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ON UNIFICATION OF RARELY CONTINUOUS FUNCTIONS *

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Abstract. In 1979, V. Popa [23] first introduced the concept of rare continuity. In this paper, we introduce a new class of functions, termed rarely μ-continuous functions, which unifies different weak forms of rarely continuous functions and investigate some of its properties.

Keywords: μ-open set; rare set; rarely μ-continuous function

1. Introduction

The notion of rare continuity was first introduced by V. Popa in [23] which was further studied by Long and Herrington [18] and Jafari [13, 14]. Certain weak forms of rarely continuous functions, for example, rare quasi-continuity, rare α-continuity, rare δs-continuity, rare pre-continuity, rare δ-continuity, rare g-continuity have been introduced and studied by Popa and Noiri [24], Jafari [16], Caldas, Jafari, Moshokoa and Noiri [4], Jafari [15], Caldas and Jafari [3], Caldas and Jafari [2] respectively.

The notion of generalized topological space was first introduced by A. Császár. After that a large number of papers have been devoted for the investigation of different properties of such spaces. We recall some notions defined in [5]. Let X be a non-empty set, $expX$ denotes the power set of X. We call a class $\mu \subsetneqq expX$ a generalized topology [5], (GT for short) if $\emptyset \in \mu$ and union of elements of μ belongs to μ. A set X, with a GT μ on it is said to be a generalized topological space (GTS for short) and is denoted by (X, μ).

For a GTS (X, μ), the elements of μ are called μ-open sets and the complement of μ-open sets are called μ-closed sets. For $A \subsetneqq X$, we denote by $c_\mu(A)$ the intersection of all μ-closed sets containing A, i.e., the smallest μ-closed set containing A; and by $i_\mu(A)$ the union of all μ-open sets contained in A, i.e., the largest μ-open set contained in A (see [5, 6]).

It is easy to observe that i_μ and c_μ are idempotent and monotonic, where γ : $expX \to expX$ is said to be idempotent iff $A \subsetneqq B \subsetneqq X$ implies that $\gamma(\gamma(A)) = \gamma(A)$ and

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monotonic iff $\gamma(A) \subsetneqq \gamma(B)$. It is also well known from [6, 7] that if μ is a GT on X and $A \subsetneqq X$, $x \in X$, then $x \in c_\mu(A)$ iff $x \in M \in \mu \Rightarrow M \cap A \neq \varnothing$ and $c_\mu(X \setminus A) = X \setminus i_\mu(A)$.

The purpose of this paper is to introduce the concept of rare μ-continuity which unifies the existing class of weak forms of rarely continuous functions by a particular choice of GT. We have also investigated several properties of rarely μ-continuous functions. In this sequel the notion of I.μ-continuity has been introduced which is then shown to be weaker than (μ, σ)-continuity and stronger than rarely μ-continuous function. It can also be observed that the results obtained in some other papers can be obtained from our results for a suitably chosen GT.

Hereafter, throughout the paper we shall use (X, μ) to refer to a generalized topological space and (X, τ), (Y, σ) to be topological spaces unless otherwise stated.

2. Preliminaries

Let (X, τ) be a topological space. The δ-closure [28] of a subset A of (X, τ) is denoted by $cl_\delta(A)$ and is defined by

$$\{x \in X : A \cap U \neq \varnothing \text{ for all regular open sets } U \text{ containing } x\},$$

where a subset A is called regular open [27] if $A = int(cl(A))$. The set A is called δ-closed if $cl_\delta(A) = A$. The complement of a δ-closed set is called δ-open. It is known from [28] that the family of all δ-open sets form a topology on X which is smaller than the original topology τ. A subset A of X is called semi-open [17] (resp. preopen [20], α-open [21], δ-semiopen [22]) if $A \subsetneqq cl(int(A))$ (resp. $A \subsetneqq int(cl(A))$, $A \subsetneqq int(cl(int(A)))$, $A \subseteq cl(int_\delta(A))$). The complement of a semi-open (resp. preopen, α-open, δ-semiopen) set is called a semi-closed (resp. preclosed, α-closed, δ-semiclosed) set. The collection of all semi-open (resp. preopen, α-open, δ-open, δ-semiopen) sets in a topological space is denoted by $SO(X)$ (resp. $PO(X)$, $\alpha O(X)$, $\delta O(X)$, $\delta SO(X)$). We note that each of these collections forms a GT on X. A subset A of a space X is called a $\bigwedge$-set [19] if it is equal to its kernel i.e., intersection of all open superset of A. A is called a λ-closed set [1] if $A = U \cap V$ where U is a $\bigwedge$-set and V is a closed set. The complement of a λ-closed set is called a λ-open set. The family of all λ-open sets of a topological space is denoted by $\lambda O(X)$. A subset A of X is called rare if $int(A) = \varnothing$. We shall use the symbol $O(f(x), Y)$ to refer to the collection of all open sets in the topological space Y containing $f(x)$.

Definition 2.1. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be rarely continuous [23] if for each x in X and $G \in O(Y, f(x))$ there exists a rare set R_G with $G \cap cl(R_G) = \varnothing$, and an open set U in X containing x such that $f(U) \subsetneqq G \cup R_G$.

Definition 2.2. A function $f : (X, \mu) \to (Y, \sigma)$ is said to be (μ, σ)-continuous [25] if for each $x \in X$ and each $G \in O(Y, f(x))$, there exists a μ-open set U containing x in X such that $f(U) \subsetneqq G$.

3. Rarely μ-continuous functions

Definition 3.1. A function $f : (X, \mu) \to (Y, \sigma)$ is said to be rarely μ-continuous if for each x in X and $G \in O(Y, f(x))$ there exists a rare set R_G with $G \cap cl(R_G) = \emptyset$, and a μ-open set U in X containing x such that $f(U) \subseteqq G \cup R_G$.

Remark 3.1. Let μ and λ be two GT's on the set X such that $\mu \subseteqq \lambda$. If $f : (X, \mu) \to (Y, \sigma)$ is rarely μ-continuous then $f : (X, \lambda) \to (Y, \sigma)$ is rarely λ-continuous.

Example 3.1. Let

$$X = \{a, b, c\},\ \sigma = \{\emptyset, \{a\}, \{a, b\}, X\},\ \mu = \{\emptyset, \{a, b\}, \{a, c\}, X\}.$$

Then μ is a GT on the topological space (X, σ). It can be easily verified that the identity function $f : (X, \mu) \to (X, \sigma)$ is rarely μ-continuous.

Theorem 3.1. *For a function $f : (X, \mu) \to (Y, \sigma)$ the followings are equivalent:*

(i) f is rarely μ-continuous at x.

(ii) For each set $G \in O(Y, f(x))$, there exists a μ-open set U in X containing x such that $int[f(U) \cap (Y \setminus G)] = \emptyset$.

(iii) For each $G \in O(Y, f(x))$, there exists a μ-open set U in X containing x such that $int[f(U)] \subseteqq cl(G)$.

(iv) For each $G \in O(Y, f(x))$, there exists a rare set R_G with $G \cap cl(R_G) = \emptyset$ such that $x \in i_\mu(f^{-1}(G \cup R_G))$.

(v) For each $G \in O(Y, f(x))$, there exists a rare set R_G with $cl(G) \cap R_G = \emptyset$ such that $x \in i_\mu(f^{-1}(cl(G) \cup R_G))$.

(vi) For each $G \in RO(Y, f(x))$, there exists a rare set R_G with $G \cap cl(R_G) = \emptyset$ such that $x \in i_\mu(f^{-1}(G \cup R_G))$.

Proof. (i) $\Rightarrow$ (ii) : Let $G \in O(Y, f(x))$. Then $f(x) \in G \subseteqq int(cl(G))$ and $int(cl(G)) \in O(Y, f(x))$. Thus by (i) there exists a rare set R_G with $int(cl(G)) \cap cl(R_G) = \emptyset$ and a μ-open set U in X containing x such that $f(U) \subseteqq int(cl(G)) \cup R_G$. We have

$$\begin{aligned} int[f(U) \cap (Y \setminus G)] &= int[f(U)] \cap int(Y \setminus G) \\ &\subseteqq int[int(cl(G)) \cup R_G] \cap (Y \setminus cl(G)) \\ &\subseteqq (cl(G) \cup int(R_G)) \cap (Y \setminus cl(G)) \qquad = \emptyset. \end{aligned}$$

(ii) $\Rightarrow$ (iii) : It is straightforward.

(iii) $\Rightarrow$ (i) : Let $G \in O(Y, f(x))$. Then by (iii), there exists a μ-open set U in X containing x such that $int[f(U)] \subseteqq cl(G)$. We have

$$\begin{aligned} f(U) &= [f(U) \setminus int(f(U))] \cup int(f(U)) \subseteqq [f(U) \setminus int(f(U))] \cup cl(G) \\ &= [f(U) \setminus int(f(U))] \cup G \cup (cl(G) \setminus G) \\ &= [(f(U) \setminus int(f(U))) \cap (Y \setminus G)] \cup G \cup (cl(G) \setminus G). \end{aligned}$$

Set $R^* = [f(U) \setminus int(f(U))] \cap (Y \setminus G)$ and $R^{**} = (cl(G) \setminus G)$. Then R^* and R^{**} are rare sets. More $R_G = R^* \cup R^{**}$ is a rare set such that $cl(R_G) \cap G = \varnothing$ and $f(U) \subseteqq G \cup R_G$. This shows that f is rarely μ-continuous.

(i) $\Rightarrow$ (iv) : Suppose that $G \in O(Y, f(x))$. Then there exists a rare set R_G with $G \cap cl(R_G) = \varnothing$ and a μ-open set U in X containing x, such that $f(U) \subseteqq G \cup R_G$. It follows that $x \in U \subseteqq f^{-1}(G \cup R_G)$. This implies that $x \in i_\mu(f^{-1}(G \cup R_G))$.

(iv) $\Rightarrow$ (v): Suppose that $G \in O(Y, f(x))$. Then there exists a rare set R_G with $G \cap cl(R_G) = \varnothing$ such that $x \in i_\mu(f^{-1}(G \cup R_G))$. Since $G \cap cl(R_G) = \varnothing$, $R_G \subseteqq Y \setminus G$, where $Y \setminus G = (Y \setminus cl(G)) \cup (cl(G) \setminus G)$. Now, we have

$$R_G \subseteqq (R_G \cup (Y \setminus cl(G)) \cup (cl(G) \setminus G)).$$

Set $R^* = R_G \cap (Y \setminus cl(G))$. It follows that R^* is a rare set with $cl(G) \cap R^* = \varnothing$. Therefore

$$x \in i_\mu[f^{-1}(G \cup R_G)] \subseteqq i_\mu[f^{-1}(cl(G) \cup R^*)].$$

(v) $\Rightarrow$ (vi) : Assume that $G \in RO(Y, f(x))$. Then there exists a rare set R_G with $cl(G) \cap R_G = \varnothing$ such that $x \in i_\mu[f^{-1}(cl(G) \cup R_G)]$. Set $R^* = R_G \cup (cl(G) \setminus G)$. It follows that R^* is a rare set and $G \cap cl(R^*) = \varnothing$. Hence

$$x \in i_\mu[f^{-1}(cl(G) \cup R_G)] = i_\mu[f^{-1}(G \cup (cl(G) \setminus G) \cup R_G)] = i_\mu[f^{-1}(G \cup R^*)].$$

(vi) $\Rightarrow$ (ii): Let $G \in O(Y, f(x))$. By $f(x) \in G \subseteqq int(cl(G))$ and the fact that $int(cl(G)) \in RO(Y)$, there exists a rare set R_G with $int(cl(G)) \cap cl(R_G) = \varnothing$ such that $x \in i_\mu[f^{-1}(int(cl(G)) \cup R_G)]$. Let $U = i_\mu[f^{-1}(int(cl(G)) \cup R_G)]$. Hence, $x \in U \in \mu$ and, therefore $f(U) \subseteqq int(cl(G)) \cup R_G$. Hence, we conclude

$$int[f(U) \cap (Y \setminus G)] = \varnothing.$$

□

Theorem 3.2. *A function $f : (X, \mu) \to (Y, \sigma)$ is rarely μ-continuous if and only if for each open set $G \subseteqq Y$, there exists a rare set R_G with $G \cap cl(R_G) = \varnothing$ such that $f^{-1}(G) \subseteqq i_\mu[f^{-1}(G \cup R_G)]$.*

Proof. It follows from Theorem 3.1. □

Definition 3.2. A function $f : (X, \mu) \to (Y, \sigma)$ is said to be $I.\mu$-continuous at $x \in X$ if for each set $G \in O(Y, f(x))$, there exists a μ-open set U in X containing x such that $int[f(U)] \subseteqq G$. If f has this property at each point $x \in X$, then we say that f is I.μ-continuous on X.

Remark 3.2. It should be noted that (μ, σ)-continuity implies I.μ-continuity and I.μ-continuity implies rare μ-continuity. But the converses are not true as shown by the following examples.

Example 3.2. Let

$$X = \{a, b, c\},\ \mu = \{\emptyset, \{b\}, \{b, c\}, \{a, c\}, X\},\ \sigma = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}.$$

Then the identity function $f : (X, \mu) \to (X, \sigma)$ is I.μ-continuous but not (μ, σ)-continuous.

Example 3.3. Let

$$X = \{a, b, c\},\ \mu = \{\emptyset, \{a, b\}, \{a, c\}, X\},\ \sigma = \{\emptyset, \{c\}, \{a, c\}, \{b, c\}, X\}.$$

Then μ is a GT on the topological space (X, σ). It can be easily verified that the identity function $f : (X, \mu) \to (X, \sigma)$ is rarely μ-continuous but not I.μ-continuous.

Theorem 3.3. *Let Y be a regular space. Then $f : (X, \mu) \to (Y, \sigma)$ is I.μ-continuous on X if and only if f is rarely μ-continuous on X.*

Proof. We prove only the sufficient condition since as the converse part follows from Remark 3.2. Let f be rarely μ-continuous on X and $x \in X$. Suppose that $f(x) \in G$, where G is an open set in Y. By the regularity of Y, there exists an open set $G_1 \in O(Y, f(x))$ such that $cl(G_1) \subsetneqq G$. Since f is rarely μ-continuous, there exists a μ-open set U in X containing x such that $int[f(U)] \subsetneqq cl(G_1)$. This implies that $int[f(U)] \subsetneqq G$ and therefore f is I.μ-continuous on X. □

Definition 3.3. A function $f : (X, \mu) \to (Y, \sigma)$ is said to be μ-open if the image of a μ-open set is open.

Definition 3.4. A function $f : (X, \mu) \to (Y, \sigma)$ is said to be almost weakly μ-continuous if for each open set G in Y containing $f(x)$ there exists a μ-open set U in X containing x such that $f(U) \subsetneqq cl(G)$.

It also follows that every almost weakly μ-continuous function is rarely μ-continuous. For the converse we have the next theorem.

Theorem 3.4. *If $f : (X, \mu) \to (Y, \sigma)$ be a μ-open rarely μ-continuous function, then f is almost weakly μ-continuous.*

Proof. Suppose that $x \in X$ and $G \in O(Y, f(x))$. Since f is rarely μ-continuous, there exists a μ-open set U in X such that $int(f(U)) \subsetneqq cl(G)$. Since f is μ-open, $f(U)$ is open and hence $f(U) \subsetneqq cl(int(f(U))) \subsetneqq cl(G)$. This shows that f is almost weakly μ-continuous. □

Thus it follows that if $f : (X, \tau) \to (Y, \sigma)$ be a λ-open, rarely λ-continuous function, then f is also a weakly λ-continuous function [12].

Example 3.4. (a) Let

$$X = \{a, b, c\},\ \sigma = \{\emptyset, \{a\}, \{b, c\}, X\},\ \mu = \{\emptyset, \{a, b\}, \{a, c\}, X\}.$$

Then μ is a GT on the topological space (X,σ). It is easy to check that the identity function $f:(X,\mu)\to(X,\sigma)$ is rarely μ-continuous but not almost weakly μ-continuous. It can also be shown that f is not μ-open.

(b) Let

$$X=\{a,b,c\},\ \mu=\{\varnothing,\{a\},\{a,b\}\},\ \sigma=\{\varnothing,X,\{b\},\{b,c\}\}.$$

Then the function $f:(X,\mu)\to(X,\sigma)$ defined by $f(a)=b$, $f(b)=c$, $f(c)=a$ is μ-open but not almost weakly μ-continuous. Also it is easy to check that f is not rarely μ continuous.

Definition 3.5. Let $A=\{G_i\}$ be a class of subsets of a topological space (X,τ). By rarely union sets [13] of A we mean $\{G_i\cup R_{G_i}\}$, where each R_{G_i} is a rare set such that each of $\{G_i\cap cl(R_{G_i})\}$ is empty. Recall that a subset B of X is said to be rarely almost compact relative to X [13] if for every cover of B by open sets of X, there exists a finite subfamily whose rarely union sets cover B. A topological space X is said to be rarely almost compact if the set X is rarely almost compact relative to X.

Definition 3.6. [26] A subset K of a GTS (X,μ) is said to be μ-compact relative to X if every cover of K by μ-open sets in X has a finite subcover. A space X is said to be μ-compact if X is μ-compact relative to X.

Theorem 3.5. *Let $f:(X,\mu)\to(Y,\sigma)$ be rarely μ-continuous and K be μ-compact relative to X. Then $f(K)$ is rarely almost compact relative to Y.*

Proof. Suppose that Ω is an open cover of $f(K)$. Let B be the set of all V in Ω such that $V\cap f(K)\neq\varnothing$. Then B is an open cover of $f(K)$. Hence for each $k\in K$, there is some $V_k\in B$ such that $f(k)\in V_k$. Since f is rarely μ-continuous, there exists a rare set R_{V_k} with $V_k\cap cl(R_{V_k})=\varnothing$ and a μ-open set U_k containing k such that $f(U_k)\subsetneqq V_k\cup R_{V_k}$. Hence there is a finite subfamily $\{U_k:k\in\Delta\}$ which covers K, where Δ is a finite subset of K. The subfamily $\{V_k\cup R_{V_k}:k\in\Delta\}$ also covers $f(K)$. $\square$

Theorem 3.6. *Let $f:(X,\tau)\to(Y,\sigma)$ be rarely continuous and μ be a GT on X such that $\tau\subsetneqq\mu$. Then $f:(X,\mu)\to(Y,\sigma)$ is rarely μ-continuous.*

Proof. Suppose that $x\in X$ and $G\in O(Y,f(x))$. Since f is rarely continuous, by Theorem 1 of [23] there exists an open set U in X containing x such that $int(f(U))\subsetneqq cl(G)$. Since $\tau\subsetneqq\mu$, U is a μ-open set containing x. It then follows from Theorem 3.1 that f is rarely μ-continuous. $\square$

Example 3.5. Let

$$X=\{a,b,c\},\ \tau=\{\varnothing,X,\{a\},\{a,b\}\},\ \sigma=\{\varnothing,X,\{a\},\{b\},\{a,b\},X\}.$$

Then the identity function $f:(X,\tau)\to(Y,\sigma)$ is rarely continuous. If we take

$$\mu=\{\varnothing,X,\{a,b\},\{a,c\},X\}$$

then f is not rarely μ-continuous.

Lemma 3.1. [18] *If $g : Y \to Z$ is continuous and one-to-one, then g preserves rare sets.*

Theorem 3.7. *If $f : (X,\mu) \to (Y,\sigma)$ is rarely μ-continuous and $g : (Y,\sigma) \to (Z,\tau)$ is a continuous injection, then $g \circ f : (X,\mu) \to (Z,\tau)$ is rarely μ-continuous.*

Proof. Suppose that $x \in X$ and $(g \circ f)(x) \in V$, where V is an open set in Z. By hypothesis, g is continuous, therefore $G = g^{-1}(V)$ is an open set in Y containing $f(x)$ such that $g(G) \subseteqq V$. Since f is rarely μ-continuous, there exists a rare set R_G with $G \cap cl(R_G) = \varnothing$ and a μ-open set U containing x such that $f(U) \subseteqq G \cup R_G$. It follows from Lemma 3.1 that $g(R_G)$ is a rare set in Z. Since R_G is a subset of $Y \setminus G$ and g is injective, we have $cl(g(R_G)) \cap V = \varnothing$. This implies that

$$(g \circ f)(U) \subseteqq V \cup g(R_G).$$

Hence we obtain the result. □

Example 3.6. Let

$$X = \{a,b,c\},\ \sigma = \{\varnothing,\{b\},\{a,c\},X\},\ \mu = \{\varnothing,\{a,b\},\{a,c\},X\}.$$

Then μ is a GT on the topological space (X,σ). It can be easily verified that the identity function $f : (X,\mu) \to (X,\sigma)$ is rarely μ-continuous. Let $\tau = \{\varnothing, X, \{a\}, \{b,c\}\}$. Then $g : (X,\sigma) \to (X,\tau)$ defined by $g(a) = g(c) = a$, $g(b) = b$ is continuous but

$$g \circ f : (X,\mu) \to (X,\tau)$$

is not rarely μ-continuous.

Definition 3.7. A topological space (X,τ) is called r-separated [14] if for every pair of distinct points x and y in X, there exist open sets U_x and U_y containing x and y, respectively, and rare sets R_{U_x}, R_{U_y} with

$$U_x \cap cl(R_{U_x}) = \varnothing \text{ and } U_y \cap cl(R_{U_y}) = \varnothing$$

such that $(U_x \cup R_{U_x}) \cap (U_y \cup R_{U_y}) = \varnothing$.

Definition 3.8. A GTS (X,μ) is said to be μ-T_2 [8] if for any distinct pair of points x and y in X, there exist disjoint μ-open sets U and V in X containing x and y, respectively.

Theorem 3.8. *If (Y,σ) is r-separated and $f : (X,\mu) \to (Y,\sigma)$ is a rarely μ-continuous injection, then (X,μ) is μ-T_2.*

Proof. Let x and y be any distinct points in X. Then $f(x) \neq f(y)$ (as f is injective). Thus there exist open sets G_x and G_y in Y containing $f(x)$ and $f(y)$, respectively, and rare sets R_{G_x} and R_{G_y} with

$$G_x \cap cl(R_{G_x}) = \varnothing \text{ and } G_y \cap cl(R_{G_y}) = \varnothing$$

such that $(G_x \cup R_{G_x}) \cap (G_y \cap R_{G_y}) = \emptyset$. Therefore

$$i_\mu[f^{-1}(G_x \cup R_{G_x})] \cap i_\mu[f^{-1}(G_y \cup R_{G_y})] = \emptyset.$$

By Theorem 3.5 we have

$$x \in f^{-1}(G_x) \subseteqq i_\mu[f^{-1}(G_x \cup R_{G_x})] \text{ and } y \in f^{-1}(G_y) \subseteqq i_\mu[f^{-1}(G_y \cup R_{G_y})].$$

Since $i_\mu[f^{-1}(G_x \cap R_{G_x})]$ and $i_\mu[f^{-1}(G_y \cap R_{G_y})]$ are two μ-open sets, (X, μ) is a μ-T_2 space. □

4. Conclusion

Let μ be a GT on the topological space (X, τ). Then the definitions of various types of rarely continuous functions $f : (X, \tau) \to (Y, \sigma)$ may be introduced from the definition of rarely μ-continuous function by replacing the generalized topologies μ on X suitably. In fact if μ is replaced by τ (resp. $SO(X)$, $PO(X)$, $\alpha O(X)$, $\delta O(X)$, $\delta SO(X)$, $\lambda O(X)$) then we can obtain almost all the results of [23] (resp. [24, 15, 16, 3, 4, 9]). We also observe that every rarely s-precontinuous function [11] is weakly s-precontinuous [11, 12] if f is r-preopen [11]. If $\mu = PO(X)$, then every rarely s-precontinuous and hence every weakly s-precontinuous function is rarely μ-precontinuous.

Also if we take $\mu = \lambda O(X)$ then almost weakly μ-continuity reduces to weakly λ-continuity of [10]. Thus every weakly λ-continuous function [10] is rare λ-continuous [9] and hence rare μ-continuous.

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