SOME SEMISYMMETRY CONDITIONS ON RIEMANNIAN MANIFOLDS

Ahmet Yıldız and Azime Çetinkaya

Abstract. We study a Riemannian manifold $M$ admitting a semisymmetric metric connection $\tilde{\nabla}$ such that the vector field $U$ is a parallel unit vector field with respect to the Levi-Civita connection $\nabla$. Firstly, we show that if $M$ is projectively flat with respect to the semisymmetric metric connection $\tilde{\nabla}$ then $M$ is a quasi-Einstein manifold. Also we prove that if $R \cdot \tilde{P} = 0$ if and only if $M$ is projectively semisymmetric; if $\tilde{P} \cdot R = 0$ or $R \cdot \tilde{P} - \tilde{P} \cdot R = 0$ then $M$ is conformally flat and quasi-Einstein manifold. Here $R$, $P$ and $\tilde{P}$ denote Riemannian curvature tensor, the projective curvature tensor of $\nabla$ and the projective curvature tensor of $\tilde{\nabla}$, respectively.

1. Introduction

Let $\tilde{\nabla}$ be a linear connection in an $n$-dimensional differentiable manifold $M$. The torsion tensor $T$ is given by

$$T(X,Y) = \tilde{\nabla}_X Y - \tilde{\nabla}_Y X - [X,Y].$$

The connection $\tilde{\nabla}$ is symmetric if its torsion tensor $T$ vanishes, otherwise it is non-symmetric. If there is a Riemannian metric $g$ in $M$ such that $\tilde{\nabla} g = 0$, then the connection $\tilde{\nabla}$ is a metric connection, otherwise it is non-metric [24]. It is well known that a linear connection is symmetric and metric if and only if it is the Levi-Civita connection.

Hayden [13] introduced a metric connection $\tilde{\nabla}$ with a non-zero torsion on a Riemannian manifold. Such a connection is called a Hayden connection. In [12] and [18], Friedmann and Schouten introduced the idea of a semisymmetric linear connection in a differentiable manifold. A linear connection is said to be a semisymmetric connection if its torsion tensor $T$ is of the form

$$(1.1) \quad T(X,Y) = \omega(Y)X - \omega(X)Y,$$

where the 1-form $\omega$ is defined by

$$\omega(X) = g(X, U),$$

Received September 23, 2013.; Accepted December 30, 2013.
2010 Mathematics Subject Classification. 53C05; 53C07, 53C25
and $U$ is a vector field. In [17], Pak showed that a Hayden connection with the torsion tensor of the form (1.1) is a semisymmetric metric connection. In [23], Yano considered a semisymmetric metric connection and studied some of its properties. He proved that in order that a Riemannian manifold admits a semisymmetric metric connection whose curvature tensor vanishes, it is necessary and sufficient that the Riemannian manifold be conformally flat. For some properties of Riemannian manifolds with a semisymmetric metric connection (see also [1], [6], [4], [5], [7], [14], [21], [22]). Then, Murathan and Özgür [16] studied Riemannian manifolds admitting a semisymmetric metric connection $\tilde{\nabla}$ such that the vector field $U$ is a parallel vector field with respect to the Levi-Civita connection $\nabla$.

On the other hand, if a Riemannian manifold satisfying the condition $R \cdot R = 0$, then the manifold is called semisymmetric ([19], [20]). It is well known that the class of semisymmetric manifolds includes the set of locally symmetric manifolds ($\nabla R = 0$) as a proper subset. A Riemannian manifold is said to be Ricci-semisymmetric if $R \cdot S = 0$. The class of semisymmetric manifolds includes the set of Ricci-semisymmetric manifolds ($\nabla S = 0$) as a proper subset. Evidently, the condition $R \cdot R = 0$ implies condition $R \cdot S = 0$. The converse is in general not true. Also, a Riemannian manifold satisfying the condition $R \cdot P = 0$, then the manifold is called projectively semisymmetric.

Motivated by the studies of the above authors, in this paper we consider Riemannian manifolds $(M, g)$ admitting a semisymmetric metric connection such that $U$ is a unit parallel vector field with respect to the Levi-Civita connection $V$. The paper is organized as follows: In Section 2 and Section 3, we give the necessary notions and results which will be used in the next section. In the last section, firstly we show that if $M$ is projectively flat with respect to the semisymmetric metric connection $\tilde{\nabla}$ then $M$ is a quasi-Einstein manifold. Then we prove that if $R \cdot \tilde{P} = 0$ if and only if $M$ is projectively semisymmetric; if $\tilde{P} \cdot R = 0$ or $R \cdot \tilde{P} - \tilde{P} \cdot R = 0$ then $M$ is conformally flat and quasi-Einstein manifold, where $R$, $P$ and $\tilde{P}$ denote Riemannian curvature tensor, the projective curvature tensor of $V$ and the projective curvature tensor of $\tilde{V}$, respectively.

2. Preliminaries

An $n$-dimensional Riemannian manifold $(M^n, g)$, $n > 2$, is said to be an Einstein manifold if its Ricci tensor $S$ satisfies the condition $S = \frac{\tau}{n} g$, where $\tau$ denotes the scalar curvature of $M$. If the Ricci tensor $S$ is of the form

$$S(X, Y) = ag(X, Y) + bA(X)A(Y),$$

where $a, b$ are smooth functions and $A$ is a non-zero 1-form such that

$$g(X, U) = A(X),$$

for all vector fields $X$. Then $M$ is called a quasi-Einstein manifold [3].
Lemma 2.1. A flat manifold is called conformally flat, where

\[(2.4) \quad (X \wedge E)Y = E(Y, Z)X - E(X, Z)Y,\]

In addition, if \(E\) is a symmetric \((0, 2)\)-tensor field, then we define the \((0, k + 2)\)-tensor \(Q(E, T)\) (see [9]) by

\[(2.3) \quad Q(E, T)(X_1, \ldots, X_k; X, Y) = -T((X \wedge E)Y)X_1, \ldots, X_k) - \cdots - T(X_1, \ldots, X_k, (X \wedge E)Y)X_k),\]

where \(X \wedge E\) is defined by

\[(X \wedge E)Y = E(Y, Z)X - E(X, Z)Y.\]

The Weyl tensor and the projective tensor of a Riemannian manifold \((M, g)\) are defined by

\[C(X, Y, Z, W) = R(X, Y, Z, W) - \frac{1}{n - 2} [S(Y, Z)g(X, W) - S(X, Z)g(Y, W)] + g(Y, Z)S(X, W) - g(X, Z)S(Y, W) + \frac{\tau}{(n - 1)(n - 2)} [g(Y, Z)g(X, W) - g(X, Z)g(Y, W)],\]

and

\[P(X, Y, Z, W) = R(X, Y, Z, W) - \frac{1}{n - 1} [S(Y, Z)g(X, W) - S(X, Z)g(Y, W)].\]

respectively, where \(\tau\) denotes the scalar curvature of \(M\). For \(n \geq 4\), if \(C = 0\), the manifold is called conformally flat [24]. If \(P = 0\), the manifold is called projectively flat.

Now we give the Lemmas which will be used in the last section.

**Lemma 2.1.** [10] Let \((M^n, g), n \geq 3\), be a semi-Riemannian manifold. Let at a point \(x \in M\) be given a non-zero symmetric \((0, 2)\)-tensor \(E\) and a generalized curvature tensor \(B\) such that at \(x\) the following condition is satisfied \(Q(E, B) = 0\). Moreover, let \(V\) be a vector at \(x\) such that the scalar \(\rho = a(V)\) is non-zero, where \(a\) is a covector defined by \(a(X) = E(X, V)\), \(X \in T_x M\).

i) If \(E = \frac{1}{\rho} a \otimes a\), then at \(x\) we have \(X, Y, Z)B(Y, Z) = 0\), where \(X, Y, Z) \in T_x M\).

ii) If \(E = \frac{1}{\rho} a \otimes a\) is non-zero, then at \(x\) we have \(B = \frac{1}{2} E \wedge E, \gamma \in \mathbb{R}\). Moreover, in both cases, at \(x\) we have \(B \cdot B = Q(\text{Ric}(B), B)\).

**Lemma 2.2.** [11] Let \((M^n, g), n \geq 4\), be a semi-Riemannian manifold and \(E\) be the symmetric \((0, 2)\)-tensor at \(x \in M\) defined by \(E = \rho g + \beta \omega \otimes \omega\), \(\omega \in T^*_x M\), \(\alpha, \beta, \gamma \in \mathbb{R}\). If at \(x\) the curvature tensor \(R\) is expressed by \(R = \frac{1}{2} E \wedge E, \gamma \in \mathbb{R}\), then the Weyl tensor vanishes at \(x\).
3. Semisymmetric metric connection

Let $\nabla$ be the Levi-Civita connection of a Riemannian manifold $M$. It is known [23] that if $\tilde{\nabla}$ is a semisymmetric metric connection then

$$\tilde{\nabla}_X Y = \nabla_X Y + \omega(Y)X - g(X, Y)U,$$

where

$$\omega(X) = g(X, U),$$

and $X, Y, U$ are vector fields on $M$. Let $R$ and $\tilde{R}$ denote the Riemannian curvature tensor of $\nabla$ and $\tilde{\nabla}$, respectively. Then we know [23] that

$$\tilde{R}(X, Y, Z, W) = R(X, Y, Z, W) - \theta(Y, Z)g(X, W) + \frac{1}{2}g(X, Y).$$

Now assume that $U$ is a parallel unit vector field with respect to the Levi-Civita connection $\nabla$, i.e., $\nabla U = 0$ and $\|U\| = 1$. Then

$$(\nabla_X \omega)Y = \nabla_X \omega(Y) - \omega(X)\omega(Y) + \frac{1}{2}g(X, Y).$$

So $\theta$ is a symmetric $(0, 2)$-tensor field. Hence equation (3.1) can be written as

$$(\tilde{\nabla}_X U) = X - \omega(X)U,$$

where $\tilde{\nabla}$ is Kulkarni-Nomizu product, which is defined by

$$(g \tilde{\nabla} \theta)(X, Y, Z, W) = \theta(Y, Z)g(X, W) - \theta(X, Z)g(Y, W) + g(Y, Z)\theta(X, W) - g(X, Z)\theta(Y, W).$$

Since $U$ is a parallel unit vector field, it is easy to see that $\tilde{R}$ is a generalized curvature tensor and it is trivial that $R(X, Y)U = 0$. Hence by a contraction we find $S(Y, U) = \omega(SY)$, where $S$ denotes the Ricci tensor of $\nabla$ and $S$ is the Ricci operator defined by $g(SX, Y) = S(X, Y)$. It is easy to see that we also have the following relations [16]:

$$\tilde{\nabla}_X U = X - \omega(X)U,$$

$$(3.5) \quad \tilde{R}(X, Y)U = 0, \quad \tilde{R} \cdot \theta = 0,$$

$$\theta^2(X, Y) := g(AX, AY) = \frac{1}{4}g(X, Y),$$

and

$$(3.6) \quad \tilde{S} = S - (n - 2)(g - \omega \otimes \omega),$$
(3.7) \[ \tilde{\tau} = \tau - (n - 2)(n - 1). \]

Using (2.4), (3.1), (3.6) and (3.7), we get
\[ \tilde{\mathcal{C}} = C, \]
and
\[ (3.8) \]
\[ \tilde{P} = P - \frac{1}{n-1} g \tilde{\nabla} \theta + G, \]
where \( \tilde{P} \) denotes the projective curvature tensor with respect to semisymmetric metric tensor \( \tilde{\nabla} \) and \( G \) is defined by
\[ (3.9) \]
\[ G(X, Y, Z, W) = \frac{n-2}{n-1} \{ g(Y, Z)\omega(X)\omega(W) - g(X, Z)\omega(Y)\omega(W) \}. \]

We also have the followings:
\[ (3.10) \]
\[ \tilde{P}(X, Y)U = 0, \]
\[ (3.11) \]
\[ R \cdot G = 0, \quad G \cdot R = 0. \]

4. Main results

In this section, the tensors \( \tilde{P}, \tilde{P} \cdot R \) and \( Q(\theta, T) \) are defined in the same way with (3.8), (2.2) and (2.3). Let \( \tilde{P}_{hijkl}, (R \cdot \tilde{P})_{hijklm}, (\tilde{P} \cdot R)_{hijklm} \) denote the local components of the tensors \( \tilde{P}, R \cdot \tilde{P} \) and \( \tilde{P} \cdot R \), respectively.

Theorem 4.1. Let \( (M, g) \) be a Riemannian manifold admitting a semisymmetric metric connection. If \( M \) is projectively flat with respect to semisymmetric metric tensor \( \tilde{\nabla} \), then \( M \) is a quasi-Einstein manifold.

Proof. Let \( (M, g) \) be a Riemannian manifold admitting a semisymmetric metric connection. Then using (2.4) and (3.1) we have
\[ \tilde{P}_{hijk} = R_{hijk} - (g \tilde{\nabla} \theta)_{hijk} \]
\[ (4.1) \]
\[ - \frac{1}{n-1} \{ S_{ij}g_{hk} - (n-2)[g_{ij}g_{hk} - g_{hk}(\omega \otimes \omega)_{ij}] \]
\[ - S_{hjk}g_{ik} + (n-2)[g_{hj}g_{k} - g_{k}(\omega \otimes \omega)_{hj}] \}. \]

Now if \( M \) is projectively flat with respect to semisymmetric metric tensor \( \tilde{\nabla} \), then from (4.1) we have
\[ \tilde{R}_{hijk} = (g \tilde{\nabla} \theta)_{hijk} \]
\[ + \frac{1}{n-1} S_{ij}g_{hk} - S_{ij}g_{hk} \]
\[ + \frac{n-2}{n-1} \{ g_{hk}(\omega \otimes \omega)_{ij} - g_{ik}(\omega \otimes \omega)_{hj} - g_{ij}g_{hk} + g_{ij}g_{hk} \} \].
Help of (3.4), we get
\[
R_{hijk} = \left\{ g_{ij}g_{hk} - g_{ik}g_{hj} \right\}
+ \frac{1}{n-1} \left\{ S_{ij}g_{hk} - S_{ij}g_{lk} \right\}
\frac{n-2}{n-1} \left\{ g_{ij}g_{hk} - g_{ij}g_{hk} - g_{hk}(\omega \otimes \omega)_{ij} + g_{ik}g_{hj} \right\}
+ g_{ik}(\omega \otimes \omega)_{jk} - g_{ik}(\omega \otimes \omega)_{ik}
+ g_{ij}(\omega \otimes \omega)_{hk} - g_{ij}(\omega \otimes \omega)_{hi}.
\] (4.2)

Contracting (4.2) with \(g_{hi}^j\), we obtain
\[
S_{ik} = \frac{n + \tau - 2}{n} g_{ik} + (2 - n)(\omega \otimes \omega)_{ik},
\]
which gives us that \(M\) is a quasi-Einstein manifold. \(\square\)

**Proposition 4.1.** Let \((M, g)\) be a Riemannian manifold admitting a semisymmetric metric connection \(\tilde{\nabla}\). If \(U\) is a parallel unit vector field with respect to the Levi-Civita connection \(\nabla\), then
\[
(R \cdot \tilde{P})_{hijklm} = (R \cdot P)_{hijklm}
\] (4.3)

\[
(P \cdot R)_{hijklm} = (P \cdot R)_{hijklm} - \frac{1}{n-1} Q(g - \omega \otimes \omega, R)_{hijklm}.
\] (4.4)

**Proof.** Since \(U\) is parallel, we have \(R \cdot \theta = 0\) and \(R \cdot G = 0\). So from (3.8), we obtain
\[
R \cdot \tilde{P} = R \cdot P - \frac{1}{n-1} g \tilde{\nabla} R \cdot \theta + R \cdot G = R \cdot P.
\] (4.5)

Applying (3.1) in (2.2) and using (2.3) and (3.11), we get
\[
(P \cdot R)_{hijklm} = (P \cdot R)_{hijklm} - \frac{1}{n-1} Q(\theta, R)_{hijklm}
- \frac{1}{2(n-1)} (g_{ih}R_{mjlk} - g_{im}R_{hjlk} - g_{ij}R_{mhkl})
+ g_{im}R_{ihjk} - g_{ij}R_{mkhi} - g_{jm}R_{likh}
- g_{ik}R_{mjhl} + g_{km}R_{ihlj} + (G \cdot R)_{hijklm}
\]
(4.6)

\[= (P \cdot R)_{hijklm} - \frac{1}{n-1} Q(g - \omega \otimes \omega, R)_{hijklm}
= (P \cdot R)_{hijklm} - \frac{1}{n-1} Q(\theta + \frac{1}{2} g, R)_{hijklm}.
\]

This completes the proof of the Proposition. \(\square\)
As an immediate consequence of Proposition 4.1, we have the followings:

**Theorem 4.2.** Let \((M, g)\) be a Riemannian manifold admitting a semisymmetric metric connection \(\tilde{\nabla}\) and \(U\) be a parallel unit vector field with respect to the Levi-Civita connection \(\nabla\). Then \(R \cdot \tilde{\nabla} = 0\) if and only if \(M\) is projectively semisymmetric.

**Theorem 4.3.** Let \((M^n, g)\) be a semisymmetric \(n > 3\) dimensional Riemannian manifold admitting a semisymmetric metric connection \(\tilde{\nabla}\) and \(S\) be the symmetric \((0, 2)\)-tensor defined by \(S = \alpha g + \beta \omega \otimes \omega\). If \(U\) is a parallel unit vector field with respect to the Levi-Civita connection \(\nabla\) and \(\tilde{\nabla} \cdot R = 0\), then \(M\) is a conformally flat quasi-Einstein manifold.

**Proof.** Since the condition \(\tilde{\nabla} \cdot R = 0\) holds on \(M\), from (4.4), we have

\[(P \cdot R)_{ijklm} = \frac{1}{n-1} Q(g - \omega \otimes \omega, R)_{ijklm} \text{ (4.7)}\]

After some calculations from (4.7), we get

\[(R \cdot R)_{ijklm} - \frac{1}{n-1} Q(S, R)_{ijklm} = \frac{1}{n-1} Q(g - \omega \otimes \omega, R)_{ijklm}. \text{ (4.8)}\]

Since \(M\) is a semisymmetric Riemannian manifold, then from (4.8), we have

\[Q(S + g - \omega \otimes \omega, R)_{ijklm} = 0. \text{ (4.9)}\]

Now let \(S = \alpha g + \beta \omega \otimes \omega\), \(\alpha, \beta \in \mathbb{R}\). Then from (4.9), we get

\[Q(\lambda_1 g - \lambda_2 \omega \otimes \omega, R)_{ijklm} = 0, \text{ (4.10)}\]

where \(\lambda_1 = \alpha + 1, \lambda_2 = \beta + 1\). So we have two possibilities:

\[\text{rank}(\lambda_1 g - \lambda_2 \omega \otimes \omega) = 1 \text{ (4.11)}\]

or

\[\text{rank}(\lambda_1 g - \lambda_2 \omega \otimes \omega) > 1. \text{ (4.12)}\]

Suppose that (4.11) holds at a point \(x\). Thus we have

\[\lambda_1 g - \lambda_2 \omega \otimes \omega = \rho z \otimes z, \text{ where } z \in T_x^*M \text{ and } \rho \in \mathbb{R}. \]

Because of non-zero coefficient of \(g\), this relation does not occur. Thus the case (4.12) must be fulfilled at \(x\). By virtue of Lemma 2.1, (4.10) gives us

\[R = \frac{\gamma}{2} ((g - \omega \otimes \omega) \wedge (g - \omega \otimes \omega)), \gamma \neq 0, \gamma \in \mathbb{R}. \]

So again from Lemma 2.2, we obtain \(C = 0\), which give us that \(M\) is conformally flat. Moreover, contracting (4.10) with \(g^{ij}\), we get

\[Q(\lambda_1 g - \lambda_2 \omega \otimes \omega, S)_{ijklm} = 0, \text{ where } \lambda_1, \lambda_2 : M \rightarrow \mathbb{R} \text{ are functions. So by virtue of (2.1), } M \text{ is a quasi-Einstein manifold. Thus the proof of the Theorem is completed.}\]
Theorem 4.4. Let $(M^n, g)$ be a Ricci-semisymmetric $n > 3$ dimensional Riemannian manifold admitting a semisymmetric metric connection $\tilde{\nabla}$ and $U$ be a parallel unit vector field with respect to the Levi-Civita connection $\nabla$ and $R \cdot \tilde{P} - \tilde{P} \cdot R = 0$, then $M$ is a conformally flat quasi-Einstein manifold.

Proof. Using (4.3) and (4.4), we obtain

$$0 = R \cdot \tilde{P} - \tilde{P} \cdot R = R \cdot P - P \cdot R + \frac{1}{n-1} Q(g - \omega \otimes \omega, R)_{ijklm}$$

$$= \frac{1}{n-1} g_{ik}(R \cdot S)_{jilm} - \frac{1}{n-1} g_{ik}(R \cdot S)_{jilm}$$

$$+ \frac{1}{n-1} Q(S + g - \omega \otimes \omega, R)_{ijklm}.$$

Since $M$ is a Ricci-semisymmetric Riemannian manifold, (i.e. $R \cdot S = 0$), then from the above equation, we get

$$Q(S + g - \omega \otimes \omega, R)_{ijklm} = 0. \tag{4.13}$$

Using the same method in the proof of Theorem 4.3, we obtain $M$ is a conformally flat quasi-Einstein manifold. \(\square\)

Example 4.1. Let $M^{2n+1}$ be a $(2n + 1)$-dimensional almost contact manifold endowed with an almost contact structure $(\phi, \xi, \eta)$, that is, $\phi$ is a $(1, 1)$-tensor field, $\xi$ is a vector field, and $\eta$ is a 1-form such that

$$\phi^2 = I - \eta \otimes \xi \quad \text{and} \quad \eta(\xi) = 1$$

Then

$$\phi(\xi) = 0 \quad \text{and} \quad \eta \circ \xi = 0.$$

Let $g$ be a compatible Riemannian metric with $(\phi, \xi, \eta)$, that is

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y),$$

or, equivalently

$$g(X, \phi Y) = -g(\phi X, Y) \quad \text{and} \quad g(X, \xi) = \eta(X),$$

for all $X, Y \in \chi(M)$. Then $M^{2n+1}$ becomes an almost contact metric manifold equipped with an almost contact metric structure $(\phi, \xi, \eta, g)$. An almost contact metric manifold is cosymplectic [15], if $V_X \phi = 0$. From the formula $V_X \phi = 0$ it follows that

$$V_X \xi = 0, \quad V_X \eta = 0 \quad \text{and} \quad R(X, Y)\xi = 0.$$

Then we have

$$P(X, Y)\xi = 0.$$

So we have the following relations:

$$T(X, Y) = \eta(Y)X - \eta(X)Y,$$

$$\nabla_X Y = \nabla_X Y + \eta(Y)X - g(X, Y)\xi,$$
and
\[ \theta = \frac{1}{2} g - \eta \otimes \eta. \]

Hence \( \nabla \theta = 0 \) and \( R \cdot \theta = 0 \), which gives us \( R \cdot \tilde{P} = R \cdot P \).

A cosymplectic manifold \( M \) is said to be a cosymplectic space form if the \( \phi \)-sectional curvature tensor is constant \( c \) along \( M \). A cosymplectic space form will be denoted by \( M(c) \).

Then the Riemannian curvature tensor \( R \) on \( M(c) \) is given by
\[
R(X, Y, Z, W) = c4 \left\{ \frac{1}{2} g(X, W) g(Y, Z) - g(X, Z) g(Y, W) \\
+ g(X, \phi W) g(Y, \phi Z) - g(X, \phi Z) g(Y, \phi W) \\
- 2g(X, \phi Y) g(Z, \phi W) - g(X, W) \eta(Y) \eta(Z) \\
+ g(X, Z) \eta(Y) \eta(W) - g(Y, Z) \eta(X) \eta(W) + g(Y, W) \eta(X) \eta(Z) \right\}.
\]

From direct calculation we get
\[
S(X, W) = \frac{nc}{2} \left\{ g(X, W) - \eta(X) \eta(W) \right\},
\]
which gives us that \( M \) is a quasi-Einstein manifold.

5. Conclusions

Hayden [13] introduced a metric connection \( \tilde{\nabla} \) with a non-zero torsion on a Riemannian manifold. Then, Friedmann and Schouten introduced the idea of a semisymmetric linear connection in a differentiable manifold ([12], [18]). In [23], Yano proved that a Riemannian manifold admits a semisymmetric metric connection whose curvature tensor vanishes, it is necessary and sufficient that the Riemannian manifold be conformally flat. Recently, Murathan and Özgür [16] studied Riemannian manifolds admitting a semisymmetric metric connection \( \tilde{\nabla} \) such that the vector field \( U \) is a parallel vector field with respect to the Levi-Civita connection \( \nabla \). On the other hand, if a Riemannian manifold satisfying the condition \( R \cdot R = 0 \) \( (R.S = 0) \), then the manifold is called semisymmetric (Ricci semisymmetric) ([19], [20]). In this paper, firstly we show that if \( M \) is projectively flat with respect to the semisymmetric metric connection \( \tilde{\nabla} \) then \( M \) is a quasi-Einstein manifold. Then we prove that if \( R \cdot \tilde{P} = 0 \) if and only if \( M \) is projectively semisymmetric; if \( \tilde{P} \cdot R = 0 \) or \( R \cdot \tilde{P} - \tilde{P} \cdot R = 0 \) then \( M \) is conformally flat and quasi-Einstein manifold.

REFERENCES


Ahmet YILDIZ
Faculty of Education
Inonu University