnim kritikama. Stoga su razvijeni modeli, koji na adekvatniji način inkorporiraju rizik u model optimizacije. Cilj ovog rada je da ukaže na efikasnost ovako formiranih modela optimizacije i opravdanost njihove primene u upravljanju portfoliom hartija od vrednosti na Beogradskoj berzi.

Ključne reči: *optimizacija portfolija, alternativne mere rizika, Beogradska berza.*