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ON $(LCS)_n$-MANIFOLDS

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Abstract. In the present paper we studied the Pseudo projectively flat $(LCS)_n$-manifold with several properties. Among other interesting results we obtained necessary and sufficient conditions for a 3-dimensional $(LCS)_n$-manifold in the space form.
Keywords: Lorentzian manifold, Pseudo projective curvature tensor, Lorentzian metric, characteristic vector

1. Introduction

An n-dimensional Lorentzian manifold M is a smooth connected paracompact Hausdroff manifold with a Lorentzian metric of type $(0, 2)$ such that for each point $p \in M$, the tensor $g_p : T_pM \times T_pM \to \mathbb{R}$ is a non-degenerate inner product of signature $(-, +, ..., +)$ where T_pM denotes the tangent vector space of M at p and $\mathbb{R}$ is the real number space. A non-zero vector $\nu \in T_pM$ is said to be timelike (resp. non-spacelike, null, spacelike,) if it satisfies $g_p(\nu, \nu) < 0 (res. \leq 0, = 0, > 0)$ ([1],[6]).

Let M be the Lorentzian manifold admitting a unit timelike concircular vector field ξ, called the characteristic vector field of the manifold. Then we have

$$g(\xi, \xi) = -1. \tag{1.1}$$

Since ξ is a unit concircular vector field, there exists a non-zero 1-form η such that for

$$g(X, \xi) = \eta(X), \tag{1.2}$$

the equation of the following form holds

$$(\nabla_X \eta)(Y) = \alpha\{g(X, Y) + \eta(X)\eta(Y)\}, (\alpha \neq 0), \tag{1.3}$$

for all vector fields X,Y where ∇ denotes the operator of covariant differentiation with respect to the Lorentzian metric g and α is a non-zero scalar function satisfying

$$\nabla_X \alpha = (X\alpha) = \alpha(X) = \rho\eta(X), \tag{1.4}$$

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ρ being a certain scalar function.
If we put

$$\varphi X = \frac{1}{\alpha}\nabla_X \xi, \tag{1.5}$$

then from (1.3) and (1.5) we have

$$\varphi X = X + \eta(X)\xi, \tag{1.6}$$

from which it follows that φ is a symmetric $(1,1)$ tensor. From (1.3) and (1.5) we have

$$\varphi^2 X = X + \eta(X)\xi. \tag{1.7}$$

Hence M is a manifold with a Lorentzian almost paracontact structure (φ, ξ, η, g) introduced by Matsumoto [4], Mihai and Rosca [5]. Thus the Lorentzian manifold M together with the unit timelike vector field ξ, its associated 1-form η and $(1,1)$ tensor field φ is said to be a Lorentzian almost paracontact manifold with a structure of the concircular type and such a manifold is said to be a $(LCS)_n$-manifold ([8], [10]).

2. Preliminaries

A differentiable manifold M of dimension n is called $(LCS)_n$-manifold if it admits a $(1,1)$ tensor field φ, a contravariant vector field ξ, a covariant vector field η and a Lorentzian metric g which satisfy

$$\eta(\xi) = -1, \tag{2.1}$$

$$\phi^2(X) = X + \eta(X)\xi, \tag{2.2}$$

$$g(\phi X, \phi Y) = g(X,Y) + \eta(X)\eta(Y), \tag{2.3}$$

$$g(X,\xi) = \eta(X), \tag{2.4}$$

$$\phi\xi = 0, \qquad \eta(\phi X) = 0, \tag{2.5}$$

for all $X, Y \epsilon\, TM$.

Also in a $(LCS)_n$-manifold M the following relations are satisfied [9]

$$\eta(R(X,Y)Z) = g(R(X,Y)Z,\xi) = (\alpha^2 - \rho)[g(Y,Z)\eta(X) - g(X,Z)\eta(Y)], \tag{2.6}$$

$$R(\xi,Y)Z = (\alpha^2 - \rho)[g(Y,Z)\xi - \eta(Z)Y] \tag{2.7}$$

$$R(X,Y)\xi = (\alpha^2 - \rho)[\eta(Y)X - \eta(X)Y], \tag{2.8}$$

$$R(\xi,X)\xi = (\alpha^2 - \rho)[\eta(X)\xi + X], \tag{2.9}$$

$$(\nabla_X \varphi)(Y) = \alpha[g(X,Y)\xi + \eta(Y)X + 2\eta(X)\eta(Y)\xi], \tag{2.10}$$

$$S(X,\xi) = (\alpha^2 - \rho)(n-1)\eta(X), \tag{2.11}$$

$$S(\varphi X, \varphi Y) = S(X,Y) + (\alpha^2 - \rho)(n-1)\eta(X)\eta(Y), \tag{2.12}$$

where S is the Ricci curvature and Q is the Ricci operator given by
$S(X,Y) = g(QX,Y)$.

3. Pseudo projectively flat $(LCS)_n$-manifold

The Pseudo projective curvature tensor is given by [7].

$$\widetilde{P}(X,Y)Z = aR(X,Y)Z + b[S(Y,Z)X - S(X,Z)Y] - \frac{r}{n}\left(\frac{a}{n-1}+b\right)[g(Y,Z)X - g(X,Z)Y], \tag{3.1}$$

where a and b are constants such that $a, b \neq 0$, R is the curvature tensor, S is the Ricci tensor and r is the scalar curvature.

If the pseudo projective curvature tensor vanishes, then from (3.1), we have

$$'R(X,Y,Z,W) = -\frac{b}{a}[S(Y,Z)g(X,W) - S(X,Z)g(Y,W)] + \frac{r}{n}\left(\frac{1}{n-1}+\frac{b}{a}\right)[g(Y,Z)g(X,W) - g(X,Z)g(Y,W)]. \tag{3.2}$$

where $'R(X,Y,Z,W) = g(R(X,Y)Z,W)$.
Putting ξ for W in (3.2) and using (2.6) and (2.11), we get

$$(\alpha^2-\rho)[g(Y,Z)\eta(X) - g(X,Z)\eta(Y)] = -\frac{b}{a}[S(Y,Z)X - S(X,Z)Y] + \frac{r}{n}\left(\frac{1}{n-1}+\frac{b}{a}\right)[g(Y,Z)\eta(X) - g(X,Z)\eta(Y)]. \tag{3.3}$$

Again if we put ξ for X in (3.3) and using (2.5) and (2.11), we obtain

$$S(Y,Z) = \frac{a}{b}\left[\frac{r}{n}\left(\frac{1}{n-1}+\frac{b}{a}\right) - (\alpha^2-\rho)\right]g(Y,Z) + \frac{a}{b}\left[\frac{r}{n}\left(\frac{1}{n-1}+\frac{b}{a}\right) - n(\alpha^2-\rho)\right]\eta(Y)\eta(Z). \tag{3.4}$$

Differentiating (3.4) covariantly along X, we get

$$(\nabla_X S)(Y,Z) = \frac{a}{b}\left[\frac{dr(X)}{n}\left(\frac{1}{n-1}+\frac{b}{a}\right)\right][g(Y,Z)+\eta(Y)\eta(Z)] + \frac{a}{b}\left[\frac{r}{n}\left(\frac{1}{n-1}+\frac{b}{a}\right) - n(\alpha^2-\rho)\right]\Big[(\nabla_X\eta)(Y)\eta(Z) + (\nabla_X\eta)(Z)\eta(Y)\Big].$$

where $dr(X) = \nabla_X r$. On using (1.3) this implies that

$$(\nabla_X S)(Y,Z) - (\nabla_Y S)(X,Z) = \frac{a}{b}\left[\frac{dr(X)}{n}\left(\frac{1}{n-1}+\frac{b}{a}\right)\right]\Big[g(Y,Z) \tag{3.5}$$

$$+\eta(Y)\eta(Z)\Big] - \frac{a}{b}\left[\frac{dr(Y)}{n}\left(\frac{1}{n-1}+\frac{b}{a}\right)\right]\Big[g(X,Z)$$
$$+\eta(X)\eta(Z)\Big] + \frac{a\alpha}{b}\left[\frac{r}{n}\left(\frac{1}{n-1}+\frac{b}{a}\right) - n(\alpha^2-\rho)\right]$$
$$[g(X,Z)\eta(Y) - g(Y,Z)\eta(X)]$$

On the other hand, in our case, since we have $(\nabla_X \widetilde{P})(X,Y)Z = 0$, we get $div\widetilde{P} = 0$, where div denotes the divergence. So for $n > 1$, $div\widetilde{P} = 0$ gives

$$(3.6) \quad (\nabla_X S)(Y,Z) - (\nabla_Y S)(X,Z) = \frac{1}{n(a+b)}\left[\frac{a+(n-1)b}{n-1}\right][g(Y,Z)dr(X) - g(X,Z)dr(Y)].$$

It follows from (3.5) and (3.6) that

$$(3.7) \quad \frac{1}{n(a+b)}\left[\frac{a+(n-1)b}{n-1}\right][g(Y,Z)dr(X) - g(X,Z)dr(Y)]$$
$$= \frac{a}{b}\left[\frac{dr(X)}{n}\left(\frac{1}{n-1}+\frac{b}{a}\right)\right][g(Y,Z)+\eta(Y)\eta(Z)]$$
$$-\frac{a}{b}\left[\frac{dr(Y)}{n}\left(\frac{1}{n-1}+\frac{b}{a}\right)\right][g(X,Z)+\eta(X)\eta(Z)]$$
$$+\frac{a\alpha}{b}\left[\frac{r}{n}\left(\frac{1}{n-1}+\frac{b}{a}\right) - n(\alpha^2-\rho)\right][g(X,Z)\eta(Y) - g(Y,Z)\eta(X)].$$

If r is constant, then from (3.7), we obtain

$$\frac{a\alpha}{b}\left[\frac{r}{n}\left(\frac{1}{n-1}+\frac{b}{a}\right) - n(\alpha^2-\rho)\right] = 0.$$

Since $\frac{a\alpha}{b} \neq 0$, the above equation gives

$$(3.8) \quad r = \frac{an^2(\alpha^2-\rho)(n-1)}{a+(n-1)b}.$$

Now substituting (3.4) in (3.2), we get

$$(3.9) \quad {}'R(X,Y,Z,W) = (\alpha^2-\rho)[g(Y,Z)g(X,W) - g(X,Z)g(Y,W)]$$
$$-\left[\frac{r}{n}\left(\frac{1}{n-1}+\frac{b}{a}\right) - n(\alpha^2-\rho)\right][g(X,W)\eta(Y) - g(Y,W)\eta(X)]\eta(Z).$$

On using (3.8) in (3.9), we have

$${}'R(X,Y,Z,W) = (\alpha^2-\rho)[g(Y,Z)g(X,W) - g(X,Z)g(Y,W)].$$

This shows that the manifold is of constant curvature. Thus we can state the following:

Theorem 3.1. *In a pseudo projectively flat $(LCS)_n$-manifold $M(n > 1)$ if the scalar curvature r is constant, then M is of constant curvature.*

Now putting $X = W = \xi$ in (3.2) and using (2.1) and (2.11), we get

$$'R(\xi, Y, Z, \xi) = \frac{b}{a}[S(Y, Z) + (n-1)(\alpha^2 - \rho)\eta(Y)\eta(Z)] - \frac{r}{n}\left(\frac{1}{n-1} + \frac{b}{a}\right)[g(Y, Z) + \eta(Y)\eta(Z)]. \tag{3.10}$$

In view of (2.7) and (3.10), we get

$$\left[(\alpha^2 - \rho)\left(\frac{a + b(n-1)}{a}\right) - \frac{r}{n}\left(\frac{a + b(n-1)}{a(n-1)}\right)\right] g(\varphi Y, \varphi Z) = 0 \tag{3.11}$$

Since $$g(\varphi Y, \varphi Z) \neq o,$$

hence from (3.11), we get

$$r = n(n-1)(\alpha^2 - \rho).$$

Hence we can state the following:

Theorem 3.2. *The scalar curvature r of a pseudo projective flat $(LCS)_n$-manifold M is constant, given by*

$$r = n(n-1)(\alpha^2 - \rho),$$

provided that $(\alpha^2 - \rho)$ is constant.

Contracting (3.1) with respect to X, we get

$$(C_1^1\widetilde{P})(Y, Z) = [a + b(n-1)]S(Y, Z) - \frac{r}{n}[a + b(n-1)]g(Y, Z), \tag{3.12}$$

where $(C_1^1\widetilde{P})(Y, Z)$ denotes the contraction of $\widetilde{P}(X, Y)Z$ with respect to X.

Let us assume that in an $(LCS)_n$-manifold,

$$(C_1^1\widetilde{P})(Y, Z) = 0. \tag{3.13}$$

In view of (3.12) and (3.13), we have

$$[a + b(n-1)]\left[S(Y, Z) - \frac{r}{n}g(Y, Z)\right] = 0. \tag{3.14}$$

If $[a + b(n-1)] \neq 0$, then from (3.14), we get

$$S(Y, Z) = \frac{r}{n}g(Y, Z),$$

which shows that M is an Einstein manifold.
On putting ξ for Z in (3.14), we get

$$[a+b(n-1)]\left[(n-1)(\alpha^2-\rho)-\frac{r}{n}\right]\eta(Y)=0. \tag{3.15}$$

Since $\eta(Y)\neq 0,$
hence from (3.15), we have

$$r=n(n-1)(\alpha^2-\rho)$$

Hence we can state the following:

Theorem 3.3. *If in an $(LCS)_n$-manifold M the relation $(C_1^1\widetilde{P})(Y,Z)=0$ holds, then M is an Einstein manifold with scalar curvature*

$$r=n(n-1)(\alpha^2-\rho), \qquad \textit{provided that } [a+b(n-1)]\neq 0.$$

4. An Einstein $(LCS)_n$-manifold Satisfying $R(X,Y)\cdot\widetilde{P}=0$

In this section we assume that

$$(R(X,Y)\cdot\widetilde{P})(U,V)W=0. \tag{4.1}$$

Let an $(LCS)_n$-manifold be an Einstein manifold, then its Ricci tensor S is of the form

$$S(X,Y)=kg(X,Y), \tag{4.2}$$

where k is a constant.
From (3.1) and (4.2), we have

$$\begin{aligned}\widetilde{P}(X,Y)Z&=aR(X,Y)Z+bk[g(Y,Z)X-g(X,Z)Y]\\&-\frac{r}{n}\left(\frac{a}{n-1}+b\right)[g(Y,Z)X-g(X,Z)Y].\end{aligned}$$

It can be written as

$$\begin{aligned}{}'\widetilde{P}(X,Y,Z,W)=a\,{}'R(X,Y,Z,W)+\left[bk-\frac{r}{n}\left(\frac{a}{n-1}+b\right)\right]\\ [g(Y,Z)g(X,W)-g(X,Z)g(Y,W)].\end{aligned} \tag{4.3}$$

Putting ξ for W in (4.3) and using (2.6), we get

$$\begin{aligned}\eta(\widetilde{P}(X,Y)Z)=\left[a(\alpha^2-\rho)+bk-\frac{r}{n}\left(\frac{a}{n-1}+b\right)\right][g(Y,Z)\eta(X)\\ -g(X,Z)\eta(Y)].\end{aligned} \tag{4.4}$$

Putting ξ for X in (4.4), we get

$$\eta(\widetilde{P}(\xi, Y)Z) = \left[a(\alpha^2 - \rho) + bk - \frac{r}{n}\left(\frac{a}{n-1} + b\right)\right][-g(Y, Z) - \eta(Y)\eta(Z)]. \tag{4.5}$$

Again putting ξ for Z in (4.4), we get

$$\eta(\widetilde{P}(X, Y)\xi) = 0. \tag{4.6}$$

Now,

$$(R(X, Y) \cdot \widetilde{P})(U, V)W = R(X, Y)\widetilde{P}(U, V)W - \widetilde{P}(R(X, Y)U, V)W - \widetilde{P}(U, R(X, Y)V)W - \widetilde{P}(U, V)R(X, Y)W.$$

In view of (4.1), we get

$$R(X, Y)\widetilde{P}(U, V)W - \widetilde{P}(R(X, Y)U, V)W - \widetilde{P}(U, R(X, Y)V)W - \widetilde{P}(U, V)R(X, Y)W = 0. \tag{4.7}$$

Therefore,

$$g(R(\xi, Y)\widetilde{P}(U, V)W, \xi) - g(\widetilde{P}(R(\xi, Y)U, V)W, \xi) - g(\widetilde{P}(U, R(\xi, Y)V)W, \xi) - g(\widetilde{P}(U, V)R(\xi, Y)W, \xi) = 0.$$

From this it follows that

$$-'\widetilde{P}(U, V, W, Y) - \eta(Y)\eta(\widetilde{P}(U, V)W) + \eta(U)\eta(\widetilde{P}(Y, V)W) + \eta(V)\eta(\widetilde{P}(U, Y)W) + \eta(W)\eta(\widetilde{P}(U, V)Y) - g(Y, U)\eta(\widetilde{P}(\xi, V)W) - g(Y, V)\eta(\widetilde{P}(U, \xi)W) - g(Y, W)\eta(\widetilde{P}(U, V)\xi) = 0. \tag{4.8}$$

Putting U for Y, in (4.8), we get

$$-'\widetilde{P}(U, V, W, U) - \eta(U)\eta(\widetilde{P}(U, V)W) + \eta(U)\eta(\widetilde{P}(U, V)W) + \eta(V)\eta(\widetilde{P}(U, U)W) + \eta(W)\eta(\widetilde{P}(U, V)U) - g(U, U)\eta(\widetilde{P}(\xi, V)W) - g(U, V)\eta(\widetilde{P}(U, \xi)W) - g(U, W)\eta(\widetilde{P}(U, V)\xi) = 0. \tag{4.9}$$

Let $\{e_i\}, i = 1, 2, \cdots, n$ be an orthonormal basis of the tangent space at any point. Then the sum $1 \leq i \leq n$ of the relation (4.9) for $U = e_i$ gives

$$\eta(\widetilde{P}(\xi, V)W) = \frac{1}{(n-1)}\Big[-aS(V, W) - \left\{bk - \frac{r}{n}\left(\frac{a}{n-1} + b\right)\right\}(n-1)g(V, W) - \left\{a(\alpha^2 - \rho) + bk - \frac{r}{n}\left(\frac{a}{n-1} + b\right)\right\}(n-1)\eta(V)\eta(W)\Big]. \tag{4.10}$$

Using (4.4) and (4.10), it follows from (4.8) that

$$'\widetilde{P}(U,V,W,Y) = \left[a(\alpha^2-\rho) + bk - \frac{r}{n}\left(\frac{a}{n-1}+b\right)\right] [g(V,W)g(Y,U) - g(U,W)g(V,Y)]. \tag{4.11}$$

From (4.3) and (4.11), we get

$$'R(U,V,W,Y) = (\alpha^2-\rho)[g(V,W)g(Y,U) - g(U,W)g(V,Y)], \quad a \neq 0.$$

Therefore, we can state

Theorem 4.1. *If in an Einstein $(LCS)_n$-manifold , the relation $(R(X,Y)\cdot\widetilde{P}) = 0$ hold, then M is of constant curvature, provided $a \neq 0$.*

5. 3-dimentional $(LCS)_n$-manifold

Let us consider a 3-dimensional $(LCS)_n$-manifold. In a 3-dimensional Riemannian manifold we have

$$R(X,Y)Z = g(Y,Z)QX - g(X,Z)QY + S(Y,Z)X - S(X,Z)Y - \frac{r}{2}[g(Y,Z)X - g(X,Z)Y]. \tag{5.1}$$

Since the dimension of manifold is 3, so the equation (2.11) reduces to

$$S(X,\xi) = 2(\alpha^2-\rho)\eta(X). \tag{5.2}$$

Putting ξ for Z in (5.1) and using (2.8), we have

$$\eta(Y)QX - \eta(X)QY = \left[-(\alpha^2-\rho) + \frac{r}{2}\right][\eta(Y)X - \eta(X)Y]. \tag{5.3}$$

Putting ξ for Y in (5.3) and using (5.2) and (2.1), we get

$$QX = \left[-(\alpha^2-\rho) + \frac{r}{2}\right] X + \left[-3(\alpha^2-\rho) + \frac{r}{2}\right]\eta(X)\xi$$

i.e.,

$$S(X,Y) = \left[-(\alpha^2-\rho) + \frac{r}{2}\right] g(X,Y) + \left[-3(\alpha^2-\rho) + \frac{r}{2}\right]\eta(X)\eta(Y) \tag{5.4}$$

Therefore a 3-dimensional manifold is η-Einstein.
Hence we can state the following:

Theorem 5.1. *A 3-dimensional $(LCS)_n$-manifold is an η-Einstein manifold.*

Using (5.4) in (5.1), we get

$$R(X,Y)Z = \left[-2(\alpha^2-\rho)+\frac{r}{2}\right][g(Y,Z)X - g(X,Z)Y] + \left[-3(\alpha^2-\rho)+\frac{r}{2}\right][g(Y,Z)\eta(X)\xi - g(X,Z)\eta(Y)\xi + \eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y]. \tag{5.5}$$

An $(LCS)_n$-manifold M is said to be a manifold of quasi-constant curvature if its curvature tensor R satisfies

$$R(X,Y)Z = A[g(Y,Z)X - g(X,Z)Y] + B[g(Y,Z)\eta(X)\xi - g(X,Z)\eta(Y)\xi + \eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y]. \tag{5.6}$$

where A and B are smooth functions on M and B is not identically zero on M. The notion of a manifold of quasi-constant curvature was first introduced by Chen and Yano in 1972 for a Riemannian manifold [2].

Hence in view of (5.5) and (5.6), we have the following theorem:

Theorem 5.2. *A 3-dimensional $(LCS)_n$-manifold is a manifold of quasi-constant curvature.*

An $(LCS)_n$-manifold is said to be a space form if the manifold is a space of constant curvature[4].

Hence from (5.5), we have the following:

Theorem 5.3. *A 3-dimensional $(LCS)_n$-manifold is a space form if and only if*

$$r = 6(\alpha^2-\rho).$$

Next, we consider a 3-dimensional $(LCS)_n$-manifold which satisfies the condition

$$R(X,Y)\cdot S = 0. \tag{5.7}$$

From (5.7), we have

$$S(R(X,Y)U,V) + S(U,R(X,Y)V) = 0. \tag{5.8}$$

Again from (2.6), we get

$$R(X,\xi)Z = (\alpha^2-\rho)[\eta(Z)X - g(X,Z)\xi]. \tag{5.9}$$

Putting ξ for Y in (5.8) and using (5.9), we get

$$\eta(U)S(X,V) - g(X,U)S(V,\xi) + \eta(V)S(U,X) - 2(\alpha^2-\rho)g(X,V)\eta(U) = 0. \tag{5.10}$$

Taking a frame field and contracting over X and U from (5.10), we obtain

$$S(\xi, V) + [r - 8(\alpha^2 - \rho)]\eta(V) = 0. \tag{5.11}$$

Using (5.2) in (5.11), we obtain

$$[r - 6(\alpha^2 - \rho)]\eta(V) = 0.$$

This gives $r = 6(\alpha^2 - \rho)$ (since $\eta(V) \neq 0$), which implies by Theorem 5.3 that the manifold is a space form.
Hence we can state the following:

Theorem 5.4. *A 3-dimensional Ricci semi-symmetric $(LCS)_n$-manifold is a space form.*

Since $\nabla S = 0$ implies $R(X,Y) \cdot S = 0$, we get the following:

Corollary 5.1. *A 3-dimensional Ricci semi-symmetric $(LCS)_n$-manifold is a space form.*

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