FACTA UNIVERSITATIS (NIŠ)
Ser. Math. Inform. Vol. 30, No 5 (2015), 719–729

A COMMON FIXED POINT THEOREM FOR WEAKLY SUBSEQUENTIALLY CONTINUOUS MAPPINGS SATISFYING IMPLICIT RELATION IN MENGER SPACES

Said Beloul

Abstract. The aim of this paper is to prove a common fixed point theorem for two pairs of self mappings satisfying an implicit relation by using the weak subsequential continuity with compatibility of type (E) in Menger spaces. We illustrate with two examples to support the main result.

keywords: weakly subsequentially continuous, compatible of type (E), implicit relation, Menger spaces.

1. Introduction

In recent years, many authors have studied the existence of fixed point or common fixed point in different metric structure spaces. One of these types are the Menger metric spaces, which were introduced by Menger in 1942. The idea of Menger was to use distribution functions instead of non-negative real numbers as values of the metric. The notion of PM-space corresponds to situations when we do not know exactly the distance between two points, but we know probabilities of possible values of this distance. In fact, the study of such spaces received an impetus with the pioneering work of Schweizer and Sklar [30]. In order to establish a common fixed point in metric spaces, Jungck[17] introduced commuting mappings, which generalized to weak commuting mappings by Sessa[31]. Jungck[18] Later generalized the two last notions to compatibility property. Jungck and Rhoades[19] weakened the concept of compatibility to the weak compatibility, and recently Al-Thagafi and Shahzad[3] gave a generalization, which is called the occasionally weak compatibility property. This notion is weaker than the weak compatibility due to Jungck and Rhoades[19]. Recently, Doric et al.[13] mentioned that the condition of occasionally weak compatibility reduces to weak compatibility, in the case where the two mappings have a unique point of coincidence (or a unique common fixed point). In 2009 Bouhadjera and Godet Tobie[7] introduced

Received July 13, 2015; Accepted October 20, 2015
2010 *Mathematics Subject Classification.* 47H10; 54H25

the concepts of subcompatibility and subsequential continuity which are more general than occasional weak compatibility and reciprocal continuity due to Pant[26]. Later, Imdad et al.[14] improved the results of Bouhadjera and Godet Thobie[7] by using subcompatibility with reciprocal continuity or subsequential continuity with compatibility. The concept of implicit relation has been introduced by Popa[29] who established some fixed point results by using this concept. There are also some of the interesting references concerning fixed point and common fixed results involving the notion of implicit relation in Menger spaces as in papers [6, 8, 15, 21].

2. Preliminaries

Definition 2.1. A mapping $\triangle : [0,1]\times[0,1]\times[0,1]$ is a t-norm (or a triangular norm) if it satisfies the following conditions:

1. $\triangle(a,1) = a$, for all $a \in [0,1]$,
2. $\triangle(a,b) = \triangle(b,a)$,
3. $\triangle(a,b) \leq \triangle(c,d)$ for all $a \leq c$ and $b \leq d$,
4. $\triangle(\triangle(a,b),c) = \triangle(a,\triangle(b,c))$.

Example 2.1. Let (X,d) be a metric space, define $\triangle(a,b) = \min\{a,b\}$ for all $a,b \in [0,1]$, then $\triangle$ is a t-norm.
Also $\triangle(a,b) = ab$ and $\triangle(a,b) = \max\{0, a+b-1\}$ are t-norms.

Definition 2.2. A real valued mapping $F : \mathbb{R} \to \mathbb{R}_+$ is called a distribution function, if it is non-decreasing and left-continuous with:

$$\inf F(x) = 0, \sup_{x\in} F(x) = 1$$

We denote by $\mathfrak{F}$ a set of all distribution functions, and denote by H the Heaviside distribution function defined by:

$$H(t) = \begin{cases} 0, & t \leq 0 \\ 1, & t > 0 \end{cases}$$

Definition 2.3. Let X be a non-empty set, an order pair (X,F) is called a probabilistic metric space if F is a mapping from $X \times X$ into $\{f \in \mathfrak{F}, f(0) = 0\}$ and satisfying the following conditions:

1. $F_{xy} = H$, if and only if $x = y$,
2. $F_{xy} = F_{yx}$, for all $x, y \in X$,

3. if $F_{xy}(t) = 1$ and $F_{yz}(s) = 1$, then $F_{xz}(t + s) = 1$ for all $x, y, z \in X$ and $t, s \geq 0$.

If F satisfies only (1) and (2), the pair (X, F) is called a probabilistic semi-metric space.

Definition 2.4. A triplet $(X, F, \triangle)$ is called to be a Menger space if (X, F) is a probabilistic metric space and $\triangle$ is a t-norm such for all $x, y \in X$ and $t, s \geq 0$ the following inequality holds:

$$F_{xz}(t + s) \geq \triangle(F_{xy}, F_{yz})$$

If (X, d) is a metric space, by taking $F_{xy} = H(t - d(x, y)$, it becomes (X, F) probabilistic metric space, so every metric space can be realized as a probabilistic metric space.

Definition 2.5. Let $(X, F, \triangle)$ be a Menger space with a continuous t-norm

- A sequence $\{x_n\}$ in X is said to be convergent to $x \in X$ if and only if for every $\varepsilon > 0$ and $\lambda \in (0, 1)$, there exists an integer N such $F_{x_n x}(\varepsilon) > 1 - \lambda$ for all $n \geq N$.
- A sequence $\{x_n\}$ in X is called a Cauchy one, if and only if for every $\varepsilon > 0$ and $\lambda \in (0, 1)$, there exists an integer N such $F_{x_n x_m}(\varepsilon) > 1 - \lambda$ for all $n, m \geq N$.
- A Menger space is said to be complete if every sequence in it, is is convergent.

Definition 2.6. A pair $\{A, S\}$ of selfmappings from a Menger space $(X, F, \triangle)$ into itself is compatible if and only if

$$\lim_{n\to\infty} F_{ASx_n, SAx_n} = 1,$$

for all $t \geq 0$, whenever $\{x_n\}$ is a sequence in X such

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z,$$

for some $z \in X$.

Definition 2.7. Two selfmappings A, S of a Menger space $(X, F, \triangle)$ into itself are called to be weakly compatible if and only if they commute at their coincidence points, i.e if $Ax = Sx$ for some $x \in X$, then $ASx = SAx$

Kumar and Pant[21] generalized the reciprocal continuity concept due to Pant[26]in the setting of Menger space as follows:

Definition 2.8. Two selfmappings A and S of a Menger space $(X, F, \triangle)$ are called reciprocally continuous if $\lim_{n\to\infty} ASx_n = Az$ and $\lim_{n\to\infty} SAx_n) = Sz$, whenever $x_n\}$ is a sequence in X such $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} SAx_n = z$, for some $z \in X$.

Bouhadjera and Ghodet Tobie[7] introduced the concept of subsequential continuity in metric spaces, and in the setting of Menger spaces it becomes:

Definition 2.9. Let $(X, F, \triangle)$ be a Menger space, the pair of selfmappings $\{A, S\}$ is said to be subsequentially continuous, if there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$, for some $z \in X$ and $\lim_{n\to\infty} ASx_n = Az$.

Motivated by the above definition, we define:

Definition 2.10. The pair $\{A, S\}$ is said to be weakly subsequentially continuous (wsc), if there exists a sequence $\{x_n\}$ such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$, for some $z \in X$ and $\lim_{n\to\infty} ASx_n = Az$, or $\lim_{n\to\infty} SAx_n = Sz$

The pair $\{A, S\}$ is said to be A-subsequentially continuous(S-subsequentially continuous), if there exists a sequence $\{x_n\}$ such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z, \ \lim_{n\to\infty} SAx_n = Sz.$$

Example 2.2. Let $X = [0, 2]$ and let a continuous t-norm: $\triangle(x, y) = \frac{t}{t+|x-y|}$ for all $t > 0$, define A, S as follows:

$$Ax = \begin{cases} 1 + x, & 0 \leq x \leq 1 \\ 0, & 1 < x \leq 2 \end{cases}, \quad Sx = \begin{cases} 1 - x, & 0 \leq x \leq 1 \\ \frac{x}{2}, & 1 < x \leq 2 \end{cases}$$

Clearly, A and S are discontinuous at $\frac{1}{2}$.

Consider a sequence $\{x_n\}$ such that for each $n \geq 1 : x_n = \dfrac{1}{n}$,
it is clear that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = 1$, also we have:

$$\lim_{n\to\infty} ASx_n = \lim_{n\to\infty} A(1 - \frac{1}{n}) = A(1) = 1,$$

$$\lim_{n\to\infty} SAx_n = \lim_{n\to\infty} S(1 + \frac{1}{n}) = \frac{1}{4} \neq S(1),$$

then $\{A, S\}$ is A-subsequentially continuous, so it is wsc.

Singh and Mahendra Singh [32, 33] introduced the notion of compatibility of type (E) in metric spaces, in the setting of the Menger spaces, it becomes:

Definition 2.11. Self-maps A and S of a Menger space $(X, F, \triangle)$ are said to be compatible of type (E), if $\lim_{n\to\infty} S^2x_n = \lim_{n\to\infty} SAx_n = Az$ and $\lim_{n\to\infty} A^2x_n = \lim_{n\to\infty} ASx_n = Sz$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$, for some $z \in X$.

Definition 2.12. Two self-maps A and S of a Menger space $(X, M, \triangle)$ into itself are said to be A-compatible of type (E), if

$$\lim_{n\to\infty} A^2x_n = \lim_{n\to\infty} ASx_n = Sz,$$

for some $z \in X$.

pair $\{A, S\}$ is said to be S-compatible of type (E), if $\lim_{n\to\infty} S^2x_n = \lim_{n\to\infty} SAx_n = Az$, for some $z \in X$.

Notice that if A and S are compatible of type (E), then they are A-compatible and S-compatible of type (E), but the converse is not true.

Example 2.3. Let $X = [0, \infty)$ with the continuous t-norm $\triangle(x, y) = \frac{t}{t+|x-y|}$ for all $t \geq 0$, define A, S as follows:

$$Ax = \begin{cases} \frac{x+2}{2}, & 0 \leq x \leq 2 \\ \frac{x}{2}, & x > 1 \end{cases} \qquad Sx = \begin{cases} 4 - x, & 0 \leq x \leq 2 \\ \frac{x}{2} + 1 & x > 2 \end{cases}$$

Consider a sequence $\{x_n\}$ which is defined by: $x_n = 2 - \dfrac{1}{n}$, for all $n \geq 1$, we have:

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = 2,$$

$$\lim_{n\to\infty} SAx_n = \lim_{n\to\infty} S(2 - \frac{1}{2n}) = A(2) = 2,$$

$$\lim_{n\to\infty} S^2x_n = \lim_{n\to\infty} S(2 + \frac{1}{n}) = A(2)$$

then the pair $\{A, S\}$ is S-compatible of type (E), but never compatible of type (E) since:

$$\lim_{n\to\infty} ASx_n = \lim_{n\to\infty} S(\frac{1}{2} + \frac{1}{2(n+1)}) = 1 \neq S(2)$$

3. Implicit relations

Let Φ be a set of all continuous functions $\phi : [0, 1]^6 \to [0, 1]$ satisfying:
$F(u, u, 1, 1, u, u) < 0$, for all $u, v \in (0, 1)$

Example 3.1.

$$\phi(t_1, t_2, t_3, t_4, t_5, t_6) = t_1 - \psi(\min\{t_2, t_3, t_4, t_5, t_6\}),$$

where $\psi : ([0, 1])^6 \to [0, 1]$ is an increasing and continuous function such $\psi(t) > t$, for all $t \in (0, 1)$

Example 3.2.

$$\phi(t_1, t_2, t_3, t_4, t_5, t_6) = t_1 + t_2 - at_3 - bt_4 + \frac{1}{2}(t_1 + t_2),$$

where a, b are two positive numbers such $a + b \geq 1$.

Example 3.3.

$$\phi(t_1, t_2, t_3, t_4, t_5) = 2t_1^2 - t_3t_4 - t_5t_6,$$

Example 3.4.

$$\phi(t_1, t_2, t_3, t_4, t_5, t_6) = 14t_1 - 8t_2 + 6t_3 - 9t_4 - 3t_6,$$

The aim of this paper is to prove the existence and the uniqueness of common fixed point for two pairs of selfmappings in a Menger metric space, which satisfy implicit relation by using weak subsequential continuity with compatibility of type (E). To illustrate our results we give two examples.

4. Main results

Theorem 4.1. *Let $(X, F, \triangle)$ be a Menger space, A, B, S are four selfmappings a on X such for all $x, y \in X$ and each $t > 0$, we have:*

$$\phi(F_{Sx,Ty}t), F_{Ax,By}(t), F_{Ax,Sx}t), F_{By,Ty}(t), F_{Ax,Ty}(t), F_{By,Sx}(t)) \geq 0, \tag{4.1}$$

where $\phi \in \oplus$, if the two pairs $\{A, S\}$ and $\{B, T\}$ are weakly subsequentially continuous (wsc) and compatible of type (E), then A, B, S and T have a unique common fixed point in X.

Proof. Since $\{A, S\}$ is wsc, there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$$

for some $z \in X$ and

$$\lim_{n\to\infty} ASx_n = Az, \quad \lim_{n\to\infty} SAx_n = Sz,$$

the compatibility of type (E) of $\{A, S\}$ implies that

$$\lim_{n\to\infty} ASx_n = \lim_{n\to\infty} A^2x_n = Sz$$

and

$$\lim_{n\to\infty} SAx_n = \lim_{n\to\infty} S^2x_n = Az,$$

then $Az = Sz$ and z is a coincidence point for A and S. Similarly, for B and T, since $\{B, T\}$ is wsc (suppose that it is B-subsequentially continuous) there exists a sequence $\{y_n\}$ such

$$\lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = w$$

for some $w \in X$ and

$$\lim_{n\to\infty} BTy_n = Bw,$$

also the pair $\{B, T\}$ is compatible of type (E) implies that

$$\lim_{n\to\infty} BTy_n = \lim_{n\to\infty} B^2y_n = Tw$$

$$\lim_{n\to\infty} TBy_n = \lim_{n\to\infty} T^2y_n = Bw,$$

so we have $Bw = Tw$.

We claim $Az = Bw$, if not by using (4.1) we get:

$$\phi(F_{Sz,Tw}t), F_{Az,Bw}t), F_{Az,Sz}t), F_{Bw,Tw}t), F_{Az,Tw}t), F_{Bw,Sz}t)) =$$

$$\phi(F_{Az,Bw}(t), F_{Az,Bw}(t), 1, 1, F_{Az,Bw}(t), F_{Az,Bw}(t)) \geq 0,$$

which implies that $F_{Az,Bw}(t) = 1$, then $Az = Bw$.

Now we will prove $z = Az$, if not by using(4.1) we get:

$$\phi(F_{Sx_n,Tw}(t), F_{Ax_n,Bw}(t), F_{Ax_n(t),Sx_n}(t), F_{Bw,Tw}(t), F_{Ax_n,Tw}(t), F_{Bw,Sx_n}(t)) \geq 0,$$

letting $n \to \infty$ we get:

$$\phi(F_{z,Tw}(t), F_{z,Bw}(t), 1, 1, F_{z,Tw}(t), F_{Bw,z}(t)) =$$

$$\phi(F_{z,Az}(t), F_{z,Az}(t), 1, 1, F_{z,Az}(t), F_{z,Az}(t)) \geq 0,$$

and so $F_{z,Az}(t) = 1$, then $z = Az = Sz$.

Next, we shall prove $z = t$, if not by using (4.1) we get:

$$\phi(F_{Sx_n,Tyn}(t), F_{Ax_n,Byn}(t), F_{Ax_n,Sx_n}(t), F_{Byn,Tyn}(t), F_{Ax_n,Tyn}(t), F_{Byn,Sx_n}(t)) \geq 0,$$

letting $n \to \infty$ we get:

$$\phi(F_{z,w}(t), F_{z,w}(t), 1, 1, F_{z,w}(t), F_{w,z}(t)) \geq 0,$$

which implies that $F_{z,w}(t) = 1$, then z is a fixed point for A, B, S and T.
For the uniqueness, if q is another fixed point q, by using (4.1) we get:

$$\phi(F_{Sz,Tq}(t), F_{Az,Bq}(t), F_{Az,Sq}(t), F_{(Bq,Tq}(t), F_{Az,Tq}(t), F_{Bq,Sz}(t)) =$$

$$\phi(F_{z,q}(t), F_{z,q}(t), 1, 1, F_{z,q}(t), F_{z,q}(t)) \geq 0,$$

then $z = q$, and z is unique. □

If $A = B$ and $S = T$, we get the following corollary:

Corollary 4.1. *Let $(X, F, \triangle)$ be a Menger space and let A, B, S and T be four self mappings on X such for all $x, y \in X$ we have:*

$$\phi(F_{Sx,Sy}(t), F_{Ax,Ay}(t), F_{Ax,Sx}(t), F_{Ay,Sy}(t), F_{Ax,Sy}(t), F_{Ay,Sx}(t)) \geq 0,$$

where $\phi \in \Phi$, if the pair $\{A, S\}$ is weakly subsequentially continuous and compatible of type (E), then A, B, S and T have a unique common fixed point.

Corollary 4.2. *For four self mappings A, B, S and T on Menger space $(X, F, \triangle)$ such for all $x, y \in X$ we have:*

$$F_{Sx,Ty}(t) \geq \psi(\min\{(F_{Ax,By}(t), F_{Ax,Sx}(t), F_{By,Ty}(t), F_{Ax,Ty}(t), F_{By,Sx}(t)),$$

assume that the following conditions hold:

1. *$\{A, S\}$ is A-subsequentially continuous and A-compatible of type (E),*
2. *$\{B, T\}$ is B-subsequentially continuous and B-compatible of type (E),*

then A, B, S and T have a unique common fixed point.

If we combine Theorem 4.1 with Example 3.1, we obtain:

Corollary 4.3. *Let $(X, F, \triangle)$ be a Menger space and let $A, B, S, T : X \to X$ two self mappings such for all $x, y \in X$ and every $t > 0$, we have:*

$$M(Sx, Ty, t) \geq aM(By, Ty, t) + bM(Ax, Ty, t),$$

where $a + b \geq 1$, suppose that the following conditions satisfy:

1. *$\{A, S\}$ is S-subsequentially continuous and S-compatible of type (E),*
2. *$\{B, T\}$ is T-subsequentially continuous and T-compatible of type (E),*

then A, B, S and T have a unique common fixed point.

Example 4.1. Let $(X, F, \triangle)$ be a Menger metric space such

$$X = [0, 2], \quad \triangle(x, y) = \min(x, y)$$

and

$$F_{x,y} = \frac{t}{t + |x - y|}, \text{for all } t \in [0, 1],$$

define mappings A, B, S and T as follows:

$$Ax = Bx = \begin{cases} \frac{x+1}{2}, & 0 \leq x \leq 1 \\ 2, & 1 < x \leq 2 \end{cases} \qquad Sx = Tx = \begin{cases} 1, & 0 \leq x \leq 1 \\ \frac{3}{4}, & 1 < x \leq 2 \end{cases}$$

We consider a sequence $\{x_n\}$ which is defined for each $n \geq 1$ by:$x_n = 1 - \frac{1}{n}$,
Clearly $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = 1$, also we have:

$$\lim_{n\to\infty} ASx_n = A(1) = S(1) = 1$$

$$\lim_{n\to\infty} A^2 x_n = S(1) = 1,$$

then $\{A, S\}$ is A-subsequentially continuous and A-compatible of type (E).

Taking

$$\phi(t_1, t_2, t_3, t_4, t_5, t_6) = t_1 - \psi(\min\{t_2, t_3, t_4, t_5, t_6\}),$$

with $\psi(t) = \sqrt{t}$ and we will show that the following inequality hold:

$$F_{Sx,Ty}(t) \geq (\min\{F_{Ax,By}(t), F_{Ax,Sx}(t), F_{By,Ty}(t), F_{Ax,Ty}(t)F_{By,Sx}(t)\})^{\frac{1}{2}},$$

we have the following cases:

1. For $x, y \in [0, 1]$, we have

$$F_{Sx,Ty}(t) = 1 \geq \frac{t}{t + x} = \psi(F_{Ax,By}(t))$$

2. For $x \in [0, 1]$ and $1 < y \leq 2$, we have

$$F_{Sx,Ty}(t) = \frac{t}{t + 0.25} \geq (\frac{t}{t + 1})^{\frac{1}{2}} = F_{By,Sx}(t)$$

3. For $x \in (1,\infty)$ and $y \in [0,1]$, we have

$$F_{Sx,Ty}(t) = \frac{t}{t+0.25} \geq (\frac{t}{t+1,25})^{\frac{1}{2}} = F_{By,Ty}(t)$$

4. For $x, y \in (1,\infty)$, we have

$$F_{Sx,Ty}(t) = 1 \geq (\frac{t}{t+1,25} = F_{Ax,Ty}(t),$$

then all hypotheses of Corollary 4.3 satisfy, and the point 1 is the unique common fixed for A, B, S and T.

Example 4.2. Let $(X, F, \triangle)$ be the probabilistic metric space as defined in Example 4.1 with $X = \mathbb{R}_+$, define mappings A, B, S and T as follows:

$$Ax = Bx = \begin{cases} \frac{x}{4} + \frac{3}{2}, & 0 \leq x \leq 2 \\ 2x - 3, & x > 2 \end{cases} \qquad Sx = Tx = \begin{cases} \frac{x}{2} + 1, & 0 \leq x \leq 2 \\ \frac{x}{2}, & x > 2 \end{cases}$$

We consider a sequence $\{x_n\}$ which is defined for each $n \geq 1$ by:
$x_n = 2 - \frac{1}{n}$. Clearly $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = 1$, also we have:

$$\lim_{n\to\infty} ASx_n = A(1) = S(2) = 2$$

$$\lim_{n\to\infty} A^2 x_n = S(2) = 2,$$

then $\{A, S\}$ is A-subsequentially continuous and A-compatible of type (E).

Taking F and ψ as in Example 4.1, for the inequality (4.1) we have the following cases:

1. For $x, y \in [0,2]$, we have

$$F_{Sx,Ty}(t)) = \frac{t}{t+0.5|x-y|} \geq \left(\frac{t}{t+0.25|x-y|}\right)^{\frac{1}{2}} = \psi(F_{Ax,By}(t)$$

2. For $x \in [0,2]$ and $y > 2$, we have

$$F_{Sx,Ty}(t) = \frac{t}{t+0.5|x-y|} \geq \left(\frac{t}{t+0.25x+0.5}\right)^{\frac{1}{2}} = \psi(F_{By,Sx}(t))$$

3. For $x \in (1,\infty)$ and $y \in [0,1]$, we have

$$F_{Sx,Ty}(t) = \frac{t}{t+0.5y|} \geq (\frac{t}{t+0.25y+0.5|})^{\frac{1}{2}} = \psi(F_{By,Ty}(t))$$

4. For $x, y \in (1,\infty)$, we have

$$F_{Sx,Ty}(t) = 1 \geq \left(\frac{t}{t+2(x-1)}\right)^{\frac{1}{2}} = \psi(F_{Ax,Ty}(t)),$$

then all hypotheses of Corollary 4.3 are satisfied, and the point 2 is the unique common fixed for A and S.

REFERENCES

1. M. Aamri and D. El Moutawakil, *Some new common fixed point theorems under strict contractive conditions*, Math. Anal. Appl, 270 (2002), 181-188.
2. J. Ali, M. Imdad, D. Miheu and M. Tanveer,*Common fixed points of strict contractions in Menger spaces*, Acta Math. Hungar. 132(4) (2011), 367-386.
3. M. A. Al-Thagafiand N. Shahzad, *Generalized I-nonexpansive selfmaps and invariant approximations*, Acta Math. Sinica 24, no5, (2008), 867-876.
4. I. Beg and M. Abbas,*Common fixed points of weakly compatible and noncommuting mappings in Menger spaces*, Int. J. Mod. Math. 3(3) (2008), 261-269.
5. I. Beg and M. Abbas,*Existence of common fixed points in Menger spaces*, Commun. Appl. Nonlinear Anal. 16(4) (2009), 93-100.
6. I. Beg and S. Chauhan,*Fixed point for compatible and subsequentially continuous mappings in menger spaces and applications*, Novi Sad J. Math. Vol. 43, No. 1, 2013, 131-144.
7. H. Bouhadjera and C. Godet Thobie,Common fixed point theorems for pairs of subcompatible maps,arXiv:0906.3159v1 [math.FA],(2009).
8. S.Chauhan and D. Pant,*Fixed point theorems for compatible and subsequentially continuous mappings in Menger spaces*,J. Nonlinear Sci. Appl. 7 (2014), 78-89.
9. Deepmala and A. K. Das, *On Solvability for Certain Functional Equations Arising in Dynamic Programming*, Mathematics and Computing, Springer Proceedings in Mathematics Statistics, Volume 139 (2015), 79-94.
10. Deepmala and H. K. Pathak,*Some common fixed point theorems for D-operator pair with applications to nonlinear integral equations*, Nonlinear Funct. Anal. Appl., 18 (2013), no. 2, 205-218.
11. Deepmala and H. K. Pathak, *Remarks on occasionally weakly compatible mappings versus occasionally weakly compatible mappings*, Demonstratio Mathematica 47 (3) (2014), 695-703.
12. Deepmala and H. K. Pathak, *Common fixed points for hybrid strict contractions in symmetric spaces under relaxed conditions*, Antarct. J. Math., 10 (6) (2013), 579-588.
13. D. Dorić, Z. Kadelburg and S. Radenović, *A note on occasionally weakly compatible mappings and common fixed point*, Fixed Point Theory 13 (2012), 475-480.
14. M. Imdad, J. Ali and M. Tanveer, *Remarks on some recent metrical common fixed point theorems*, Appl. Math. Lett. 24 (2011), 1165-1169.
15. M. Imdad, M. Tanveer and M. Hassan,*Some common fixed point theorems in Menger PM spaces*, Fixed Point Theory Appl. Vol. 2010, Article ID 819269, 14 pp.
16. M. Imdad, J. Ali and M. Tanveer, *Coincidence and common fixed point theorems for nonlinear contractions in Menger PM spaces*, Chaos, Solitons Fractals 42(5) (2009), 3121-3129.
17. G. Jungck,*Commuting mappings and fixed points*, Amer. Math. Monthly, 83(4) (1976), 261-263.
18. G. Jungck,*Compatible mappings and common fixed points*, Int. J. Math. Math. Sci. 9 (1986), 771-779.
19. G. Jungck and B. E. Rhoades, *Fixed point for set valued functions without continuity*, Indian J. Pure Appl. Math. 29 (3),(1998) 227-238.

20. Kubiaczyk and S. SharmaI, *Some common fixed point theorems in Menger space under strict contractive conditions*, Southeast Asian Bull. Math. 32 (2008), 117-124.

21. S. Kumarand B. D. Pant, *A Common fxed point theorem in probabilistic metric spaces using implicit relation*, Filomat 22(2) (2008), 43-52.

22. K. Menger, *Statistical metrics*, Proc. Nat. Acad. Sci. U.S.A. 28 (1942), 535-537.

23. L. N. Mishra, S. K. Tiwari, V. N. Mishra,and I. A. Khan, *Unique Fixed Point Theorems for Generalized Contractive Mappings in Partial Metric Spaces*, Journal of Function Spaces, Volume 2015 (2015), Article ID 960827, 8 pages.

24. L. N. Mishra, S. K. Tiwari and V. N. Mishra, *Fixed point theorems for generalized weakly S-contractive mappings in partial metric spaces*, Journal of Applied Analysis and Computation, l5(4)(2015), 600-612.

25. B. D. Pant, M. Abbas and S. Chauhan,*Coincidences and common fxed points of weakly compatible mappings in Menger Spaces*, J.Indian Math. Soc. 80(1-4) (2013), 127-139.

26. R. P . Pant,*A common fixed point theorem under a new condition*, Indian. J. Pure Appl.Math. 30 (2) (1999), 147-152.

27. R. P. Pant, R. K. Bisht and D. Arora,*Weak reciprocal continuity and fixed point theorems*, Ann Univ Ferrara ,57,(2011) 181-190.

28. H. K. Pathak and Deepmala,*Some existence theorems for solvability of certain functional equations arising in dynamic programming*, Bull. Cal. Math. Soc. 104 (3) (2012), 237-244.

29. V. Popa, *A general fixed point theorem for four weakly compatible mappings satisfying an implicit relation,* Filomat n 19 (2005), 45-51.

30. B. Schweizerand S. Sklar,*Statistical metric spaces*, Pacific J. Math. 10 (1960) 313-334.

31. S. Sessa, *On a weak commutativity condition of mappings in fixed point considerations*, Publ. Inst. Math. Beograd 32 (46) (1982) 149-153.

32. M. R. Singh and Y. Mahendra Singh, *Compatible mappings of type (E) and common fixed point theorems of Meir-Keeler type,* International J. Math. Sci. Engg. Appl. 1 (2) (2007), 299-315.

33. M. R. Singh and Y. Mahendra Singh, *On various types of compatible maps and common fixed point theorems for non-continuous maps*, Hacet.J.Math.Stat. 40(4) (2011), 503-513.

Said Beloul
Department of Mathematics and Informatics
Faculty of Sciences and Technology
University of Eloued
P.O.Box 789, Eloued 39000, Algeria
beloulsaid@gmail.com