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# REPRODUCED PRINCIPAL IDEAL DOMAIN ON GENERAL HYPERRING $\mathbb{Z}_{p^nq^m}$

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**Abstract.** Every classical algebra is a set equipped with binary operations that operate under certain axiom principles. The generalization of classical algebras to hyperalgebras has been created with the aim of generalizing operations to hyperoperations that apply to specific subject principles. This paper introduces the concept of reproduced general hyperrings as a generalization of rings and investigates and analyzes some of their essential properties. This study defines the notation of reproduced hyperideals in reproduced general hyperrings, consider the ideals of finite rings and obtain the finite and cyclic hyperideals. In the endl, we introduce and show that a principal Ideal domain finite reproduced general hyperring is Ideal-absorbing.
**Keywords**: hyperring, principal ideal domain, axioms.

## 1. Introduction

A ring is an algebraic structure that is equipped with two binary operations and in this regard it can connect two elements to only one element at the same time. From the practical point of view, connecting two elements to one element is a limitation because in practice we may need to connect a group of elements. Because algebraic structures are regular systems and their elements are related under specific subject principles, these structures can have many applications in the real world. Therefore, developing and removing the limitations of algebraic structures such as rings is

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very important. A hyperring is just a ring, that is equipped with a hyperaddition, and hyperrings are considered in spaces of signs, also known as abstract real spectra and objects which arise naturally in the study of constructible sets in real geometry. Indeed, hyperrings as a generalization of rings are equipped with two hyperoperations and operation or two hyperoperations. Since hyperoperations are maps with a nonempty set range, can be helpful in the application of a group of elements. The theory of hyperrings as a generalization of rings can be considered as a type of elimination of the limitation of the connection of elements under the principles of a special subject. The concept of Krasner hyperring was introduced by Krasner [14], who used it as a tool for the approximation of valued fields or the second type of a hyperring as multiplicative hyperring (the multiplication is a hyperoperation, while the addition is an operation was introduced by R. Rota in 1982 [17]. Today, some researchers have investigated some works in hyperrings such as a study on special kinds of derivations in ordered hyperrings [16], the reducibility concept in general hyperrings [6], regular parameter elements and regular local hyperrings [4], hyperideals of (finite) general hyperrings [2], direct limit of Krasner $(m, n)$-hyperrings [1], a generalization of graded prime hyperideals over graded multiplicative hyperrings [11], extended centroid of hyperrings [18], weakly $(k, n)$-absorbing (primary) hyperideals of a Krasner $(m, n)$-hyperring [8] and contribution to study special kinds of hyperideals in ordered semihyperrings [15]. Fundamental relations are basic tools in algebraic hyperstructures theory and some researchers worked on fundamental relations of hyperrings such as Boolean rings based on hyperrings [3], commutative rings obtained from hyperrings (Hv-rings) with $\alpha^*$–relations [9], Boolean rings obtained from hyperrings with $\eta^*_{1,m}$ relations [10], fundamental relation and automorphism group of very thin $H_v$–groups [12], height of prime hyperideals in Krasner hyperrings[5] and The fundamental Relations in Hyperrings [19]. Hamidi et al. constructed multigroups and hyperrings on every non–empty set, introduced and analyzed a special relation on hyperrings and extended it to the smallest strongly regular equivalence binary relation in such a way that the quotient of each given hyperring on this relation is a commutative Boolean ring with identity. They try to generalize the concept of rings to general hyperrings, to describe some of their properties and the differences between hyperrings and general hyperrings.

**Motivation and advantage:** Algebraic structures as one of the important branches of mathematics have many applications in the real world. These structures as an algebraic system equipped with several algebraic operations can be used as a mathematical model. In algebraic structures, under each operation, only two elements can be equalized to one element, and this limits the algebraic structures. Of course, this limitation can be overcome and covered by generalizing algebraic structures to algebraic superstructures. The advantage of algebraic superstructures is that, in addition to covering algebraic structures, they can relate both elements to a set of elements. This advantage allows us to connect a network of elements in the modeling of real-world problems. In this research, by developing rings into hyperrings within the context of a ring, we create a new achievement in hybrid substructures.

This paper introduces and works on the construction of reproduced general hyperrings and shows that this class of hyperstructures has some identity elements while having a unique zero element. It is natural to question as to what are the relationships between elements whence are considered in the same set concerning algebraic operations. Since any operation at most connects three elements, we need to extend more elements in defined axioms. It motivates us to introduce the concept of two algebraic hyperoperations in an underlying set. So the main motivation is to introduce some identity elements concerning algebraic hyperproducts and to consider the differences between other hyperstructures and structures. We obtained some theorems and corollaries that in special conditions are similar to corresponded theorems in (non-associative)rings, so we conclude that reproduced general hyperrings are a generalization of (non-associative)rings. Also, the concept of reproduced ideals is presented in this work and we analyze the hyperideals on principal ideal domain reproduced general hyperideals.

### 1.1. Preliminaries

In this section, we review some definitions and results from hyperstructures from [7, 13], which we need in what follows. Let $R$ be a nonempty set, $\mathcal{P}^*(R) = \{S \mid \emptyset \neq\subseteq R\}$ and $\varrho = \{(x, X) \mid x \in R, X \in \mathcal{P}^*(R)\}$ be a map. Then $\varrho$ is called a *hyperoperation* (*hypercomposition*), an *algebraic hypercompositional structure* $(R, \varrho)$ is called a *hypergroupoid* and for all nonempty subsets $S, T$ of $R$, $\varrho(S, T) = \bigcup\limits_{s\in S, t\in T} \varrho(s, t)$. An algebraic hypercompositional structure $(R, \varrho)$, where $\varrho$ is a binary hyperoperation, is called a *hypergroupoid* and a recall that a *hypergroupoid* $(R, \varrho)$ is called a *semihypergroup*, if for all $x, y, z \in R$, $\varrho(\varrho(x, y), z) = \varrho(x, \varrho(y, z))$ and a semihypergroup $(R, \varrho)$ is called a *hypergroup*, if for all $x \in R$, $\varrho(x, R)R = \varrho(R, x)$(*reproduction axiom*). A *general hyperring* is an algebraic hypercompositional structure $(R, \varrho, \varsigma)$, where $(i)$ $(R, \varrho)$ is a hypergroup, $(ii)$ $(R, \varsigma)$ is a semihypergroup and $(iii)$ for any $x, y, z \in R$: $\varsigma(x, \varsigma(y, z)) \subseteq \varrho(\varsigma(x, y), \varsigma(x, z))$ and $\varsigma(\varrho(x, y), z) \subseteq \varrho(\varsigma(x, z), \varsigma(y, z)$. A general hyperring $(R, \varrho, \iota, \varsigma)$ is called *commutative* (with unit element), if for all $x, y \in R$, $\varsigma(x, y) = \varsigma(y, x)$ (if there exists an element $\epsilon \in R$ such that for all $x \in R, \varsigma(\epsilon, x) = \varsigma(x, \epsilon) = \{x\}$). A nonempty subset $I$ of $R$ is called a (*right*)*left hyperideal*, if (1), $(I, \varrho)$ is a hypergroup and (2), $(\varsigma(R, I) \subseteq I)(\varsigma(I, R) \subseteq I$. A hyperideal $I$ is a both left and right hyperideal.

## 2. Hyperideals of general hyperrings

In this section, we apply the structure of rings and extend them to general hyperrings. Also the concept of reproduced ideals is introduced and investigated.

**Definition 2.1.** Let $(R, +, \cdot)$ be a ring. Then $R$ is said to be a $(\varrho, \varsigma)$-reproduced general hyperring, if there are hyperoperations "$\varrho$" and "$\varsigma$", that $(R, \varrho, \varsigma)$ is a general hyperring and $\varrho, \varsigma$ are are dependent to $+$ and $\cdot$, respectively.

**Theorem 2.1.** *Assume* $k \in \mathbb{N}$. *Then* $(\mathbb{Z}_{2k}, +, \cdot)$ *is a* $(\varrho, \varsigma)$*-reproduced general hyperring.*

*Proof.* Fix $\overline{0} \neq \overline{a} \in \mathbb{Z}_{2k}$, where $\overline{2a} = \overline{0}$. Clearly, $(\mathbb{Z}_{2k}, \varrho)$ is a hypergroup, where for any $\overline{x}, \overline{y} \in \mathbb{Z}_{2k}$, $\varrho(\overline{x}, \overline{y}) = \{\overline{x+y}, \overline{x+y+a}\}$. Now for any $\overline{x}, \overline{y} \in \mathbb{Z}_{2k}$, define $\varsigma(\overline{x}, \overline{y}) = \{\overline{xy}, \overline{xy+a}\}$. Simple computations show that $(\mathbb{Z}_{2k}, \varrho, \varsigma)$ is a general hyperring. □

**Example 2.1.** By Theorem 2.1, $(R = \{a, b, c, d, e, f\}, \varrho, \varsigma)$ is a $(\varrho, \varsigma)$-reproduced general hyperring by the following hyperoperations:

| $\varrho$ | $a$ | $b$ | $c$ | $d$ | $e$ | $f$ |
|---|---|---|---|---|---|---|
| $a$ | $\{a,d\}$ | $\{b,e\}$ | $\{c,f\}$ | $\{d,a\}$ | $\{e,b\}$ | $\{f,c\}$ |
| $b$ | $\{b,e\}$ | $\{c,f\}$ | $\{d,a\}$ | $\{e,b\}$ | $\{f,c\}$ | $\{a,d\}$ |
| $c$ | $\{c,f\}$ | $\{d,a\}$ | $\{e,b\}$ | $\{f,c\}$ | $\{a,d\}$ | $\{b,e\}$ |
| $d$ | $\{d,a\}$ | $\{e,b\}$ | $\{f,c\}$ | $\{a,d\}$ | $\{b,e\}$ | $\{c,f\}$ |
| $e$ | $\{e,b\}$ | $\{f,c\}$ | $\{a,d\}$ | $\{b,e\}$ | $\{c,f\}$ | $\{d,a\}$ |
| $f$ | $\{f,c\}$ | $\{a,d\}$ | $\{b,e\}$ | $\{c,f\}$ | $\{d,a\}$ | $\{b,e\}$ |

,

| $\varsigma$ | $a$ | $b$ | $c$ | $d$ | $e$ | $f$ |
|---|---|---|---|---|---|---|
| $a$ | $\{a,d\}$ | $\{a,d\}$ | $\{a,d\}$ | $\{a,d\}$ | $\{a,d\}$ | $\{a,d\}$ |
| $b$ | $\{a,d\}$ | $\{b,e\}$ | $\{c,f\}$ | $\{d,a\}$ | $\{e,b\}$ | $\{f,c\}$ |
| $c$ | $\{a,d\}$ | $\{c,f\}$ | $\{e,b\}$ | $\{a,d\}$ | $\{c,f\}$ | $\{e,b\}$ |
| $d$ | $\{d,a\}$ | $\{a,d\}$ | $\{d,a\}$ | $\{a,d\}$ | $\{d,a\}$ | $\{a,d\}$ |
| $e$ | $\{a,d\}$ | $\{e,b\}$ | $\{c,f\}$ | $\{a,d\}$ | $\{e,b\}$ | $\{c,f\}$ |
| $f$ | $\{a,d\}$ | $\{f,c\}$ | $\{e,b\}$ | $\{d,a\}$ | $\{c,f\}$ | $\{b,e\}$ |

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**Theorem 2.2.** *Let $p$ be a prime and $k \in \mathbb{N}$. Then $\mathbb{Z}_{p^k} \cup \{\sqrt{p}\}$ is a $(\varrho_{\sqrt{p}}, \varsigma_{\sqrt{p}})$-reproduced general hyperring.*

*Proof.* Let $x, y \in \mathbb{Z}_{p^k} \cup \{\sqrt{p}\}$. Define

$$\varrho_{\sqrt{p}}(x,y) = \varrho_{\sqrt{p}}(y,x) = \begin{cases} \{0, \sqrt{p}\} & x = -y \text{ or } x = y = \sqrt{p}, \\ x + y & x, y \in \mathbb{Z}_{p^k}, x \neq -y, \\ y & x = 0 \text{ or } (\ x = \sqrt{p} \text{ and } y \notin \{0, \sqrt{p}\}) \end{cases}$$

and

$$\varsigma_{\sqrt{p}}(x,y) = \varsigma_{\sqrt{p}}(y,x) = \begin{cases} x.y & x, y \in \mathbb{Z}_{p^k}, \\ \sqrt{p} & x \in \mathbb{Z}_{p^k} \smallsetminus \{mp\}, y = \sqrt{p} (m \in \mathbb{N}), \\ 0 & x = mp, y = \sqrt{p} (m \in \mathbb{N}), \\ \{0, \sqrt{p}\} & x = y = \sqrt{p}. \end{cases}$$

Computations show that $(\mathbb{Z}_{p^k} \cup \{\sqrt{p}\}, \varrho_{\sqrt{p}}, \varsigma_{\sqrt{p}})$ is a general hyperring. □

**Example 2.2.** By Theorem 2.1, $(\mathbb{Z}_4 \cup \{\sqrt{2}\}, \varrho, \varsigma)$ is a $(\varrho, \varsigma)$-reproduced general hyperring by the following hyperoperations:

| $\varrho$ | $0$ | $1$ | $2$ | $3$ | $\sqrt{2}$ |
|---|---|---|---|---|---|
| $0$ | $0$ | $1$ | $2$ | $3$ | $\sqrt{2}$ |
| $1$ | $1$ | $2$ | $3$ | $\{0, \sqrt{2}\}$ | $1$ |
| $2$ | $2$ | $3$ | $\{0, \sqrt{2}\}$ | $1$ | $2$ |
| $3$ | $3$ | $\{0, \sqrt{2}\}$ | $1$ | $2$ | $3$ |
| $\sqrt{2}$ | $\sqrt{2}$ | $1$ | $2$ | $3$ | $\{0, \sqrt{2}\}$ |

,

| $\varsigma$ | 0 | 1 | 2 | 3 | $\sqrt{2}$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 2 | 3 | $\sqrt{2}$ |
| 2 | 0 | 2 | 0 | 2 | 0 |
| 3 | 0 | 3 | 2 | 1 | $\sqrt{2}$ |
| $\sqrt{2}$ | 0 | $\sqrt{2}$ | 0 | $\sqrt{2}$ | $\{0, \sqrt{2}\}$ |

.

## 2.1. On hyperideals

Now, we present hyperideals of a general hyperring. In particular, we determine hyperideals of finite commutative general hyperrings.

**Definition 2.2.** Let $(R, \varrho, \varsigma)$ be a general hyperring and $\emptyset \neq I \subseteq R$. We say

$(i)$ $I$ is a general subhyperring of $R$, if $(I, \varrho, \varsigma)$ is a general hyperring;

$(ii)$ $I$ is a hyperideal of $R$, if $\varsigma(R, I) \cup \varsigma(I, R) \subseteq I$.

**Theorem 2.3.** *Suppose $(R, +, \cdot)$ is a general hyperring and $\emptyset \neq I \subseteq R$. Then $I$ is a hyperideal of $R$ if and only if the following hold:*

$(i)$ *for any $x \in I, \varrho(x, I) = \varrho(I, x) = I$;*

$(ii)$ *for any $r \in R$ and $x \in I$, we have $\varsigma(r, x) \cup \varsigma(x, r) \subseteq I$.*

*Proof.* Immediate by definition. □

**Theorem 2.4.** *Let $(R, \varrho, \varsigma)$ be a general hyperring and $I$ be a hyperideal of $R$. Then*

$(i)$ *$\forall\, r \in R, x \in I, n \in \mathbb{N}$, we have* $\varrho\underbrace{(\varsigma(r, x), \varsigma(r, x), \ldots \varsigma(r, x))}_{n\ times} \subseteq I$;

$(iv)$ *if $x \in I$, then $\varsigma(x) \in I$.*

*Proof.* Immediate. □

Assume $(R, +, \cdot)$ is a general hyperring. We symbolize the set hyperideals of $R$ by $\mathcal{I}(R)$. Clearly, $R \in \mathcal{I}(R) \neq \emptyset$ and will call $R$ as a non-proper hyperideal of any general hyperring.

**Example 2.3.** Let $R = \{e, \iota, a, b\}$. Then $(R, \varrho, \varsigma)$ is a general hyperring as follows.

| $\varrho$ | $e$ | $\iota$ | $a$ | $b$ |
|---|---|---|---|---|
| $e$ | $e$ | $\iota$ | $a$ | $b$ |
| $\iota$ | $\iota$ | $R$ | $\{\iota, a\}$ | $\{\iota, b\}$ |
| $a$ | $a$ | $\{\iota, a\}$ | $R$ | $\{a, b\}$ |
| $b$ | $b$ | $\{\iota, b\}$ | $\{a, b\}$ | $R$ |

*and*

| $\varsigma$ | $e$ | $\iota$ | $a$ | $b$ |
|---|---|---|---|---|
| $e$ | $e$ | $e$ | $e$ | $e$ |
| $\iota$ | $e$ | $\iota$ | $a$ | $b$ |
| $a$ | $e$ | $a$ | $a$ | $a$ |
| $b$ | $e$ | $b$ | $a$ | $\{a, b\}$ |

.

Then $\mathcal{I}(R) = \{I = \{e\}, J = R\}$.

**Theorem 2.5.** *Assume $(R,+,\cdot)$ is a commutative general hyperring and $I, I' \in \mathcal{I}(R)$. Then*

*(i)* $\varrho(I, I') \in \mathcal{I}(R)$.

*(ii)* *if* $I \cap I' \neq \emptyset$, *then* $I \cap I' \in \mathcal{I}(R)$.

*Proof.* $(i)$ Clearly, $\varrho(I, I') \neq \emptyset$. Let $a \in I$ and $a' \in I'$. Then for any $z \in \varrho(\varrho(a,a'), \varrho(I+I'))$, there is $b \in I, b' \in I'$, that $z \in \varrho(\varrho(a+a'), \varrho(b,b')) = \varrho(\varrho(a,b), \varrho(a',b')) \subseteq \varrho(I, I')$. If $z \in \varrho(I, I')$ be an arbitrary element in $\varrho(I, I')$, then there are $a, b, c \in I$, and $a', b', c' \in I'$, that $z \in \varrho(c, c') \subseteq \varrho(\varrho(a,b), \varrho(a',b')) = \varrho((a,a'), \varrho(b,b')) \subseteq \varrho(\varrho(a,a'), \varrho(I, I'))$. Hence $\varrho(\varrho(a,a'), \varrho(I,I')) = \varrho(I, I')$. Now, for any $r \in R, a \in I$ and $a' \in I'$, one obtains $\varsigma(r, \varrho(a,a')) \cup \varsigma(\varrho(a,a'), r) = \varrho(\varsigma(r,a), \varsigma(r,a'))) \cup \varrho(\varsigma(a,r), \varsigma(a',r)) = \varrho(\varsigma(r,a), \varsigma(r,a')) \subseteq \varrho(I, I')$. Hence $\varrho(I, I') \in \mathcal{I}(R)$.

$(ii)$ Since $I \cap I' \subseteq I$, we get that $I \cap I' \in \mathcal{I}(R)$. □

**Theorem 2.6.** *Assume $(R,+,\cdot)$, $(S,+,\cdot)$ are general hyperrings, $f: R \to S$ be a homomorphism, and $I \in \mathcal{I}(R)$ and $J \in \mathcal{I}(S)$.*

*(i)* *If $f$ is an epimorphism, then* $f(I) \in \mathcal{I}(S)$.

*(ii)* $f^{-1}(J) \in \mathcal{I}(R)$.

*Proof.* $(i)$ Since $\emptyset \neq I$, we have $f(I) \neq \emptyset$. Let $f(a) \in f(I)$. Then for every $f(b) \in f(I)$, there is $a' \in I$, that $b \in \varrho(a,a')$, and so $f(b) \in f(\varrho(a,a')) = \varrho(f(a), f(a')) \subseteq \varrho(f(a), I)$. Hence $f(I) \subseteq \varrho(f(a), f(I))$. If $c \in \varrho(f(a), f(I))$ is an arbitrary element, then there is $a' \in I$, that $c \in \varrho(f(a), f(a')) = f(\varrho(a,a')) \subseteq f(I)$. Hence, $\varrho(f(a), f(I)) = f(I)$. Now, for any $s \in S$ and $f(a) \in f(I)$, there is $r \in R$, that

$$\begin{aligned} \varsigma(s, f(a)) \cup \varsigma(f(a), s) &= \varsigma(f(r), f(a)) \cup \varsigma(f(a), f(r)) = (f(\varsigma(r,a))) \cup (f(\varsigma(a,r))) \\ &= f(\varsigma(r,a) \cup \varsigma(a,r)) \subseteq f(I). \end{aligned}$$

$(ii)$ It is straightforward. □

### 2.2. Reproduced ideals in reproduced general hyperring $(\mathbb{Z}_n, \varrho, \varsigma)$

In this subsection, all reproduced ideals of finite reproduced general hyperring $R = (\mathbb{Z}_n, \varrho, \varsigma)$ are computed and it is proved that every reproduced ideal of the reproduced general hyperring $(\mathbb{Z}_n, \varrho, \varsigma)$ is characterized by the divisors of $n$.

**Definition 2.3.** Let $(R,+,\cdot)$ be a ring and $I \subseteq R$ be an ideal of $R$. We will call $I$ as a reproduced ideal of $(\varrho, \varsigma)$-reproduced general hyperring $(R, \varrho, \varsigma)$, if $I$ is extended to a hyperideal of $(R, \varrho, \varsigma)$. We will denote the $\mathcal{RI}$ by the set of all reproduced ideals of $(\varrho, \varsigma)$-reproduced general hyperring $(R, \varrho, \varsigma)$.

**Theorem 2.7.** *Let $n, d \in \mathbb{N}$ and $\overline{x}, \overline{y} \in R$. Then*

(*i*) $\langle \overline{x} \rangle \in \mathcal{I}(\mathbb{Z}_n, \varrho, \varsigma)$,

(*ii*) $\langle \overline{0} \rangle = \{\overline{0}\}$,

(*iii*) $\langle \overline{x} \rangle = \langle \overline{y} \rangle \Leftrightarrow \gcd(x, n) = \gcd(y, n) = d$.

*Proof.*

(*i*) Let $\overline{x} \in R$. By definition, we have $\langle \overline{x} \rangle = \bigcup_{k \in \mathbb{N}} \{\overline{kx}, \overline{0}\}$ and show that it is a hyperideal of $R$. Let $\overline{y} \in \langle \overline{x} \rangle$ and $\overline{z} \in \varrho(\overline{y}, \langle \overline{x} \rangle)$. Thus there is $k, k' \in \mathbb{N}$ and $\overline{w} \in \langle \overline{x} \rangle$ that $\overline{z} \in \varrho(\overline{y}, \overline{w})$ and so $\overline{z} \in \varrho(\overline{kx}, \overline{k'x}), \overline{z} \in \overline{kk'x}$ and $\overline{z} \in \langle \overline{0} \rangle$. There is $k'' \in \mathbb{N}$ that $\overline{z} \in \{\overline{k''x}, \overline{0}\} \subseteq \langle \overline{x} \rangle$. In a similar way, for any $\overline{r} \in \mathbb{Z}_n$ and $\overline{y} \in \langle \overline{x} \rangle$, we have $\varrho(\overline{r}, \langle \overline{x} \rangle) \subseteq \langle \overline{x} \rangle$. Hence $\langle \overline{x} \rangle \in \mathcal{I}(\mathbb{Z}_n, \varrho, \varsigma)$.

(*ii*) One can see that $\langle \overline{0} \rangle = \bigcup_{k \in \mathbb{N}} \{\overline{k0}, \overline{0}\} = \{\overline{0}\}$.

(*iii*) Let $\overline{z} \in \langle \overline{x} \rangle$. Then there is $k \in \mathbb{N}$, that $\overline{z} = \overline{kx}$ or $\overline{z} = \overline{0}$. Since $\gcd(x, n) = d$ and by item (*i*), there is $k' \in \mathbb{Z}$, that $x = k'd$. If $\overline{z} = \overline{kx}$, then $\overline{z} = \overline{kx} = \overline{kk'd} \in \langle \overline{x} \rangle$ and if $\overline{z} = \overline{0}$, then $\overline{z} = \overline{k0} = \overline{0} \in \langle \overline{x} \rangle$. Hence $\langle \overline{x} \rangle \subseteq \langle \overline{d} \rangle$. Let $\overline{z} \in \langle \overline{d} \rangle$. Then there is $k \in \mathbb{N}$, that $\overline{z} = \overline{kd}$ or $\overline{z} = \overline{0}$. Since $\gcd(x, n) = d$ and by item (*i*), there is $r, s \in \mathbb{Z}$ that $rx + ns = d$, and so $\overline{rkx} + \overline{nks} = \overline{kd}$. Applying Theorem 2.1, we get that $\overline{z} = \overline{krx}$ or $\overline{z} = \overline{0}$. Hence $\langle \overline{d} \rangle \subseteq \langle \overline{x} \rangle$. Also for $\gcd(y, n) = d$ the proof is similarly, then $\langle \overline{d} \rangle = \langle \overline{y} \rangle$, there for $\langle \overline{x} \rangle = \langle \overline{y} \rangle$. $\square$

**Example 2.4.** Consider the general hyperring $(\mathbb{Z}_{100}, \varrho, \varsigma)$. By Theorem 2.7, we have the reproduced ideals of $R = (\mathbb{Z}_{100}, \varrho, \varsigma)$ as follows:

$$\begin{aligned}\mathcal{RI}(\mathbb{Z}_{100}, \varrho, \varsigma) = \{&I_1 = \{\overline{0}\}, I_2 = \{\overline{0}, \overline{2}, \overline{4}, ..., \overline{98}\}, I_3 = \{\overline{0}, \overline{4}, \overline{8}, ..., \overline{96}\},\\ &I_4 = \{\overline{0}, \overline{5}, \overline{10}, \overline{15}, \ldots, \overline{95}\}, I_5 = \{\overline{0}, \overline{25}, \overline{50}\}, I_6 = \{\overline{0}, \overline{10}, \overline{20}, \ldots, \overline{90}\},\\ &I_7 = \{\overline{0}, \overline{50}\}, I_8 = \{\overline{0}, \overline{20}, \ldots, \overline{80}\}, I_9 = \mathbb{Z}_{45}\}.\end{aligned}$$

**Theorem 2.8.** *Let $n \in \mathbb{N}$. Then*

(*i*) $|\mathcal{RI}(\mathbb{Z}_n, \varrho, \varsigma)| = |Div(n)|$.

(*ii*) *for any* $\overline{x}, \overline{y} \in R, \langle \overline{x} \rangle \cap \langle \overline{y} \rangle = \langle \overline{lcm(x, y)} \rangle$.

*Proof.* (*i*) By Theorem 2.7, $I \in \mathcal{RI}$ if and only if there is $s\overline{x} \in R$, that $I = \langle \overline{x} \rangle$. Also for any $\overline{x} \in R, gcd(x, n) = d$ if and only if $\langle \overline{x} \rangle = \langle \overline{d} \rangle$. Thus $|\mathcal{RI}(\mathbb{Z}_n, \varrho, \varsigma)| = |Div(n)|$.

(*ii*) Let $\overline{x} \in R$. By definition, we have $\langle \overline{x} \rangle = \bigcup_{k \in \mathbb{N}} \{\overline{kx}, 0\}$. Clearly, there is $k_1, k_2 \in \mathbb{N}$ that $lcm(x, y) = k_1 x$ and $lcm(x, y) = k_2 y$. Hence $\overline{lcm(x, y)} \in \langle \overline{x} \rangle \cap \langle \overline{y} \rangle$ and so $\langle \overline{lcm(x, y)} \rangle \subseteq \langle \overline{x} \rangle \cap \langle \overline{y} \rangle$. Conversely. let $\overline{a} \in \langle \overline{x} \rangle \cap \langle \overline{y} \rangle$. Then $\overline{a} = \overline{0}$ or there is $k_1, k_2 \in \mathbb{N}$ that $\overline{a} = \overline{k_1 x} = \overline{k_2 y}$. Thus $x \mid a$ and $y \mid a$ and so $lcm(x, y) \mid a$. Hence there is $k \in \mathbb{N}$ that $\overline{a} = \overline{k \times lcm(x, y)}$ and so $\langle \overline{a} \rangle \subseteq \langle \overline{x} \rangle \cap \langle \overline{y} \rangle$. $\square$

**Corollary 2.1.** *Let $n \in \mathbb{N}$. Then $\mathcal{RI}(\mathbb{Z}_n, \varrho, \varsigma) = \{\langle \overline{d} \rangle \mid d \in Div(n)\}$.*

**Example 2.5.** Let $p, q, r$ be primes, $m, l, k \in \mathbb{N}$ and $n = p^m q^l r^k$. Then

$$\mathcal{I}(\mathbb{Z}_n, \varrho, \varsigma) = \{\langle \overline{p^{t_1} q^{t_2} r^{t_3}} \rangle \mid 0 \leq t_1 \leq m, 0 \leq t_2 \leq l, 0 \leq t_3 \leq k\}\rangle.$$

**Corollary 2.2.** *Assume $p_1, p_2, \ldots, p_k$ are primes, $k, \alpha_1, \alpha_2, \ldots, \alpha_k \in \mathbb{N}$ and $n = \prod_{i=1}^{k} p_i^{\alpha_i}$. Then $|\mathcal{RI}(\mathbb{Z}_n, \varrho, \varsigma)| = \sum_{i=1}^{k} (\alpha_i + 1)$.*

### 2.3. Reproduced ideals in $(\mathbb{Z}_{p^k} \cup \{\sqrt{p}\}, \varrho_{\sqrt{p}}, \varsigma_{\sqrt{p}},)$

In this subsection, all reproduced ideals of finite reproduced general hyperring $(\mathbb{Z}_{p^k} \cup \{\sqrt{p}\}, \varrho_{\sqrt{p}}, \varsigma_{\sqrt{p}},)$ are computed and it is proved that every reproduced ideal of the reproduced general hyperring $(\mathbb{Z}_{p^k} \cup \{\sqrt{p}\}, \varrho_{\sqrt{p}}, \varsigma_{\sqrt{p}},)$ is characterized by the divisors of $n$.

**Theorem 2.9.** *Let $p$ be a prime. Then in reproduced general hyperring $(\mathbb{Z}_p \cup \{\sqrt{p}\}, \varrho_{\sqrt{p}}, \varsigma_{\sqrt{p}})$, we have $\mathcal{RI}(R) = \{R, \{\overline{0}\}, \{\overline{0}, \sqrt{p}\}\}$;*

*Proof.* Let $I \in \mathcal{RI}(R) \setminus \{\{\overline{0}\}, R\}$. Since $\emptyset \neq I$ is a hyperideal of $R$, there exists $a \in I$ and so $\{a, 2a, 3a, \ldots, (p-1)a, \overline{0}\} \subseteq I$. In addition, $\forall\ r \neq \sqrt{p}$ we have $\upsilon_{\sqrt{p}}(r, \{a, 2a, 3a, \ldots, (p-1)a, \overline{0}\}) \subseteq \{a, 2a, 3a, \ldots, (p-1)a, \overline{0}\}$. Also for $r = \sqrt{p}$, we have $\upsilon_{\sqrt{p}}(r, \{a, 2a, 3a, \ldots, (p-1)a, \overline{0}\}) \subseteq \{\sqrt{p}, \overline{0}\}$. Thus $I = \{\sqrt{p}, \overline{0}\}$. □

**Theorem 2.10.** *Let $p$ be a prime and $k \in \mathbb{N}$. Then in reproduced general hyperring $(\mathbb{Z}_{p^k} \cup \{\sqrt{p}\}, \varrho_{\sqrt{p}}, \varsigma_{\sqrt{p}})$, we have*

- $(i)$ *if $I$ is a nontrivial hyperideal of $R$, then $\sqrt{p} \in I$;*
- $(ii)$ *$\forall\ 1 \leq m \leq p^{k-1}$, $I_p^{(m)} = \{m\overline{p}, 2m\overline{p}, \ldots, tm\overline{p}, \sqrt{p} \mid t \in \mathbb{N}$ is the smallest s.t $tm \equiv 0 \pmod{p^{k-1}}\}$ is a hyperideal of $R$.*

*Proof.* $(i)$ Let $\overline{0} \neq x \in I$. Since $I$ is a hyperideal of $R$ and $\sqrt{p} \in R$, we get that $\upsilon_{\sqrt{p}}(\sqrt{p}, x) \subseteq I$. On other hand $\forall\ x \in I, \varsigma_{\sqrt{p}}(\sqrt{p}, x) = \overline{0}, \sqrt{p}$ or $\{\overline{0}, \sqrt{p}\}$. If $\upsilon_{\sqrt{p}}(\sqrt{p}, x) = \overline{0}$, then by definition there exists $m \in \mathbb{N}$ that $x = mp$. Hence there is $n \in \mathbb{N}$ that $\{\overline{0}, \sqrt{p}\} = \varrho_{\sqrt{p}} \underbrace{(x, x, \ldots x)}_{n\ times} \subseteq I$ and so in any case $\sqrt{p} \in I$.

$(ii)$ Let $1 \leq m \leq p^{k-1}$ and $x, y \in I_p^{(m)} \smallsetminus \{\sqrt{p}\}$. Then there exists $1 \leq k_1, k_2 \leq t \in \mathbb{N}$ that $x + y = (k_1 + k_2)(m\overline{p}) \subseteq I_p^{(m)}$, because of $\overline{0} \leq (k_1 + k_2)(m\overline{p}) \leq \overline{p^{k-1}}$. In addition $\forall\ \overline{x} \in I_p^{(m)}$, $\varrho_{\sqrt{p}}(\sqrt{p}, \overline{x}) = \{\overline{x}\} \subseteq I_p^{(m)}$ and $\varrho_{\sqrt{p}}(\sqrt{p}, \sqrt{p}) = \{\overline{0}, \sqrt{p}\} \subseteq I_p^{(m)}$, imply that $\forall\ x, y \in I_p^{(m)}$, $\varrho_{\sqrt{p}}(x, y) \subseteq I_p^{(m)}$. Also $\forall\ r \in R \smallsetminus \{\sqrt{p}\}$ and $x \in I_p^{(m)} \smallsetminus \{\sqrt{p}\}$ there exists $1 \leq k \leq t \in \mathbb{N}$ that $\upsilon_{\sqrt{p}}(r, x) = rk(m\overline{p}) \subseteq I_p^{(m)}$, because of $\overline{0} \leq (rkm)\overline{p} \leq \overline{p^{k-1}}$. On the other hand, $\upsilon_{\sqrt{p}}(\sqrt{p}, \overline{x}) \subseteq \{\overline{0}, \sqrt{p}\}$, implies that $\forall\ r \in R$ and $x \in I_p^{(m)}$, we have $\upsilon_{\sqrt{p}}(r, x) \subseteq I_p^{(m)}$. □

**Theorem 2.11.** *Let $p$ be a prime and $k \in \mathbb{N}$. Then in reproduced general hyperring $(\mathbb{Z}_{p^k} \cup \{\sqrt{p}\}, \varrho_{\sqrt{p}}, \varsigma_{\sqrt{p}})$, we have*

(*i*) $\forall\ 1 \leq m \leq p^{k-1}$, *we have* $|I_p^{(m)}| = 1 + \dfrac{p^{k-1}}{\gcd(m, p^{k-1})}$;

(*ii*) $\forall\ 1 \leq m, m' \leq p^{k-1}$, $I_p^{(m)} = I_p^{(m')}$ *if and only if* $\gcd(p^{k-1}, m) = \gcd(p^{k-1}, m')$.

*Proof.* (*i*) Let $1 \leq m \leq p^{k-1}$. Using Theorem 2.10 (*i*), $\sqrt{p} \in I_p^m$, so $|I_p^m| = 1 + |\{t \in \mathbb{N} \mid t$ is the smallest s.t $tm \equiv 0 \pmod{p^{k-1}}\}| = q$. Suppose $t \in \mathbb{Z}$ is the smallest that $tm \equiv 0 \pmod{p^{k-1}}$. Thus $p^{k-1} \mid tm$. If $\gcd(p^{k-1}, m) = 1$, then $p^{k-1} \mid t$ and because $t$ is the smallest, we obtain that $t = p^{k-1}$. But for $\gcd(p^{k-1}, m) = d \neq 1$, have $\dfrac{p^{k-1}}{d} \mid t$. Since $p^{k-1}m \equiv 0 \pmod{p^{k-1}}$ and $t \in \mathbb{N}$ is the smallest that $tm \equiv 0 \pmod{p^{k-1}}$, we get that $\dfrac{p^{k-1}}{\gcd(m, p^{k-1})} = t$.

(*ii*) Let $1 \leq m, m' \leq p^{k-1}$. Then by item (*ii*), $I_p^{(m)} = I_p^{(m')}$ if and only if $1 + \dfrac{p^{k-1}}{\gcd(m, p^{k-1})} = 1 + \dfrac{p^{k-1}}{\gcd(m', p^{k-1})} \iff \gcd(p^{k-1}, m) = \gcd(p^{k-1}, m')$. □

**Theorem 2.12.** *Let $p$ be a prime, $k \in \mathbb{N}$ and $1 \leq j \leq p^{k-1}$. Then in reproduced general hyperring $(\mathbb{Z}_{p^k} \cup \{\sqrt{p}\}, \varrho_{\sqrt{p}}, \varsigma_{\sqrt{p}})$, we have*

(*i*) $I_p^{(p^i)} = I_p^{(rp^i)}$, *where* $rp^i \neq p^j$;

(*ii*) $\forall\ 1 \leq m \leq p^{k-1}$, $I_p^{(p^{k-1})} \subseteq I_p^{(m)}$;

(*iii*) $I_p^{(p^{k-1})} \subseteq I_p^{(p^{k-2})} \subseteq I_p^{(p^{k-3})} \subseteq I_p^{(p^{k-4})} \subseteq \ldots \subseteq I_p^{(p)}$.

*Proof.* The proof is similar to Theorem 2.11. □

**Theorem 2.13.** *Let $p$ be a prime and $k \in \mathbb{N}$. Then in reproduced general hyperring $(\mathbb{Z}_{p^k} \cup \{\sqrt{p}\}, \varrho_{\sqrt{p}}, \varsigma_{\sqrt{p}})$, we have*

(*i*) $\mathcal{RI}(R) = \{R, \{\overline{0}\}, I_p^{(m)} \mid 1 \leq m \leq p^{k-1}\}$;

(*ii*) $|\mathcal{RI}(R)| = k$.

*Proof.* (*i*) Clearly $R, \{\overline{0}\} \in \mathcal{RI}(R)$. Let $I$ be a nontrivial hyperideal of $R$, using Theorem 2.10 (*i*), $\overline{0}, \sqrt{p} \in I$. Suppose that $0 \neq a \in I$. If $\gcd(a, p^k) = 1$, then there exist $s, s' \in \mathbb{Z}$ that $1 = as + s'p^k$. It follows that $\overline{1} \in I$ and we get that $R = I$. But for $\gcd(a, p^k) = d \neq 1$, since $p$ is a prime, there exist $1 \leq i \leq k$ in such a way that $d = p^i$, consequently $p^i \in I$.

(*ii*) It is immediate by (*i*). □

**Definition 2.4.** Let $R$ be a reproduced general hyperring and $M \neq R$ be an arbitrary hyperideal of $R$.

(*i*) $M$ is called a maximal hyperideal of $R$, if the only reproduced hyperideals containing $M$ are $M$ and $R$;

(*ii*) $M$ is called a reproduced prime hyperideal of $R$, $\forall\ a, b \in R, \varsigma(a, b) \subseteq M$ implies that $a \in M$ or $b \in M$.

**Theorem 2.14.** *Let $p$ be a prime and $k \in \mathbb{N}$. Then in reproduced general hyperring $(\mathbb{Z}_{p^k} \cup \{\sqrt{p}\}, \varrho_{\sqrt{p}}, \varsigma_{\sqrt{p}})$, $I_p^{(1)}$ is the reproduced maximal hyperideal of $R$.*

*Proof.* Applying Theorem 2.11, for any $I_p^{(m)}, I_p^{(m')}, \in \mathcal{HI}(R)$, we have $|I_p^{(m)}| \geq |I_p^{(m')}|$ if and only if $\dfrac{p^{k-1}}{\gcd(m, p^{k-1})} \geq \dfrac{p^{k-1}}{\gcd(m', p^{k-1})}$. In addition, for $|\dfrac{p^{k-1}}{\gcd(m, p^{k-1})}| = s$, $s$ is maximum if and only if $\gcd(m, p^{k-1}) = 1$. Thus, $m = 1$, implies that $|I_p^{(m)}| \geq |I_p^{(m')}|$. □

**Example 2.6.** Consider the general hyperring $R = \mathbb{Z}_{125} \cup \{\sqrt{3}\}$. Computations show that

$$\begin{aligned}
&I_5^{(1)} = I_5^{(2)} = I_5^{(3)} = I_5^{(4)} = I_5^{(6)} = I_5^{(7)} = I_5^{(8)} = I_5^{(9)} = I_5^{(11)} \\
&= I_5^{(12)} = I_5^{(13)} = I_5^{(14)} = I_5^{(16)} = I_5^{(17)} = I_5^{(18)} = I_5^{(19)} = I_5^{(21)} \\
&= I_5^{(22)} = I_5^{(23)} = I_5^{(24)} = \{\overline{5}, \overline{10}, \overline{15}, \overline{20}, \ldots, \overline{115}, \overline{120}, \overline{0}, \sqrt{5}\}, \\
&I_5^{(5)} = I_5^{(10)} = I_5^{(15)} = I_5^{(20)} = \{\overline{0}, \overline{25}, \overline{50}, \overline{75}, \overline{100}, \sqrt{5}\}, I_5^{(25)} = \{\overline{0}, \sqrt{5}\}
\end{aligned}$$

and so $\mathcal{RI}(R) = \{I_5^{(1)}, I_5^{(5)}, I_5^{(25)}, \{\overline{0}\}, \mathbb{Z}_{125} \cup \{\sqrt{3}\}\}$.

Let $(R, \varrho, \varsigma)$ be a general hyperring. Then will denote $\mathcal{MRI}(R) = \{M \in \mathcal{RI}(R) \mid M$ is a maximal hyperideal$\}$ and $\mathcal{PRI}(R) = \{M \in \mathcal{RI}(R) \mid M$ is a prime hyperideal$\}$.

**Theorem 2.15.** *Let $p$ be a prime and $k \in \mathbb{N}$. Then in reproduced general hyperring $(\mathbb{Z}_{p^k} \cup \{\sqrt{p}\}, \varrho_{\sqrt{p}}, \varsigma_{\sqrt{p}})$, we have*

(*i*) $\mathcal{MRI}(R) = \{I_p^{(m)} \mid gcd(m, p) = 1\}$.

(*ii*) $|\mathcal{MRI}(R)| = p^{k-2}(p-1)$.

*Proof.* (*i*) Let $I_p^{(m)} \in \mathcal{MRI}(R)$ and $m \neq 1$. Since $gcd(m, p) = 1$, we get that $I_p^{(m)} = I_p^{(1)}$ and so by Theorem 2.14.

(*ii*) By (*i*), $|\mathcal{MRI}(R)| = \varphi(p^{k-1})$, where $\varphi$ is **Euler's phi** function. □

**Theorem 2.16.** *Let $p$ be a prime and $k \in \mathbb{N}$. Then in reproduced general hyperring $(\mathbb{Z}_{p^k} \cup \{\sqrt{p}\}, \varrho_{\sqrt{p}}, \varsigma_{\sqrt{p}})$, $I_p^{(1)}$ is the reproduced prime hyperideal of $R$.*

*Proof.* Let $p$ be a prime, $x, y \in \mathbb{Z}_{p^k} \cup \{\sqrt{p}\}$ and $\varsigma(x, y) \subseteq I_p^{(1)}$. Then $\varsigma(x, y) = \{0, \sqrt{p}\}, \varsigma(x, y) = \{\sqrt{p}\}, \varsigma(x, y) = \{0\}$ or there exists $1 \leq s \leq t$ such that $\varsigma(x, y) = \{sm\overline{p}\}$, where $tm \equiv 0 \pmod{p^{k-1}}$. If $\varsigma(x, y) = \{\sqrt{p}\}$, then $x \in \mathbb{Z}_{p^k} \setminus \{mp\}, y = \sqrt{p}(m \in \mathbb{N})$, and so $y \in I_p^{(1)}$. If $\varsigma(x, y) = \{0\}$, then $x = mp, y = \sqrt{p}(m \in \mathbb{N})$, and so $x, y \in I_p^{(1)}$. If $\varsigma(x, y) = \{0, \sqrt{p}\}$, then $x = y = \sqrt{p}$, and so $x, y \in I_p^{(1)}$. If there exists $1 \leq s \leq t$ such that $\varsigma(x, y) = \{s\overline{p} \neq \sqrt{p}\}$, where $tm \equiv 0 \pmod{p^{k-1}}$, then for $s = 1$, we have $\overline{p} \in I_p^{(1)}$. Thus $I_p^{(1)}$ is a reproduced prime hyperideal of $R$. □

**Theorem 2.17.** *Let $p$ be a prime and $k \in \mathbb{N}$. Then in reproduced general hyperring $(\mathbb{Z}_{p^k} \cup \{\sqrt{p}\}, \varrho_{\sqrt{p}}, \varsigma_{\sqrt{p}})$, we have*

- $(i)$ $\mathcal{PRI}(R) = \{I_p^{(m)} \mid gcd(m, p) = 1\}$.
- $(ii)$ $|\mathcal{PRI}(R)| = p^{k-2}(p-1)$.

*Proof.* $(i)$ Let $I_p^{(m)} \in \mathcal{PRI}(R)$ and $m \neq 1$. Since $gcd(m, p) = 1$, we get that $I_p^{(m)} = I_p^{(1)}$ and so by Theorem 2.16, $I_p^{(m)}$ is a reproduced prime hyperideal of $R$.

$(ii)$ By $(i)$, $|\mathcal{PRI}(R)| = \varphi(p^{k-1})$. □

**Definition 2.5.** Let $(R, \varrho, \varsigma)$ be a general hyperring. Then $(R, \varrho, \varsigma)$ is called an Ideal-absorbing, if the its set of all prime ideals and the its set of all maximal hyper ideals is equal.

**Corollary 2.3.** *Let $p$ be a prime and $k \in \mathbb{N}$. Then the reproduced general hyperring $(\mathbb{Z}_{p^k} \cup \{\sqrt{p}\}, \varrho_{\sqrt{p}}, \varsigma_{\sqrt{p}})$ is an Ideal-absorbing.*

## 3. Conclusion and discussion

The current paper has defined the general hyperrings as a generalization of hyperrings and presented some properties in these hyperstructures. For each ring considered, it is possible to work only with the elements of the context set. This means that if we want to add another element to a ring, it is necessary to break all the principles of the axiom and it is possible that the new complex will not become a ring. But by adding an element to an arbitrary ring, a hyperring can be formed, and this is one of the limitations of rings that is solved by hyperrings. This advantage can be applied to all substructures including ideals and substructures Also,

- $(i)$ principal ideal domain reproduced general hyperrings are constructed,
- $(ii)$ the set of all prime ideals and the set of all maximal hyper ideals of principal ideal domain reproduced general hyperrings are computed, principal ideal domain reproduced general hyperrings are constructed.

(*iii*) the concept of Ideal-absorbing reproduced general hyperrings is defined and is proved that the principal ideal domain reproduced general hyperrings are Ideal-absorbing.

We hope that these results are helpful for further studies in general hyperring theory. In our future studies, we hope to obtain more results regarding fuzzy general hyperring, soft general hyperring, tropical general multifield and their applications.

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