(ψ, γ, 2)-CHEREDNIK-OPDAM LIPSCHITZ FUNCTIONS IN THE SPACE $L^{2}_{\alpha, \beta}(\mathbb{R})$

Radouan Daher and Salah El Ouadih

Abstract. In this paper, using a generalized translation operator, we obtain an analog of Younis Theorem 5.2 in [3] for the Cherednik-Opdam transform for functions satisfying the ($\psi, \gamma, 2$)-Cherednik-Opdam Lipschitz condition in the space $L^{2}_{\alpha, \beta}(\mathbb{R})$.

Keywords: Cherednik-Opdam operator; Cherednik-Opdam transform; generalized translation.

1. Introduction and Preliminaries

Various investigators such as V.N. Mishra and L.N. Mishra [7], Mishra and al. [5, 6] have determined the degree of approximation of $2\pi$-periodic signals (functions) belonging to various classes $Lip_0, Lip(\alpha, r), Lip(\xi(t), r)$ and $W(L_r, \xi(t))$, ($r \geq 1$), of functions through trigonometric Fourier approximation using different summability matrices with monotone rows. In this direction, Younis Theorem 5.2 [3] characterized the set of functions in $L^2(\mathbb{R})$ satisfying the Cauchy Lipschitz condition by means of an asymptotic estimate growth of the norm of their Fourier transforms, namely we have

Theorem 1.1. [3] Let $f \in L^2(\mathbb{R})$. Then the following are equivalents

(i) $\|f(x + h) - f(x)\| = O\left(\frac{h^\delta}{(\log h)^\gamma}\right)$, as $h \to 0$, $0 < \delta < 1$, $\gamma \geq 0$,

(ii) $\int_{|\lambda| \geq r} |\hat{f}(\lambda)|^2 d\lambda = O\left(\frac{r^{-2\delta}}{(\log r)^{2\gamma}}\right)$, as $r \to \infty$,

where $f$ stands for the Fourier transform of $f$.

In this paper, we prove the generalization of Theorem 1.1 for the Cherednik-Opdam transform for functions satisfying the ($\psi, \gamma, 2$)-Cherednik-Opdam Lipschitz condition in the space $L^2_{\alpha, \beta}(\mathbb{R})$. For this purpose, we use the generalized translation operator. We point out that similar results have been established in the Jacobi...
In this section, we develop some results from harmonic analysis related to the differential-difference operator $T^{(\alpha,\beta)}$. Further details can be found in [1] and [2]. In the following we fix parameters $\alpha$, $\beta$ subject to the constraints $\alpha \geq \beta \geq -\frac{1}{2}$ and $\alpha > -\frac{1}{2}$.

Let $\rho = \alpha + \beta + 1$ and $\lambda \in \mathbb{C}$. The Opdam hypergeometric functions $G^{(\alpha,\beta)}_\lambda$ on $\mathbb{R}$ are eigenfunctions $T^{(\alpha,\beta)}G^{(\alpha,\beta)}_\lambda(x) = i\lambda G^{(\alpha,\beta)}_\lambda(x)$ of the differential-difference operator

$$T^{(\alpha,\beta)}f(x) = f'(x) + [(2\alpha + 1)\coth x + (2\beta + 1)\tanh x]f(x) - f(-x) - \rho f(-x),$$

that are normalized such that $G^{(\alpha,\beta)}_\lambda(0) = 1$. In the notation of Cherednik one would write $T^{(\alpha,\beta)}$ as

$$T(k_1 + k_2)f(x) = f'(x) + \left\{ \frac{2k_1}{1 + e^{-2x}} + \frac{4k_2}{1 - e^{-4x}} \right\} (f(x) - f(-x)) - (k_1 + 2k_2)f(x),$$

with $\alpha = k_1 + k_2 - \frac{1}{2}$ and $\beta = k_2 - \frac{1}{2}$. Here $k_1$ is the multiplicity of a simply positive root and $k_2$ the (possibly vanishing) multiplicity of a multiple of this root. By [1] or [2], the eigenfunction $G^{(\alpha,\beta)}_\lambda$ is given by

$$G^{(\alpha,\beta)}_\lambda = \phi^{\alpha,\beta}_\lambda(x) = \frac{1}{\rho - i\lambda} \frac{\partial}{\partial x} \phi^{\alpha,\beta}_\lambda(x) = \phi^{\alpha,\beta}_\lambda(x) + \frac{\rho}{4(\alpha + 1)} \sinh(2x)\phi^{\alpha + 1,\beta + 1}_\lambda(x),$$

where $\phi^{\alpha,\beta}_\lambda(x) = 2 F_1\left(\frac{2 + i\lambda}{2}, \frac{2 - i\lambda}{2}; \alpha + 1; -\sinh^2 x\right)$ is the classical Jacobi function.

**Lemma 1.1.** [4] The following inequalities are valid for Jacobi functions $\phi^{\alpha,\beta}_\lambda(x)$

(i) $|\phi^{\alpha,\beta}_\lambda(x)| \leq 1$.

(ii) $1 - \phi^{\alpha,\beta}_\lambda(x) \leq x^2(\lambda^2 + \rho^2)$.

(iii) There is a constant $c > 0$ such that

$$1 - \phi^{\alpha,\beta}_\lambda(x) \geq c,$$

for $\lambda x \geq 1$.

Denote $L^2_{\alpha,\beta}(\mathbb{R})$, the space of measurable functions $f$ on $\mathbb{R}$ such that

$$\|f\|_{2,\alpha,\beta} = \left( \int_{\mathbb{R}} |f(x)|^2 A_{\alpha,\beta}(x) dx \right)^{1/2} < +\infty,$$

where

$$A_{\alpha,\beta}(x) = (\sinh |x|)^{2\alpha + 1}(\cosh |x|)^{2\beta + 1}.$$

The Cherednik-Opdam transform of $f \in C_c(\mathbb{R})$ is defined by

$$\mathcal{H}f(\lambda) = \int_{\mathbb{R}} f(x)G^{(\alpha,\beta)}_{\lambda}(x)dx \quad \text{for all} \quad \lambda \in \mathbb{C}.$$
The inverse transform is given as
\[ \mathcal{H}^{-1} g(x) = \int_{\mathbb{R}} g(\lambda) G_\lambda^{(\alpha, \beta)}(x) \left(1 - \frac{\rho}{i\lambda}\right) \frac{d\lambda}{8\pi|c_{\alpha, \beta}(\lambda)|^2}, \]
here
\[ c_{\alpha, \beta}(\lambda) = \frac{2^{\rho - i\lambda} \Gamma(\alpha + 1) \Gamma(i\lambda)}{\Gamma(\frac{1}{2}(\rho + i\lambda)) \Gamma(\frac{1}{2}(\alpha - \beta + 1 + i\lambda))}. \]

The corresponding Plancherel formula was established in [1], to the effect that
\[ \int_{\mathbb{R}} |f(x)|^2 A_{\alpha, \beta}(x) dx = \int_{0}^{+\infty} \left(|\mathcal{H}f(\lambda)|^2 + |\mathcal{H}f(\lambda)|^2\right) d\sigma(\lambda), \]
where \( \mathcal{H}f = f(-x) \) and \( d\sigma \) is the measure given by
\[ d\sigma(\lambda) = \frac{d\lambda}{16\pi|c_{\alpha, \beta}(\lambda)|^2}. \]

According to [2] there exists a family of signed measures \( \mu_{\alpha, \beta}^{(x, y)} \) such that the product formula
\[ G_\lambda^{(\alpha, \beta)}(x) G_\lambda^{(\alpha, \beta)}(y) = \int_{\mathbb{R}} G_\lambda^{(\alpha, \beta)}(z) d\mu_{\alpha, \beta}^{(x, y)}(z) \]
holds for all \( x, y \in \mathbb{R} \) and \( \lambda \in \mathbb{C} \), where
\[ d\mu_{\alpha, \beta}^{(x, y)}(z) = \begin{cases} K_{\alpha, \beta}(x, y, z) A_{\alpha, \beta}(z) dz, & \text{if } xy \neq 0 \\ d\delta_x(z), & \text{if } y = 0 \\ d\delta_y(z), & \text{if } x = 0 \end{cases} \]
and
\[ K_{\alpha, \beta}(x, y, z) = M_{\alpha, \beta}| \sinh x \cdot \sinh y \cdot \sinh z |^{-2\alpha} \int_{0}^{\pi} g(x, y, z, \chi) \Gamma^{\alpha - \beta - 1} \times [1 - \sigma_{x, y, z} - \sigma_{x, z, y} - \sigma_{z, y, x} + \frac{\rho}{\beta + \frac{1}{2}} \coth x \cdot \coth y \cdot \coth z (\sin \chi)^2] \times (\sin \chi)^{2\beta} d\chi \]
if \( x, y, z \in \mathbb{R} \setminus \{0\} \) satisfy the triangular inequality \( ||x| - |y|| < |z| < |x| + |y| \), and \( K_{\alpha, \beta}(x, y, z) = 0 \) otherwise. Here
\[ \sigma_{x, y, z} = \begin{cases} \cosh z + \cosh y - \cosh x \cdot \cosh z \cdot \cosh y \cdot \sinh z \cdot \sinh y, & \text{if } xy \neq 0 \\ 0, & \text{if } xy = 0 \end{cases} \]
and \( g(x, y, z, \chi) = 1 - \cosh^2 x - \cosh^2 y \cdot \cosh^2 z + 2 \cosh x \cdot \cosh y \cdot \cosh z \cdot \cos \chi \).

**Lemma 1.2.** [2] For all \( x, y \in \mathbb{R} \), we have
(i) \( K_{\alpha, \beta}(x, y, z) = K_{\alpha, \beta}(y, x, z) \).
(ii) \( K_{\alpha, \beta}(x, y, z) = K_{\alpha, \beta}(-x, z, y) \).
(iii) \( K_{\alpha, \beta}(x, y, z) = K_{\alpha, \beta}(-z, y, -x) \).
The product formula is used to obtain explicit estimates for the generalized translation operators

$$
\tau_x^{(\alpha,\beta)} f(y) = \int_{\mathbb{R}} f(z) d\mu_x^{(\alpha,\beta)}(z).
$$

It is known from [2] that

$$
(H \tau_x^{(\alpha,\beta)} f)(\lambda) = G^{(\alpha,\beta)}(\lambda) \mathcal{H} f(\lambda),
$$

for $f \in C_c(\mathbb{R})$.

2. Main Result

In this section we give the main result of this paper. We need first to define $(\psi, \gamma, 2)$-Cherednik-Opdam Lipschitz class.

Denote $N_h$ by

$$
N_h = \tau_h^{(\alpha,\beta)} + \tau^{-\alpha,\beta} - 2I,
$$

where $I$ is the unit operator in the space $L^2_{\alpha,\beta}(\mathbb{R})$.

**Definition 2.1.** Let $\gamma \geq 0$. A function $f \in L^2_{\alpha,\beta}(\mathbb{R})$ is said to be in the $(\psi, \gamma, 2)$-Cherednik-Opdam Lipschitz class, denoted by $\text{Lip}^{(\psi, \gamma, 2)}$, if

$$
\|N_h f(x)\|_{2,\alpha,\beta} = O\left(\frac{\psi(h)}{(\log \frac{1}{h})^{1/2}}\right) \quad \text{as} \quad h \to 0,
$$

where

(a) $\psi$ is a continuous increasing function on $[0, \infty)$,

(b) $\psi(0) = 0$, $\psi(ts) = \psi(t)\psi(s)$ for all $t, s \in [0, \infty)$,

(c) and

$$
\int_0^{1/h} s^2 \psi(s^{-2}) (\log s)^{-2\gamma} ds = O\left(h^{-2}\psi(h^2) \left(\log \frac{1}{h}\right)^{-2\gamma}\right), \quad h \to 0.
$$

**Lemma 2.1.** If $f \in C_c(\mathbb{R})$, then

$$
(2.1) \quad (H \tau_x^{(\alpha,\beta)} f)(\lambda) = G^{(\alpha,\beta)}(\lambda) (-x) \mathcal{H} f(\lambda).
$$

**Proof.** For $f \in C_c(\mathbb{R})$, we have

$$
(2.1) \quad 
\begin{align*}
H \tau_x^{(\alpha,\beta)} f(\lambda) &= \int_{\mathbb{R}} \tau_x^{(\alpha,\beta)} f(-y) G^{(\alpha,\beta)}(\lambda)(-y) A_{\alpha,\beta}(y) dy \\
&= \int_{\mathbb{R}} \tau_x^{(\alpha,\beta)} f(y) G^{(\alpha,\beta)}(\lambda)(y) A_{\alpha,\beta}(y) dy \\
&= \int_{\mathbb{R}} \left[ \int_{\mathbb{R}} f(z) \mathcal{K}_{\alpha,\beta}(x, y, z) A_{\alpha,\beta}(z) dz \right] G^{(\alpha,\beta)}(\lambda)(y) A_{\alpha,\beta}(y) dy \\
&= \int_{\mathbb{R}} f(z) \left[ \int_{\mathbb{R}} G^{(\alpha,\beta)}(\lambda)(y) \mathcal{K}_{\alpha,\beta}(x, y, z) A_{\alpha,\beta}(y) dy \right] A_{\alpha,\beta}(z) dz.
\end{align*}
$$
Since \( K_{\alpha,\beta}(x, y, z) = K_{\alpha,\beta}(-x, z, y) \), it follows from the product formula that
\[
\mathcal{H}^{(\alpha,\beta)} f(\lambda) = G^{(\alpha,\beta)}_{\lambda}(-x) \int_{\mathbb{R}} f(z) G^{(\alpha,\beta)}_{\lambda}(z) A_{\alpha,\beta}(z) dz
= G^{(\alpha,\beta)}_{\lambda}(-x) \int_{\mathbb{R}} f(-z) G^{(\alpha,\beta)}_{\lambda}(-z) A_{\alpha,\beta}(z) dz
= G^{(\alpha,\beta)}_{\lambda}(-x) \mathcal{H} f(\lambda).
\]

Lemma 2.2. For \( f \in L^2_{\alpha,\beta}(\mathbb{R}) \), then
\[
\|N_h f(x)\|_{2,\alpha,\beta}^2 = 4 \int_0^{+\infty} \left| \phi^{\alpha,\beta}_{\lambda}(h) - 1 \right|^2 \left( |\mathcal{H} f(\lambda)|^2 + |\mathcal{H} f(\lambda)|^2 \right) d\sigma(\lambda).
\]

Proof. From formulas (1.1) and (2.1), we have
\[
\mathcal{H}(N_h f)(\lambda) = (G^{(\alpha,\beta)}_{\lambda}(h) + G^{(\alpha,\beta)}_{\lambda}(-h) - 2) \mathcal{H}(f)(\lambda),
\]
and
\[
\mathcal{H} (\bar{N}_h f)(\lambda) = (G^{(\alpha,\beta)}_{\lambda}(-h) + G^{(\alpha,\beta)}_{\lambda}(h) - 2) \mathcal{H}(\bar{f})(\lambda).
\]
Since
\[
G^{(\alpha,\beta)}_{\lambda}(h) = \varphi^{\alpha,\beta}_{\lambda}(h) + \frac{\rho}{4(\alpha + 1)} \sinh(2h) \varphi^{\alpha+1,\beta+1}_{\lambda}(h),
\]
and \( \varphi^{\alpha,\beta}_{\lambda} \) is even, then
\[
\mathcal{H}(N_h f)(\lambda) = 2(\varphi^{\alpha,\beta}_{\lambda}(h) - 1) \mathcal{H}(f)(\lambda)
\]
and
\[
\mathcal{H}(\bar{N}_h f)(\lambda) = 2(\varphi^{\alpha,\beta}_{\lambda}(h) - 1) \mathcal{H}(\bar{f})(\lambda).
\]

Now by Plancherel Theorem, we have the result.

Theorem 2.1. Let \( f \in L^2_{\alpha,\beta}(\mathbb{R}) \). Then the following are equivalents

(a) \( f \in \text{Lip}(\psi, \gamma, 2) \),
(b) \( \int_r^{+\infty} \left( |\mathcal{H} f(\lambda)|^2 + |\mathcal{H} f(\lambda)|^2 \right) d\sigma(\lambda) = O \left( \frac{\psi(r^{-2})}{(\log r)^{2\gamma}} \right), \quad \text{as} \quad r \to \infty.
\]

Proof. (a) \(\Rightarrow\) (b) Let \( f \in \text{Lip}(\psi, \gamma, 2) \). Then we have
\[
\|N_h f(x)\|_{2,\alpha,\beta} = O \left( \frac{\psi(h)}{(\log \frac{1}{h})^{\gamma}} \right) \quad \text{as} \quad h \to 0.
\]

From Lemma 2.2, we have
\[
\|N_h f(x)\|_{2,\alpha,\beta}^2 = 4 \int_0^{+\infty} \left| 1 - \varphi^{\alpha,\beta}_{\lambda}(h) \right|^2 \left( |\mathcal{H} f(\lambda)|^2 + |\mathcal{H} f(\lambda)|^2 \right) d\sigma(\lambda).
\]
If \( \lambda \in \left[ \frac{1}{h}, \frac{2}{h} \right] \), then \( \lambda h \geq 1 \) and (iii) of Lemma 1.1 implies that

\[
1 \leq \frac{1}{c^2} |1 - \varphi^{a,\beta}(h)|^2.
\]

Then

\[
\int_{\frac{1}{h}}^{\frac{2}{h}} (|Hf(\lambda)|^2 + |H\tilde{f}(\lambda)|^2) \, d\sigma(\lambda) \leq \frac{1}{c^2} \int_{\frac{1}{h}}^{\frac{2}{h}} |1 - \varphi^{a,\beta}(h)|^2 (|Hf(\lambda)|^2 + |H\tilde{f}(\lambda)|^2) \, d\sigma(\lambda)
\]

\[
\leq \frac{1}{c^2} \int_{0}^{+\infty} |1 - \varphi^{a,\beta}(h)|^2 (|Hf(\lambda)|^2 + |H\tilde{f}(\lambda)|^2) \, d\sigma(\lambda)
\]

\[
\leq \frac{1}{4c} \|N_h f(x)\|_{2,\alpha,\beta}^2
\]

\[
= O \left( \frac{\psi(h^2)}{(\log \frac{1}{h})^{2\gamma}} \right).
\]

We obtain

\[
\int_{r}^{2r} (|Hf(\lambda)|^2 + |H\tilde{f}(\lambda)|^2) \, d\sigma(\lambda) \leq C \frac{\psi(r^{-2})}{(\log r)^{2\gamma}}, \quad r \to \infty,
\]

where \( C \) is a positive constant. Now,

\[
\int_{r}^{+\infty} (|Hf(\lambda)|^2 + |H\tilde{f}(\lambda)|^2) \, d\sigma(\lambda) = \sum_{i=0}^{\infty} \int_{2^i r}^{2^{i+1} r} (|Hf(\lambda)|^2 + |H\tilde{f}(\lambda)|^2) \, d\sigma(\lambda)
\]

\[
\leq C \frac{\psi(r^{-2})}{(\log r)^{2\gamma}} + \frac{\psi((2r)^{-2})}{(\log 2r)^{2\gamma}} + \frac{\psi((4r)^{-2})}{(\log 4r)^{2\gamma}} + \cdots
\]

\[
\leq K_{\psi} \frac{\psi(r^{-2})}{(\log r)^{2\gamma}},
\]

where \( K_{\psi} = C(1 - \psi(2^{-2}))^{-1} \) since \( \psi(2^{-2}) < 1 \).

Consequently

\[
\int_{r}^{+\infty} (|Hf(\lambda)|^2 + |H\tilde{f}(\lambda)|^2) \, d\sigma(\lambda) = O \left( \frac{\psi(r^{-2})}{(\log r)^{2\gamma}} \right), \quad \text{as} \quad r \to \infty.
\]

(b) \( \Rightarrow \) (a). Suppose now that

\[
\int_{r}^{+\infty} (|Hf(\lambda)|^2 + |H\tilde{f}(\lambda)|^2) \, d\sigma(\lambda) = O \left( \frac{\psi(r^{-2})}{(\log r)^{2\gamma}} \right), \quad \text{as} \quad r \to \infty,
\]

and write

\[
\|N_h f(x)\|_{2,\alpha,\beta}^2 = 4(I_1 + I_2),
\]
where

\[ I_1 = \int_0^\frac{1}{h} \left| 1 - \phi_{\lambda}^{\alpha, \beta}(h) \right|^2 \left( |Hf(\lambda)|^2 + |H\tilde{f}(\lambda)|^2 \right) d\sigma \lambda, \]

and

\[ I_2 = \int_\frac{1}{h}^{+\infty} \left| 1 - \phi_{\lambda}^{\alpha, \beta}(h) \right|^2 \left( |Hf(\lambda)|^2 + |H\tilde{f}(\lambda)|^2 \right) d\sigma \lambda. \]

Firstly, we use the formula \( |\phi_{\lambda}^{\alpha, \beta}(h)| \leq 1 \) and

\[ I_2 \leq 4 \int_\frac{1}{h}^{+\infty} \left( |Hf(\lambda)|^2 + |H\tilde{f}(\lambda)|^2 \right) d\sigma (\lambda) = O \left( \frac{\psi(h^2)}{(\log \frac{1}{h})^{2\gamma}} \right), \quad \text{as} \quad h \to 0. \]

To estimate \( I_1 \), we use the inequalities (i) and (ii) of Lemma 1.1

\[ I_1 = \int_0^\frac{1}{h} \left| 1 - \phi_{\lambda}^{\alpha, \beta}(h) \right|^2 \left( |Hf(\lambda)|^2 + |H\tilde{f}(\lambda)|^2 \right) d\sigma \lambda \]

\[ \leq 2 \int_0^\frac{1}{h} \left| 1 - \phi_{\lambda}^{\alpha, \beta}(h) \right| \left( |Hf(\lambda)|^2 + |H\tilde{f}(\lambda)|^2 \right) d\sigma \lambda \]

\[ \leq 2h^2 \int_0^\frac{1}{h} (\lambda^2 + \rho^2) \left( |Hf(\lambda)|^2 + |H\tilde{f}(\lambda)|^2 \right) d\sigma \lambda. \]

Now, we apply integration by parts for a function \( \phi(s) = \int_s^{+\infty} \left( |Hf(\lambda)|^2 + |H\tilde{f}(\lambda)|^2 \right) d\sigma (\lambda) \)

to get

\[ I_1 \leq -2h^2 \int_0^{1/h} (s^2 + \rho^2) \phi'(s) ds \]

\[ \leq -2h^2 \int_0^{1/h} s^2 \phi'(s) ds \]

\[ \leq h^2 \left( -\frac{1}{h^2} \phi(\frac{1}{h}) + 2 \int_0^{1/h} s \phi(s) ds \right) \]

\[ \leq -\phi(\frac{1}{h}) + 2h^2 \int_0^{1/h} s \phi(s) ds \]

\[ \leq 2h^2 \int_0^{1/h} s \phi(s) ds. \]

Since \( \phi(s) = O \left( \frac{\psi(s^{-3})}{(\log s)^{2\gamma}} \right) \), we have \( s \phi(s) = O \left( \frac{\psi(s^{-3})}{(\log s)^{2\gamma}} \right) \) and

\[ \int_0^{1/h} s \phi(s) ds = O \left( \int_0^{1/h} \frac{s \psi(s^{-2})}{(\log s)^{2\gamma}} ds \right) = O \left( \frac{h^{-2} \psi(h^2)}{(\log \frac{1}{h})^{2\gamma}} \right), \]
so that

\[ I_1 = O \left( \frac{\psi(h^2)}{(\log \frac{1}{h})^{2\gamma}} \right). \]

Consequently,

\[ \|N_h f(x)\|_{2,\alpha,\beta} = O \left( \frac{\psi(h)}{(\log \frac{1}{h})^{\gamma}} \right) \text{ as } h \to 0, \]

and this ends the proof of the theorem. \( \square \)

### 3. Conclusion

In this work we have succeeded to generalise the theorem in [3] for the Cherednik-Opdam transform in the space \( L^2_{\alpha,\beta}(\mathbb{R}) \). We proved that \( f(x) \) belong to \( \text{Lip}(\psi, \gamma, 2) \).

Then

\[
\int_{r}^{+\infty} \left( |\mathcal{H}f(\lambda)|^2 + |\mathcal{H}\hat{f}(\lambda)|^2 \right) d\sigma(\lambda) = O \left( \frac{\psi(r^{-2})}{(\log r)^{2\gamma}} \right), \quad \text{as } r \to \infty.
\]

### Acknowledgements

The authors would like to thank the referee for his valuable comments and suggestions.

### References

5. L. N. Mishra, V. N. Mishra, K. Khatri and Deepmala, *On the trigonometric approximation of signals belonging to generalized weighted Lipschitz \( W'(L^r, \xi(t)) \), \( (r \geq 1) \) class by matrix \( (C^1, N_p) \) Operator of conjugate series of its Fourier series*, Applied Mathematics and Computation, Vol. 237, (2014), 252-263.
6. V. N. Mishra, K. Khatri, L. N. Mishra and Deepmala; *Trigonometric approximation of periodic signals belonging to generalized weighted Lipschitz \( W'(L^r, \xi(t)) \), \( (r \geq 1) \) – class by Nörlund-Euler \( (N, p_n)(E, q) \) operator of conjugate series of its Fourier series*, Journal of Classical Analysis, Vol. 5, no. 2 (2014), 91-105. doi:10.7153/jca-05-08.


Radouan Daher
Department of Mathematics
Faculty of Sciences Ain Chock
University Hassan II, Casablanca, Morocco
rjdahe024@gmail.com

Salah El Ouadih
Department of Mathematics
Faculty of Sciences Ain Chock
University Hassan II, Casablanca, Morocco
salahwadih@gmail.com