ON UNIFICATION OF RARELY CONTINUOUS FUNCTIONS

Bishwambhar Roy and Ritu Sen

Abstract. In 1979, V. Popa [23] first introduced the concept of rare continuity. In this paper, we introduce a new class of functions, termed rarely $\mu$-continuous functions, which unifies different weak forms of rarely continuous functions and investigate some of its properties.

Keywords: $\mu$-open set; rare set; rarely $\mu$-continuous function

1. Introduction

The notion of rare continuity was first introduced by V. Popa in [23] which was further studied by Long and Herrington [18] and Jafari [13, 14]. Certain weak forms of rarely continuous functions, for example, rare quasi-continuity, rare $\alpha$-continuity, rare $\delta s$-continuity, rare pre-continuity, rare $\delta$-continuity, rare $g$-continuity have been introduced and studied by Popa and Noiri [24], Jafari [16], Caldas, Jafari, Moshokoa and Noiri [4], Jafari [15], Caldas and Jafari [3], Caldas and Jafari [2] respectively.

The notion of generalized topological space was first introduced by A. Császár. After that a large number of papers have been devoted for the investigation of different properties of such spaces. We recall some notions defined in [5]. Let $X$ be a non-empty set, $expX$ denotes the power set of $X$. We call a class $\mu \subseteq expX$ a generalized topology [5], (GT for short) if $\emptyset \in \mu$ and union of elements of $\mu$ belongs to $\mu$. A set $X$, with a GT $\mu$ on it is said to be a generalized topological space (GTS for short) and is denoted by $(X, \mu)$.

For a GTS $(X, \mu)$, the elements of $\mu$ are called $\mu$-open sets and the complement of $\mu$-open sets are called $\mu$-closed sets. For $A \subseteq X$, we denote by $c_A$ the intersection of all $\mu$-closed sets containing $A$, i.e., the smallest $\mu$-closed set containing $A$; and by $i_A$ the union of all $\mu$-open sets contained in $A$, i.e., the largest $\mu$-open set contained in $A$ (see [5, 6]).

It is easy to observe that $i_A$ and $c_A$ are idempotent and monotonic, where $\gamma : expX \rightarrow expX$ is said to be idempotent iff $A \subseteq B \subseteq X$ implies that $\gamma(\gamma(A)) = \gamma(A)$ and

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monotonic iff $\gamma(A) \subseteq \gamma(B)$. It is also well known from [6, 7] that if $\mu$ is a GT on $X$ and $A \subseteq X$, $x \in X$, then $x \in c_\mu(A)$ iff $x \in M \in \mu \Rightarrow M \cap A \neq \emptyset$ and $c_\mu(X \setminus A) = X \setminus i_\tau(A)$.

The purpose of this paper is to introduce the concept of rare $\mu$-continuity which unifies the existing class of weak forms of rarely continuous functions by a particular choice of GT. We have also investigated several properties of rarely $\mu$-continuous functions. In this sequel the notion of $I, \mu$-continuity has been introduced which is then shown to be weaker than $(\mu, \sigma)$-continuity and stronger than rarely $\mu$-continuous function. It can also be observed that the results obtained in some other papers can be obtained from our results for a suitably chosen GT.

Hereafter, throughout the paper we shall use $(X, \mu)$ to refer to a generalized topological space and $(X, \tau), (Y, \sigma)$ to be topological spaces unless otherwise stated.

2. Preliminaries

Let $(X, \tau)$ be a topological space. The $\delta$-closure [28] of a subset $A$ of $(X, \tau)$ is denoted by $cl_\delta(A)$ and is defined by

$$\{x \in X : A \cap U \neq \emptyset \text{ for all regular open sets } U \text{ containing } x\},$$

where a subset $A$ is called regular open [27] if $A = \text{int}(cl(A))$. The set $A$ is called $\delta$-closed if $cl_\delta(A) = A$. The complement of a $\delta$-closed set is called $\delta$-open. It is known from [28] that the family of all $\delta$-open sets form a topology on $X$ which is smaller than the original topology $\tau$. A subset $A$ of $X$ is called semi-open [17] (resp. preopen [20], $\alpha$-open [21], $\delta$-semiopen [22]) if $A \subseteq \text{cl}(\text{int}(A))$ (resp. $A \subseteq \text{int}(\text{cl}(A))$, $A \subseteq \text{int}(\text{cl}(A)))$. The complement of a semi-open (resp. preopen, $\alpha$-open, $\delta$-semiopen) set is called a semi-closed (resp. preclosed, $\alpha$-closed, $\delta$-semiclosed) set. The collection of all semi-open (resp. preopen, $\alpha$-open, $\delta$-semiopen) sets in a topological space is denoted by $SO(X)$ (resp. $PO(X)$, $\alpha O(X)$, $\delta O(X)$, $\delta SO(X)$). We note that each of these collections forms a GT on $X$. A subset $A$ of a space $X$ is called a $\land$-set [19] if it is equal to its kernel i.e., intersection of all open superset of $A$. $A$ is called a $\land$-closed set [1] if $A = U \cap V$ where $U$ is a $\land$-set and $V$ is a closed set. The complement of a $\land$-closed set is called a $\land$-open set. The family of all $\land$-open sets of a topological space is denoted by $\land O(X)$. A subset $A$ of $X$ is called rare if $\text{int}(A) = \emptyset$. We shall use the symbol $O(f(x), Y)$ to refer to the collection of all open sets in the topological space $Y$ containing $f(x)$.

**Definition 2.1.** A function $f : (X, \tau) \to (Y, \sigma)$ is said to be rarely continuous [23] if for each $x$ in $X$ and $G \in O(Y, f(x))$ there exists a rare set $R_G$ with $G \cap cl(R_G) = \emptyset$, and an open set $U$ in $X$ containing $x$ such that $f(U) \subseteq G \cup R_G$.

**Definition 2.2.** A function $f : (X, \mu) \to (Y, \sigma)$ is said to be $(\mu, \sigma)$-continuous [25] if for each $x$ in $X$ and each $G \in O(Y, f(x))$, there exists a $\mu$-open set $U$ containing $x$ in $X$ such that $f(U) \subseteq G$. 
3. Rarely $\mu$-continuous functions

**Definition 3.1.** A function $f : (X, \mu) \to (Y, \sigma)$ is said to be rarely $\mu$-continuous if for each $x$ in $X$ and $G \in O(Y, f(x))$ there exists a rare set $R_G$ with $G \cap cl(R_G) = \emptyset$, and a $\mu$-open set $U$ in $X$ containing $x$ such that $f(U) \subseteq G \cup R_G$.

**Remark 3.1.** Let $\mu$ and $\lambda$ be two GT’s on the set $X$ such that $\mu \subseteq \lambda$. If $f : (X, \mu) \to (Y, \sigma)$ is rarely $\mu$-continuous then $f : (X, \lambda) \to (Y, \sigma)$ is rarely $\lambda$-continuous.

**Example 3.1.** Let

$$X = \{a, b, c\}, \sigma = \emptyset, [a], [a, b], X, \mu = \{\emptyset, [a, b], [a, c], X\}.$$

Then $\mu$ is a GT on the topological space $(X, \sigma)$. It can be easily verified that the identity function $f : (X, \mu) \to (X, \sigma)$ is rarely $\mu$-continuous.

**Theorem 3.1.** For a function $f : (X, \mu) \to (Y, \sigma)$ the followings are equivalent:

(i) $f$ is rarely $\mu$-continuous at $x$.

(ii) For each set $G \in O(Y, f(x))$, there exists a $\mu$-open set $U$ in $X$ containing $x$ such that $int[f(U) \cap (Y \setminus G)] = \emptyset$.

(iii) For each $G \in O(Y, f(x))$, there exists a $\mu$-open set $U$ in $X$ containing $x$ such that $int[f(U)] \subseteq cl(G)$.

(iv) For each $G \in O(Y, f(x))$, there exists a rare set $R_G$ with $G \cap cl(R_G) = \emptyset$ such that $x \in f^{-1}(G \cup R_G)$.

(v) For each $G \in O(Y, f(x))$, there exists a rare set $R_G$ with $cl(G) \cap R_G = \emptyset$ such that $x \in f^{-1}(cl(G) \cup R_G)$.

(vi) For each $G \in RO(Y, f(x))$, there exists a rare set $R_G$ with $G \cap cl(R_G) = \emptyset$ such that $x \in f^{-1}(G \cup R_G)$.

**Proof.** (i) $\Rightarrow$ (ii) : Let $G \in O(Y, f(x))$. Then $f(x) \in G \subseteq int(cl(G))$ and $int(cl(G)) \in O(Y, f(x))$. Thus by (i) there exists a rare set $R_G$ with $int(cl(G)) \cap cl(R_G) = \emptyset$ and a $\mu$-open set $U$ in $X$ containing $x$ such that $f(U) \subseteq int(cl(G)) \cup R_G$. We have

$$int[f(U) \cap (Y \setminus G)] = int(f(U)) \cap int(Y \setminus G) \subseteq int(int(cl(G)) \cup R_G) \cap (Y \setminus cl(G)) \subseteq (cl(G) \cup int(R_G)) \cap (Y \setminus cl(G)) = \emptyset.$$

(ii) $\Rightarrow$ (iii) : It is straightforward.

(iii) $\Rightarrow$ (i) : Let $G \in O(Y, f(x))$. Then by (iii), there exists a $\mu$-open set $U$ in $X$ containing $x$ such that $int[f(U)] \subseteq cl(G)$. We have

$$f(U) = [f(U) \setminus int(f(U))] \cup int(f(U)) \subseteq [f(U) \setminus int(f(U))] \cup cl(G) = [f(U) \setminus int(f(U))] \cup G \cup (cl(G) \setminus G) = [(f(U) \setminus int(f(U))) \cap (Y \setminus G)] \cup G \cup (cl(G) \setminus G).$$
It should be noted that (Remark 3.2. A function Definition 3.2. A function f Theorem 3.2. 264 B. Roy and R. Sen implies rare -continuity. But the converses are not true as shown by the following examples.

(i) ⇒ (iv) : Suppose that G ∈ O(Y, f(x)). Then there exists a rare set RG with G ∩ cl(RG) = ∅ and a µ-open set U in X containing x, such that f(U) ⊆ G ∪ RG. It follows that x ∈ U ⊆ f⁻¹(G ∪ RG). This implies that x ∈ i⁺(f⁻¹(G ∪ RG)).

(iv) ⇒ (v): Suppose that G ∈ O(Y, f(x)). Then there exists a rare set RG with G ∩ cl(RG) = ∅ such that x ∈ i₅(f⁻¹(G ∪ RG)). Since G ∩ cl(RG) = ∅, RG ⊆ Y \ G, where Y \ G = (Y \ cl(G)) ∪ (cl(G) \ G). Now, we have

\[ R_G \subseteq (R_G \cup (Y \setminus cl(G))) \cup (cl(G) \setminus G). \]

Set R' = R_G ∩ (Y \ cl(G)). It follows that R' is a rare set with cl(G) ∩ R' = ∅. Therefore

\[ x \in i₅[f⁻¹(G ∪ R_G)] \subseteq i₅[f⁻¹(cl(G) ∪ R')]. \]

(v) ⇒ (vi) : Assume that G ∈ RO(Y, f(x)). Then there exists a rare set R_G with cl(G) ∩ R_G = ∅ such that x ∈ i₅[f⁻¹(cl(G) ∪ R_G)]. Set R' = R_G ∪ (cl(G) \ G). It follows that R' is a rare set and G ∩ cl(R') = ∅. Hence

\[ x \in i₅[f⁻¹(cl(G) ∪ R_G)] = i₅[f⁻¹(G ∪ (cl(G) \ G) ∪ R_G)] = i₅[f⁻¹(G ∪ R')]. \]

(vi) ⇒ (ii): Let G ∈ O(Y, f(x)). By f(x) ∈ G ⊆ int(cl(G)) and the fact that int(cl(G)) ∈ RO(Y), there exists a rare set R_G with int(cl(G)) ∩ cl(R_G) = ∅ such that x ∈ i₅[f⁻¹(int(cl(G)) ∪ R_G)]. Let U = i₅[f⁻¹(int(cl(G)) ∪ R_G)]. Hence, x ∈ U ∈ µ and, therefore f(U) ⊆ int(cl(G)) ∪ R_G. Hence, we conclude

\[ int[f(U) \cap (Y \setminus G)] = ∅. \]

\[ \square \]

**Theorem 3.2.** A function f : (X, µ) → (Y, σ) is rarely µ-continuous if and only if for each open set G ⊆ Y, there exists a rare set R_G with G ∩ cl(R_G) = ∅ such that f⁻¹(G) ⊆ i₅[f⁻¹(G ∪ R_G)].

**Proof.** It follows from Theorem 3.1. \[ \square \]

**Definition 3.2.** A function f : (X, µ) → (Y, σ) is said to be I₁µ-continuous at x ∈ X if for each set G ∈ O(Y, f(x)), there exists a µ-open set U in X containing x such that int[f(U)] ⊆ G. If f has this property at each point x ∈ X, then we say that f is I₁µ-continuous on X.

**Remark 3.2.** It should be noted that (µ, σ)-continuity implies I₁µ-continuity and I₁µ-continuity implies rare µ-continuity. But the converses are not true as shown by the following examples.
Example 3.2. Let

\[ X = \{a, b, c\}, \mu = \emptyset, \{b, c\}, \{a, c\}, X \}, \sigma = \emptyset, X, \{a\}, \{b\}, \{a, b\} \].

Then the identity function \( f : (X, \mu) \to (X, \sigma) \) is \( I, \mu \)-continuous but not \( (\mu, \sigma) \)-continuous.

Example 3.3. Let

\[ X = \{a, b, c\}, \mu = \emptyset, \{a, b\}, \{a, c\}, X \}, \sigma = \emptyset, \{c\}, \{a, c\}, \{b, c\}, X \].

Then \( \mu \) is a GT on the topological space \( (X, \sigma) \). It can be easily verified that the identity function \( f : (X, \mu) \to (X, \sigma) \) is rarely \( \mu \)-continuous but not \( I, \mu \)-continuous.

Theorem 3.3. Let \( Y \) be a regular space. Then \( f : (X, \mu) \to (Y, \sigma) \) is \( I, \mu \)-continuous on \( X \) if and only if \( f \) is rarely \( \mu \)-continuous on \( X \).

Proof. We prove only the sufficient condition since as the converse part follows from Remark 3.2. Let \( f \) be rarely \( \mu \)-continuous on \( X \) and \( x \in X \). Suppose that \( f(x) \in G \), where \( G \) is an open set in \( Y \). By the regularity of \( Y \), there exists an open set \( G_1 \in O(Y, f(x)) \) such that \( cl(G_1) \subseteq G \). Since \( f \) is rarely \( \mu \)-continuous, there exists a \( \mu \)-open set \( U \) in \( X \) containing \( x \) such that \( int(f(U)) \subseteq cl(G_1) \). This implies that \( int(f(U)) \subseteq G \) and therefore \( f \) is \( I, \mu \)-continuous on \( X \). \( \square \)

Definition 3.3. A function \( f : (X, \mu) \to (Y, \sigma) \) is said to be \( \mu \)-open if the image of a \( \mu \)-open set is open.

Definition 3.4. A function \( f : (X, \mu) \to (Y, \sigma) \) is said to be almost weakly \( \mu \)-continuous if for each open set \( G \) in \( Y \) containing \( f(x) \) there exists a \( \mu \)-open set \( U \) in \( X \) containing \( x \) such that \( f(U) \subseteq cl(G) \).

It also follows that every almost weakly \( \mu \)-continuous function is rarely \( \mu \)-continuous. For the converse we have the next theorem.

Theorem 3.4. If \( f : (X, \mu) \to (Y, \sigma) \) be a \( \mu \)-open rarely \( \mu \)-continuous function, then \( f \) is almost weakly \( \mu \)-continuous.

Proof. Suppose that \( x \in X \) and \( G \in O(Y, f(x)) \). Since \( f \) is rarely \( \mu \)-continuous, there exists a \( \mu \)-open set \( U \) in \( X \) such that \( int(f(U)) \subseteq cl(G) \). Since \( f \) is \( \mu \)-open, \( f(U) \) is open and hence \( f(U) \subseteq cl(int(f(U))) \subseteq cl(G) \). This shows that \( f \) is almost weakly \( \mu \)-continuous. \( \square \)

Thus it follows that if \( f : (X, \tau) \to (Y, \sigma) \) be a \( \lambda \)-open, rarely \( \lambda \)-continuous function, then \( f \) is also a weakly \( \lambda \)-continuous function [12].

Example 3.4. (a) Let

\[ X = \{a, b, c\}, \sigma = \emptyset, \{a\}, \{b, c\}, X \}, \mu = \emptyset, \{a, b\}, \{a, c\}, X \].
Then \( \mu \) is a GT on the topological space \((X, \sigma)\). It is easy to check that the identity function \( f : (X, \mu) \to (X, \sigma) \) is rarely \( \mu \)-continuous but not almost weakly \( \mu \)-continuous. It can also be shown that \( f \) is not \( \mu \)-open.

(b) Let

\[
X = \{a, b, c\}, \quad \mu = \{\emptyset, [a], [a, b]\}, \quad \sigma = \{\emptyset, X, [b], [b, c]\}.
\]

Then the function \( f : (X, \mu) \to (X, \sigma) \) defined by \( f(a) = b, f(b) = c, f(c) = a \) is \( \mu \)-open but not almost weakly \( \mu \)-continuous. Also it is easy to check that \( f \) is not rarely \( \mu \)-continuous.

**Definition 3.5.** Let \( A = \{G_i\} \) be a class of subsets of a topological space \((X, \tau)\). By rarely union sets [13] of \( A \) we mean \( \{G_i \cup R_G \} \), where each \( R_G \) is a rare set such that each of \( G_i \cap cl(R_G) \) is empty. Recall that a subset \( B \) of \( X \) is said to be rarely almost compact relative to \( X \) [13] if for every cover of \( B \) by open sets of \( X \), there exists a finite subfamily whose rarely union sets cover \( B \). A topological space \( X \) is said to be rarely almost compact if the set \( X \) is rarely almost compact relative to \( X \).

**Definition 3.6.** A subset \( K \) of a GTS \((X, \mu)\) is said to be \( \mu \)-compact relative to \( X \) if every cover of \( K \) by \( \mu \)-open sets in \( X \) has a finite subcover. A space \( X \) is said to be \( \mu \)-compact if \( X \) is \( \mu \)-compact relative to \( X \).

**Theorem 3.5.** Let \( f : (X, \mu) \to (Y, \sigma) \) be rarely \( \mu \)-continuous and \( K \) be \( \mu \)-compact relative to \( X \). Then \( f(K) \) is rarely almost compact relative to \( Y \).

**Proof.** Suppose that \( \Omega \) is an open cover of \( f(K) \). Let \( B \) be the set of all \( V \) in \( \Omega \) such that \( V \cap f(K) \neq \emptyset \). Then \( B \) is an open cover of \( f(K) \). Hence for each \( k \in K \), there is some \( V_k \in B \) such that \( f(k) \in V_k \). Since \( f \) is rarely \( \mu \)-continuous, there exists a rare set \( R_i \) with \( V_k \cap cl(R_i) = \emptyset \) and a \( \mu \)-open set \( U_k \) containing \( k \) such that \( f(U_k) \subseteq V_k \cup R_i \). Hence there is a finite subfamily \( \{U_k : k \in \Delta\} \) which covers \( K \), where \( \Delta \) is a finite subset of \( K \). The subfamily \( \{V_k \cup R_i : k \in \Delta\} \) also covers \( f(K) \).

**Theorem 3.6.** Let \( f : (X, \tau) \to (Y, \sigma) \) be rarely continuous and \( \mu \) be a GT on \( X \) such that \( \tau \subseteq \mu \). Then \( f : (X, \mu) \to (Y, \sigma) \) is rarely \( \mu \)-continuous.

**Proof.** Suppose that \( x \in X \) and \( G \in O(Y, f(x)) \). Since \( f \) is rarely continuous, by Theorem 1 of [23] there exists an open set \( U \) in \( X \) containing \( x \) such that \( int(f(U)) \subseteq cl(G) \). Since \( \tau \subseteq \mu \), \( U \) is a \( \mu \)-open set containing \( x \). It then follows from Theorem 3.1 that \( f \) is rarely \( \mu \)-continuous.

**Example 3.5.** Let

\[
X = \{a, b, c\}, \quad \tau = \{\emptyset, X, [a], [a, b]\}, \quad \sigma = \{\emptyset, X, [b], [b, a], X\}.
\]

Then the identity function \( f : (X, \tau) \to (Y, \sigma) \) is rarely continuous. If we take

\[
\mu = \{\emptyset, X, [a, b], [a, c], X\}
\]

then \( f \) is not rarely \( \mu \)-continuous.
Theorem 3.8. If \( f : (X, \mu) \to (Y, \sigma) \) is respectively injective, we have \( \mu \)-continuous injection, then \( g \circ f : (X, \mu) \to (Z, \tau) \) is \( \mu \)-continuous.

Proof. Suppose that \( x \in X \) and \((g \circ f)(x) \in V\), where \( V \) is an open set in \( Z \). By hypothesis, \( g \) is continuous, therefore \( G = g^{-1}(V) \) is an open set in \( Y \) containing \( f(x) \) such that \( g(G) \subseteq V \). Since \( f \) is rarely \( \mu \)-continuous, there exists a rare set \( R_G \) with \( G \cap cl(R_G) = \emptyset \) and a \( \mu \)-open set \( U \) containing \( x \) such that \( f(U) \subseteq G \cup R_G \). It follows from Lemma 3.1 that \( g(R_G) \) is a rare set in \( Z \). Since \( R_G \) is a subset of \( Y \setminus G \) and \( g \) is injective, we have \( cl(g(R_G)) \cap V = \emptyset \). This implies that \( (g \circ f)(U) \subseteq V \cup g(R_G) \).

Hence we obtain the result. \( \Box \)

Example 3.6. Let

\[ X = \{a, b, c\}, \quad \sigma = \{\emptyset, \{b, \{a, c\}, X\}, \quad \mu = \{\emptyset, \{a, b\}, \{a, c\}, X\}. \]

Then \( \mu \) is a GT on the topological space \( (X, \sigma) \). It can be easily verified that the identity function \( f : (X, \mu) \to (X, \sigma) \) is rarely \( \mu \)-continuous. Let \( \tau = \{\emptyset, X, \{a, b, c\}\} \). Then \( g : (X, \sigma) \to (X, \tau) \) defined by \( g(a) = g(c) = a, g(b) = b \) is continuous but

\[ g \circ f : (X, \mu) \to (X, \tau) \]

is not rarely \( \mu \)-continuous.

Definition 3.7. A topological space \( (X, \tau) \) is called \( r \)-separated \([14]\) if for every pair of distinct points \( x \) and \( y \) in \( X \), there exist open sets \( U_x \) and \( U_y \) containing \( x \) and \( y \), respectively, and rare sets \( R_{U_x}, R_{U_y} \) with

\[ U_x \cap cl(R_{U_x}) = \emptyset \quad \text{and} \quad U_y \cap cl(R_{U_y}) = \emptyset \]

such that \( (U_x \cup R_{U_x}) \cap (U_y \cup R_{U_y}) = \emptyset \).

Definition 3.8. A GTS \( (X, \mu) \) is said to be \( \mu-T_r \) \([8]\) if for any distinct pair of points \( x \) and \( y \) in \( X \), there exist disjoint \( \mu \)-open sets \( U \) and \( V \) in \( X \) containing \( x \) and \( y \), respectively.

Theorem 3.8. If \( (Y, \sigma) \) is \( r \)-separated and \( f : (X, \mu) \to (Y, \sigma) \) is a rarely \( \mu \)-continuous injection, then \( (X, \mu) \) is \( \mu-T_r \).

Proof. Let \( x \) and \( y \) be any distinct points in \( X \). Then \( f(x) \neq f(y) \) (as \( f \) is injective). Thus there exist open sets \( G_x \) and \( G_y \) in \( Y \) containing \( f(x) \) and \( f(y) \), respectively, and rare sets \( R_{G_x} \) and \( R_{G_y} \) with

\[ G_x \cap cl(R_{G_x}) = \emptyset \quad \text{and} \quad G_y \cap cl(R_{G_y}) = \emptyset \]
such that \((G_x \cup R_Gx) \cap (G_y \cap R_Gy) = \emptyset\). Therefore
\[i_\mu[f^{-1}(G_x \cup R_Gx)] \cap i_\mu[f^{-1}(G_y \cup R_Gy)] = \emptyset.\]

By Theorem 3.5 we have
\[x \in f^{-1}(G_x) \subseteq i_\mu[f^{-1}(G_x \cup R_Gx)] \quad \text{and} \quad y \in f^{-1}(G_y) \subseteq i_\mu[f^{-1}(G_y \cup R_Gy)].\]

Since \(i_\mu[f^{-1}(G_x \cap R_Gx)]\) and \(i_\mu[f^{-1}(G_y \cap R_Gy)]\) are two \(\mu\)-open sets, \((X, \mu)\) is a \(\mu\)-\(T_2\) space.

4. Conclusion

Let \(\mu\) be a GT on the topological space \((X, \tau)\). Then the definitions of various types of rarely continuous functions \(f : (X, \tau) \to (Y, \sigma)\) may be introduced from the definition of rarely \(\mu\)-continuous function by replacing the generalized topologies \(\mu\) on \(X\) suitably. In fact if \(\mu\) is replaced by \(\tau\) (resp. \(SO(X), PO(X), \alpha O(X), \delta O(X), \lambda O(X)\)) then we can obtain almost all the results of [23] (resp. [24, 15, 16, 3, 4, 9]). We also observe that every rarely \(s\)-precontinuous function [11] is weakly \(s\)-precontinuous [11, 12] if \(f\) is \(r\)-preopen [11]. If \(\mu = PO(X)\), then every rarely \(s\)-precontinuous and hence every weakly \(s\)-precontinuous function is rarely \(\mu\)-precontinuous.

Also if we take \(\mu = \lambda O(X)\) then almost weakly \(\mu\)-continuity reduces to weakly \(\lambda\)-continuity of [10]. Thus every weakly \(\lambda\)-continuous function [10] is rare \(\lambda\)-continuous [9] and hence rare \(\mu\)-continuous.

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Bishwambhar Roy
Department of Mathematics
Women's Christian College
6, Greek Church Row
Kolkata 700 026, INDIA
bishwambhar.roy@yahoo.co.in

Ritu Sen
Department of Mathematics
S. A. Jaipuria College
10, Raja Naba Krishna Street
Kolkata 700 005, INDIA
ritu_sen29@yahoo.co.in