Abstract. In this paper we introduce some newly defined triple sequence spaces by combining the modulus function and non-negative six dimensional matrix of the form \( A = (a_{l,m,n,p,q,r}) \) and we study some of their topological properties. We also obtain and prove some inclusion relations.

1. Introduction

A triple sequence (real or complex) is a function from \( \mathbb{N} \times \mathbb{N} \times \mathbb{N} \) to \( \mathbb{R}(\mathbb{C}) \), where \( \mathbb{N} \), \( \mathbb{R} \) and \( \mathbb{C} \) denote a set of natural numbers, real numbers and complex numbers, respectively. In 2007, Sahiner et. al. [2] introduced the concept of triple sequences and established their statistical convergence. Subsequently, Dutta et. al. [3] generalized this concept by using the Orlicz function. Later on, Savas and Esi [5] introduced statistical convergence of triple sequences on probabilistic normed spaces. Recently, Debnath et. al. [13], Debnath and Das [14] generalized these concepts by using the difference operator.

In 1986 Maddox [10] introduced the strongly Cesaro summable with respect to a modulus function for the class of sequence. It was further investigated by Connor [11] in 1989 as an extended work for strong \( A \)-summability, considering \( A = (a_{n,k}) \) is a non-negative regular matrix. Pringsheim gave the definition of the convergence for double sequences in 1900. Since then, this concept has been studied by many authors and rapid development was made on this subject. In 2011, Savas and Patterson [6] introduced the definition for double sequence spaces defined by modulus function and considering the non-negative four-dimensional matrix as \( A = (a_{m,n,k,l}) \). In this paper, we have extended this concept for triple sequence spaces using the non-negative six-dimensional matrix \( A = (a_{l,m,n,p,q,r}) \) defined by
modulus function and taking $w^3$, the set of all triple sequence of complex numbers.

**Definition 1.1.** [2]: A triple sequence $(x_{lmn})$ is said to be convergent to $L$, in Pringsheim’s sense if for every $\epsilon > 0$, there exists $N(\epsilon) \in N$ such that $|x_{lmn} - L| < \epsilon$, whenever $l \geq N, m \geq N, n \geq N$ and we write $\lim_{l,m,n \to \infty} x_{lmn} = L$.

**Definition 1.2.** [2]: A triple sequence $(x_{lmn})$ is said to be bounded if there exists $M > 0$ such that $|x_{lmn}| < M$ for all $l, m, n \in N$.

**Note:** A triple sequence convergent in Pringsheim’s sense may not be bounded [15].

**Definition 1.3.** [10]: A function $f : [0, \infty) \to [0, \infty)$ is called a modulus function if it satisfies the following four conditions:

1. $f(x) = 0$ if and only if $x = 0$,
2. $f(x + y) \leq f(x) + f(y)$ for all $x \geq 0$ and $y \geq 0$,
3. $f$ is increasing,
4. $f$ is continuous from the right at 0.

**Definition 1.4.** Let $A = (a_{l,m,n,p,q,r})$ denote the six-dimensional summability method that maps the complex triple sequence $x$ into the triple sequence $Ax$. Then the $lmn$th term to $Ax$ will be $\left(\sum_{p=1}^{\infty} \sum_{q=1}^{\infty} \sum_{r=1}^{\infty} a_{l,m,n,p,q,r} x_{p,q,r}\right)$.

**Definition 1.5.** Let $f$ be a modulus function and $A=(a_{l,m,n,p,q,r})$ be a non-negative six-dimensional matrix of real entries with $\sup_{l,m,n} \sum_{p=0,q=0,r=0}^{\infty} a_{l,m,n,p,q,r} < \infty$.

Then

$$c_3^0(A,f) = \{ x \in w^3 : P - \lim_{l,m,n} \sum_{p=0,q=0,r=0}^{\infty,\infty,\infty} a_{l,m,n,p,q,r} f(|x_{p,q,r}|) = 0 \}$$

$$c_3^1(A,f) = \{ x \in w^3 : P - \lim_{l,m,n} \sum_{p=0,q=0,r=0}^{\infty,\infty,\infty} a_{l,m,n,p,q,r} f(|x_{p,q,r} - L|) = 0, \text{ for some } L \}$$

$$l_3^\infty(A,f) = \{ x \in w^3 : \sup_{l,m,n} \sum_{p=0,q=0,r=0}^{\infty,\infty,\infty} a_{l,m,n,p,q,r} f(|x_{p,q,r}|) < \infty \}$$

If $f(x) = x$ then the sequence spaces become:

$$c_3^0(A) = \{ x \in w^3 : P - \lim_{l,m,n} \sum_{p=0,q=0,r=0}^{\infty,\infty,\infty} a_{l,m,n,p,q,r} |x_{p,q,r}| = 0 \}$$

$$c_3^1(A) = \{ x \in w^3 : P - \lim_{l,m,n} \sum_{p=0,q=0,r=0}^{\infty,\infty,\infty} a_{l,m,n,p,q,r} |x_{p,q,r} - L| = 0, \text{ for some } L \}$$
\[ l_3^\infty(A) = \{ x \in w^3 : \sup_{p=0,q=0,r=0} a_{l,m,n,p,q,r} |x_{p,q,r}| < \infty \} \]

The spaces in Definition 1.5 converted to some well-known sequence spaces by specifying \( A \) and \( f \). For example, if we consider \( A = (C,1,1) \) the sequence spaces \( c_0^3(f) \), \( c^3(f) \) and \( l_3^\infty(f) \) will be of the following form:

\[ c_0^3(f) = \{ x \in w^3 : P - \lim_{l,m,n} \sum_{p=0,q=0,r=0} a_{l,m,n,p,q,r} x_{p,q,r} = 0 \} \]
\[ c^3(f) = \{ x \in w^3 : P - \lim_{l,m,n} \sum_{p=0,q=0,r=0} a_{l,m,n,p,q,r} x_{p,q,r} - L = 0, \text{ for some } L \} \]
\[ l_3^\infty(f) = \{ x \in w^3 : \sup_{p=0,q=0,r=0} a_{l,m,n,p,q,r} x_{p,q,r} < \infty \} \]

Now as a final illustration, if we consider \( A = (C,1,1) \) and \( f(x) = x \), we get the following spaces

\[ c_0^3 = \{ x \in w^3 : P - \lim_{l,m,n} \sum_{p=0,q=0,r=0} a_{l,m,n,p,q,r} |x_{p,q,r}| = 0 \} \]
\[ c^3 = \{ x \in w^3 : P - \lim_{l,m,n} \sum_{p=0,q=0,r=0} a_{l,m,n,p,q,r} |x_{p,q,r} - L| = 0, \text{ for some } L \} \]
\[ l_3^\infty = \{ x \in w^3 : \sup_{p=0,q=0,r=0} a_{l,m,n,p,q,r} |x_{p,q,r}| < \infty \} \]

2. Main Results

In this section, we shall establish the main properties of the sequence spaces in Definition 1.5

**Theorem 2.1.** The sequence spaces \( c_0^3(A, f) \), \( c^3(A, f) \) and \( l_3^\infty(A, f) \) all are linear over the complex field \( C \).

**Proof.** It is obvious. □

**Theorem 2.2.** If \( A = (a_{l,m,n,p,q,r}) \) is a non-negative six dimensional matrix of real entries with \( \sum_{p=0,q=0,r=0} a_{l,m,n,p,q,r} < \infty \), and let \( f \) be a modulus function then

1. \( c^3(A, f) \subset l_3^\infty(A, f) \)
2. \( c_0^3(A, f) \subset l_3^\infty(A, f) \)

**Proof.** Here we shall establish the inclusion (1) only.

Let \( x \in c^3(A, f) \). Now using the conditions (2) and (3) of the modulus function (Definition 1.3) we get the following:

\[ \sum_{p=0,q=0,r=0} a_{l,m,n,p,q,r} f(|x_{p,q,r}|) < \infty \]
There exists an integer $M_1$ such that $|L| \leq M_1$. We obtain

$$\sum_{p=0,q=0,r=0}^{\infty,\infty,\infty} a_{l,m,n,p,q,r} f(|x_{p,q,r} - L|) + f(|L|) \sum_{p=0,q=0,r=0}^{\infty,\infty,\infty} a_{l,m,n,p,q,r}$$

As we consider $x \in c^3(A, f)$ and $\sup_{l,m,n} \sum_{p=0,q=0,r=0}^{\infty,\infty,\infty} a_{l,m,n,p,q,r} < \infty$ we are conclude that

$$x \in l^3_\infty(A, f)$$

This completes the proof. □

**Theorem 2.3.** If $A = (a_{l,m,n,p,q,r})$ is a non-negative six dimensional matrix of real entries with $\sup_{l,m,n} \sum_{p=0,q=0,r=0}^{\infty,\infty,\infty} a_{l,m,n,p,q,r} < \infty$, and let $f$ be a modulus function then the following inclusion holds

1. $c^3(A) \subset c^3(A, f)$
2. $c^3_0(A) \subset c^3_0(A, f)$
3. $l^3_\infty(A) \subset l^3_\infty(A, f)$.

**Proof.** Here the inclusions (1) and (2) can be easily proved. Thus we will only establish the inclusion (3).

Let $x \in l^3_\infty(A)$ such that $\sup_{l,m,n} \sum_{p=0,q=0,r=0}^{\infty,\infty,\infty} a_{l,m,n,p,q,r} < \infty$. Let $\epsilon > 0$ and choose $\delta$ with $0 < \delta < 1$ such that $f(t) < \epsilon$ for $0 \leq t \leq \delta$. Now we consider the following equality

$$\sum_{p=0,q=0,r=0}^{\infty,\infty,\infty} a_{l,m,n,p,q,r} f(|x_{p,q,r}|)$$

$$= \sum_{p=0,q=0,r=0}^{\infty,\infty,\infty} a_{l,m,n,p,q,r} f(|x_{p,q,r}|) + \sum_{p=0,q=0,r=0}^{\infty,\infty,\infty} a_{l,m,n,p,q,r} f(|x_{p,q,r}|)$$

From the properties of the modulus function we have the following:

$$\sum_{p=0,q=0,r=0}^{\infty,\infty,\infty} a_{l,m,n,p,q,r} f(|x_{p,q,r}|) \leq \epsilon \sum_{p=0,q=0,r=0}^{\infty,\infty,\infty} a_{l,m,n,p,q,r} \leq \epsilon \sum_{p=0,q=0,r=0}^{\infty,\infty,\infty} a_{l,m,n,p,q,r} \leq \epsilon \sum_{p=0,q=0,r=0}^{\infty,\infty,\infty} a_{l,m,n,p,q,r}$$

For $|x_{p,q,r}| > \delta$ and the fact that
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\[ |x_{p,q,r}| < |x_{p,q,r}|/\delta < [1 + \{|x_{p,q,r}|/\delta\}] \]

Where \([t]\) denoted the integer part of \(t\) and from the conditions (2) and (3) of the modulus function we can write

\[ f(|x_{p,q,r}|) < (1 + [|x_{p,q,r}|/\delta])f(1) \leq 2f(1)\{|x_{p,q,r}|/\delta\} \]

Now

\[ \sum_{p=0, q=0, r=0}^{\infty, \infty, \infty} a_{t,m,n,p,q,r}f(|x_{p,q,r}|) \leq 2f(1)/\delta \sum_{p=0, q=0, r=0}^{\infty, \infty, \infty} a_{t,m,n,p,q,r}|x_{p,q,r}| \]

The last inequality and equation (2.1) gives us the following results

\[ \sum_{p=0, q=0, r=0}^{\infty, \infty, \infty} a_{t,m,n,p,q,r}f(|x_{p,q,r}|) \leq \epsilon \sum_{p=0, q=0, r=0}^{\infty, \infty, \infty} a_{t,m,n,p,q,r}|x_{p,q,r}| \]

Since \(\sup_{t,m,n} \sum_{p=0, q=0, r=0}^{\infty, \infty, \infty} a_{t,m,n,p,q,r} < \infty\) and \(x \in l_3^3(A)\)

we find that \(x \in l_3^3(A, f)\).

This completes the proof. \(\square\)

**Theorem 2.4.** If \(A = (a_{t,m,n,p,q,r})\) is a non-negative six dimensional matrix of real entries with \(\sup_{t,m,n} \sum_{p=0, q=0, r=0}^{\infty, \infty, \infty} a_{t,m,n,p,q,r} < \infty\), and let \(f\) be a modulus function and \(\beta = \lim_{t \to \infty} f(t)/t > 0\) then \(c^3(A) = c^3(A, f)\).

**Proof.** Let \(\beta > 0\). By definition of \(\beta\) we have \(f(t) \geq \beta t\) for all \(t \geq 0\) and since \(\beta > 0\) we have \(t \leq \{1/\beta\}f(t)\) for all \(t \geq 0\).

Now from \(x \in c^3(A, f)\) we can write the following inequality

\[ \sum_{p=0, q=0, r=0}^{\infty, \infty, \infty} a_{t,m,n,p,q,r}|x_{p,q,r} - L| \leq \{1/\beta\} \sum_{p=0, q=0, r=0}^{\infty, \infty, \infty} a_{t,m,n,p,q,r}f(|x_{p,q,r} - L|) \]

whence \(x \in c^3(A)\). In our previous theorem we have shown that \(c^3(A) \subset c^3(A, f)\).

Hence the proof of the theorem is complete. \(\square\)

**Theorem 2.5.** If \(A = (a_{t,m,n,p,q,r})\) has only positive entries and \(B = (b_{t,m,n,p,q,r})\) is a non-negative six dimensional matrix such that \(\{b_{t,m,n,p,q,r}/a_{t,m,n,p,q,r}\}\) is bounded
then $l_3^{\infty}(A,f) \subset l_3^{\infty}(B,f)$.

Proof. The proof is easy, so omitted. □

**Theorem 2.6.** If $A = (a_{l,m,n,p,q,r})$ is a non-negative six dimensional matrix of real entries with

$$\sup_{l,m,n} \sum_{p=0,q=0,r=0}^{\infty,\infty,\infty} a_{l,m,n,p,q,r} < \infty,$$

and let $f$ be a modulus function then $c_0^3(A,f)$ and $c^3(A,f)$ are complete linear topological spaces with the paranorm

$$g(x) = \sup_{l,m,n} \sum_{p=0,q=0,r=0}^{\infty,\infty,\infty} a_{l,m,n,p,q,r} f(|x_{p,q,r}|),$$

Proof. The space $c_0^3(A,f)$ is a complete linear topological space which is clear from the above statements. Let us consider $c^3(A,f)$. From Theorem 2.2 for each $x \in c^3(A,f)$, $g(x)$ exists. Clearly $g(0) = 0$, $g(-x) = g(x)$ and $g(x+y) \leq g(x) + g(y)$. We shall show now that the scalar multiplication is continuous. First, we observe the following:

$$g(\lambda x) = \sup_{l,m,n} \sum_{p=0,q=0,r=0}^{\infty,\infty,\infty} a_{l,m,n,p,q,r} f(|\lambda x_{p,q,r}|) \leq (1 + ||\lambda||) g(x),$$

where $||\lambda||$ denotes the integer part of $|\lambda|$. In addition, observe that $x$ and $\lambda \to 0$ implies $g(\lambda x) \to 0$. For fixed $\lambda$, if $x$ approaches 0 then $g(\lambda x)$ approaches 0. We have to show that for fixed $x$, $\lambda$ approaching 0 implies $g(\lambda x)$ approaching 0. Let $x \in c^3(A,f)$, so this implies that

$$P - \lim_{l,m,n} \sum_{p=0,q=0,r=0}^{\infty,\infty,\infty} a_{l,m,n,p,q,r} f(|x_{p,q,r} - L|) = 0,$$

Let $\epsilon > 0$ and choose $N$ such that

$$\sum_{p=0,q=0,r=0}^{\infty,\infty,\infty} a_{l,m,n,p,q,r} f(|x_{p,q,r} - L|) < \epsilon/4 \quad (2.2)$$

for $l,m,n > N$. Also for each $(l,m,n)$ with $1 \leq l,m,n \leq N$, since

$$\sum_{p=0,q=0,r=0}^{\infty,\infty,\infty} a_{l,m,n,p,q,r} f(|x_{p,q,r} - L|) < \infty$$

There exists an integer $M_{l,m,n}$ such that

$$\sum_{p,q,r > M_{l,m,n}} a_{l,m,n,p,q,r} f(|x_{p,q,r} - L|) < \epsilon/4.$$
Let $M = \max_{1 \leq l, m, n \leq N} \{M_{l, m, n}\}$

We have for each $(l, m, n)$ with $1 \leq l, m, n \leq N$,

$$\sum_{p, q, r > M} a_{l, m, n, p, q, r} f(|x_{p, q, r} - L|) < \epsilon/4$$

From the equation (2.2) for $l, m, n > N$ we obtain the following

$$\sum_{p, q, r > M} a_{l, m, n, p, q, r} f(|x_{p, q, r} - L|) < \epsilon/4$$

Thus $M$ is an integer which is independent of $(l, m, n)$ such that

$$\sum_{p, q, r > M} a_{l, m, n, p, q, r} f(|x_{p, q, r} - L|) < \epsilon/4 \quad (2.3)$$

Further for $|\lambda| < 1$ and for all $(l, m, n)$

$$\sum_{p=0}^{\infty} \sum_{q=0}^{\infty} \sum_{r=0}^{\infty} a_{l, m, n, p, q, r} f(|\lambda x_{p, q, r}|)$$

$$= \sum_{p=0}^{\infty} \sum_{q=0}^{\infty} \sum_{r=0}^{\infty} a_{l, m, n, p, q, r} f(|\lambda x_{p, q, r} - \lambda L + \lambda L|)$$

$$\leq \sum_{p=0}^{\infty} \sum_{q=0}^{\infty} \sum_{r=0}^{\infty} a_{l, m, n, p, q, r} f(|\lambda x_{p, q, r} - \lambda L|) + \sum_{p=0}^{\infty} \sum_{q=0}^{\infty} \sum_{r=0}^{\infty} a_{l, m, n, p, q, r} f(|\lambda L|) \quad (2.4)$$

$$\leq \sum_{p, q, r > M} a_{l, m, n, p, q, r} f(|\lambda x_{p, q, r} - \lambda L|) + \sum_{p, q, r \leq M} a_{l, m, n, p, q, r} f(|\lambda x_{p, q, r} - \lambda L|)$$

$$+ \sum_{p, q, r > M} a_{l, m, n, p, q, r} f(|\lambda x_{p, q, r} - L|) + \sum_{p, q, r \leq M} a_{l, m, n, p, q, r} f(|\lambda x_{p, q, r} - L|)$$

$$+ \sum_{p, q, r > M} a_{l, m, n, p, q, r} f(|\lambda x_{p, q, r} - \lambda L|)$$

$$+ \sum_{p, q, r \leq M} a_{l, m, n, p, q, r} f(|\lambda x_{p, q, r} - \lambda L|) + \sum_{p, q, r > M} a_{l, m, n, p, q, r} f(|\lambda x_{p, q, r} - \lambda L|)$$

$$+ f(|\lambda L|) \sum_{p=0}^{\infty} \sum_{q=0}^{\infty} \sum_{r=0}^{\infty} a_{l, m, n, p, q, r}$$

For each $(l, m, n)$ and by the continuity of modulus functions as $\lambda \to \infty$ implies

$$\sum_{p, q, r \leq M} a_{l, m, n, p, q, r} f(|\lambda x_{p, q, r} - \lambda L|) + f(|\lambda L|) \sum_{p=0}^{\infty} \sum_{q=0}^{\infty} \sum_{r=0}^{\infty} a_{l, m, n, p, q, r} \to 0$$

Using Pringsheim sense. We choose $\delta < 1$ such that $|\lambda| < \delta$ implies
\[
\sum_{p, q, r \leq M} a_{l, m, n, p, q, r} f(|\lambda x_{p, q, r} - \lambda L|) + f(|\lambda L|) \sum_{p=0, q=0, r=0}^{\infty, \infty, \infty} a_{l, m, n, p, q, r} < \epsilon/4 \quad (2.5)
\]

In a similar way, we can conclude that
\[
\sum_{p, q \geq M, r < M} a_{l, m, n, p, q, r} f(|\lambda x_{p, q, r} - \lambda L|) < \epsilon/4 \quad (2.6)
\]
\[
\sum_{p, q < M, r \geq M} a_{l, m, n, p, q, r} f(|\lambda x_{p, q, r} - \lambda L|) < \epsilon/4 \quad (2.7)
\]
\[
\sum_{p, r \geq M, q < M} a_{l, m, n, p, q, r} f(|\lambda x_{p, q, r} - \lambda L|) < \epsilon/4 \quad (2.8)
\]
\[
\sum_{p, r < M, q \geq M} a_{l, m, n, p, q, r} f(|\lambda x_{p, q, r} - \lambda L|) < \epsilon/4 \quad (2.9)
\]
\[
\sum_{q, r \geq M, p < M} a_{l, m, n, p, q, r} f(|\lambda x_{p, q, r} - \lambda L|) < \epsilon/4 \quad (2.10)
\]
\[
\sum_{q, r < M, p \geq M} a_{l, m, n, p, q, r} f(|\lambda x_{p, q, r} - \lambda L|) < \epsilon/4 \quad (2.11)
\]

It follows from (2.3) through (2.11) that
\[
\sum_{p=0, q=0, r=0}^{\infty, \infty, \infty} a_{l, m, n, p, q, r} f(|\lambda x_{p, q, r}|) < \epsilon \text{ for all } (l, m, n)
\]

Thus \(g(\lambda x)\) approaches 0 as \(\lambda\) approaches 0. Therefore \(c_0^3(A, f)\) is a paranormed
linear topological space. Now we have to show that \(c_0^3(A, f)\) is complete with respect to its paranorm topologies. Let \((x_{s, p, q, r}^n)\) be a Cauchy sequence in \(c_0^3(A, f)\).

Then we can write \(g(x_s^* - x_t^*) \to 0\) as \(s, t \to \infty\) for all \((l, m, n)\)
\[
\sum_{p=0, q=0, r=0}^{\infty, \infty, \infty} a_{l, m, n, p, q, r} f(|x_{p, q, r}^s - x_{p, q, r}^t|) \to 0 \quad (2.12)
\]

Since \(A = (a_{l, m, n, p, q, r})\) is non-negative, we conclude that \(f(|x_{p, q, r}^s - x_{p, q, r}^t|) \to 0\)
as \(s, t \to \infty\), for each fixed \(p, q, r\) and by continuity of modulus function, \((x_{s, p, q, r})\) is a
Cauchy sequence in \(C\) for each fixed \(p, q, r\). Since \(C\) is complete as \(s \to \infty\) we have
\(x_{s, p, q, r}^* \to x_{p, q, r}\) for each \((p, q, r)\). Now from (2.12) we get for for each fixed \(\epsilon > 0\),
there exists a natural number \(N\) such that
\[
\sum_{p, q, r=0, s, t> N} a_{l, m, n, p, q, r} f(|x_{p, q, r}^s - x_{p, q, r}^t|) < \epsilon \quad (2.13)
\]

For all \((l, m, n)\), Since for any fixed natural number \(M\) we have from (2.13)
\[
\sum_{p, q, r \leq M, s, t> N} a_{l, m, n, p, q, r} f(|x_{p, q, r}^s - x_{p, q, r}^t|) < \epsilon
\]
From the above inequality and supposing \( t \to \infty \), for all \((l, m, n)\), we obtain
\[
\sum_{p, q, r \leq M, s > N} a_{l, m, n, p, q, r} f(|x_{s}^{p, q, r} - x_{p, q, r}|) < \epsilon
\]
Since \( M \) is arbitrary, letting \( M \to \infty \), we get \((x^{s})\) being a sequence in
\[
\sum_{p=0, q=0, r=0}^{\infty} a_{l, m, n, p, q, r} f(|x_{p, q, r}^{s} - x_{p, q, r}|) < \epsilon
\]
for all \((l, m, n)\). Thus \( g(x^{s} - x) \to 0 \) as \( s \to \infty \). Also for \( c^{3}(A, f) \), we have by definition of \( c^{3}(A, f) \) for each \( s \) that there exists \( L^{s} \) with
\[
\sum_{p=0, q=0, r=0}^{\infty} a_{l, m, n, p, q, r} f(|x_{p, q, r}^{s} - L^{s}|) \to 0
\]
As \((l, m, n) \to \infty \) and \( \sup_{l, m, n} \sum_{p=0, q=0, r=0}^{\infty} a_{l, m, n, p, q, r} < \infty \) from the condition (2) of modulus function, we have \( f(|L^{s} - L^{t}|) \to 0 \) as \( s, t \to \infty \) and thus \( L^{s} \) converges to \( L \). Hence
\[
\sum_{p=0, q=0, r=0}^{\infty} a_{l, m, n, p, q, r} f(|x_{p, q, r} - L|) \to 0
\]
As \((l, m, n) \to \infty \), thus \( x \in c^{3}(A, f) \) and this completes the proof.\( \square \)

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