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SUBMANIFOLDS OF A RIEMANNIAN MANIFOLD ADMITTING A TYPE OF RICCI QUARTER-SYMMETRIC METRIC CONNECTION *

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Abstract. The aim of the present paper is to study submanifolds of a Riemannian manifold admitting a type of Ricci quater-symmetric metric connection. We have proved that the induced connection is also a Ricci quarter-symmetric metric connection. We have also considered the mean curvature and the shape operator of the submanifold with respect to the Ricci quarter-symmetric metric connection. We have obtained the Gauss, Codazzi and Ricci equations with respect to the Ricci quarter-symmetric metric connection. Finally, we have considered the totally geodesicness and obtained the relation between the sectional curvatures of the manifold and its submanifold with respect to the Ricci quarter-symmetric metric connection.
Keywords. Riemannian manifold; submanifolds; metric connection; curvature.

1. Introduction

Let ∇ be a linear connection in an $n-$dimensional differentiable manifold M. The torsion tensor T and the curvature tensor R of ∇ are given respectively by

$$T(X,Y) = \nabla_X Y - \nabla_Y X - [X,Y]$$

and

$$R(X,Y)Z = \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X,Y]} Z.$$

The connection ∇ is symmetric if its torsion tensor T vanishes, otherwise it is called non-symmetric. The connection ∇ is a metric connection if there is a Riemannian metric g in M such that $\nabla g = 0$, otherwise it is called non-metric. It is well known that a linear connection is symmetric and metric if and only if it is the Levi-Civita

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connection. In 1975, S. Golab[8] introduced the notion of a quarter-symmetric linear connection in a differentiable manifold. In 1980, R. S. Mishra and S. N. Pandey [11] deduced some properties of the Riemannian, Kaehlerian and Sasakian manifolds that admits quarter-symmetric metric connection. In 1972, T. Imai[10] found some properties of a Riemannian manifold and hypersurfaces of a Riemannian manifold with a semisymmetric metric connection. In 1976, Z. Nakao[13] studied submanifolds of a Riemannian manifold with semisymmetric metric connection. Also in 1994, Agashe and Chafle[1] studied submanifolds of a Riemannian manifold with a semi-symmetric non-metric connection. In 2010, C. Ozgur[14], studied on submanifolds of a Riemannian manifold with a semi-symmetric non-metric connection. Also Zhao etal([9],[16],[17])studied on Riemannian manifolds with Quarter-symmetric metric connection. De etal ([7],[5],[6],[3],[12], [15]) studied Quarter-symmetric and Ricci Quarter-symmetric metric connections in Riemannian and Contact manifolds. Later in 2000, S. Ali and R. Nivas[2] studied on submanifolds immersed in a manifold with quarter-symmetric metric connection.

A linear connection is said to be a Ricci quarter-symmetric connection if its torsion tensor T is of the form

$$T(X,Y) = \pi(Y)QX - \pi(X)QY,$$

where π is a $1-$form and Q is the Ricci tensor operator defined by

$$g(QX,Y) = S(X,Y),$$

where S is the Ricci tensor of type $(0,2)$.

Motivated by these studies, in this paper we study submanifolds of a Riemannian manifold admitting a type of Ricci quarter-symmetric metric connection.

The present paper is organized as follows: After the preliminaries, in Section 3, we consider submanifold of a Riemannian manifold endowed with a Ricci quatersymmetric metric connection and show that the induced connection on a submanifold of a Riemannian manifold with a Ricci quarter-symmetric metric connection is also a Ricci quarter-symmetric metric connection. We also show that the mean curvature vector of the Riemannian manifold with respect to the Levi-Civita connection and Ricci quarter-symmetric metric connection coincide if and only if the scalar curvature vanishes. In the last part of this section we prove that the shape operators with respect to the Levi-Civita connection are simultaneously diagonalizable if and only if the shape operators with respect to the Ricci quarter-symmetric metric connection are simultaneously diagonalizable. Finally, we study the Gauss and Codazzi equation with respect to the Ricci quarter-symmetric metric connection and prove that the normal connection $\nabla^{\perp}$ is flat if and only if all second fundamental tensors with respect to the Ricci quarter-symmetric and the Levi-Civita connection are simultaneously diagonalizable and also if the submanifold is totally geodesic with respect to the Ricci quarter-symmetric metric connection, then the sectional curvature of the manifold and its submanifold are identical.

2. Preliminaries

Let $\bar{M}$ be an $m-$dimensional manifold with a Riemannian metric g and $\bar{\nabla}$ is the Levi-Civita connection on $\bar{M}$[18]. We define a linear connection ∇^* on $\bar{M}$ by

$$\nabla^*_X Y = \bar{\nabla}_X Y + \pi(Y)QX - S(X,Y)\rho \tag{2.1}$$

for arbitrary vector fields X, Y of $\bar{M}$, where ρ is the vector field defined by $g(X,\rho) = \pi(X)$ and Q is the Ricci operator defined by $S(X,Y) = g(QX,Y)$,S is the Ricci tensor of $(0,2)-$type.

Using (2.1), the torsion tensor T^* with respect to the connection ∇^* is given by

$$T^*(X,Y) = \pi(Y)QX - \pi(X)QY. \tag{2.2}$$

A linear connection ∇^* satisfying the condition (2.2) is called a Ricci quarter-symmertic connection. Also using (2.1), we have

$$(\nabla^*_Z g)(X,Y) = (\bar{\nabla}_Z g)(X,Y) = 0.$$

Hence the connection is a metric connection.

We denote by R^* the curvature tensor of $\bar{M}$ with respect to the Ricci quarter-symmetric metric connection ∇^*. So we have

$$\begin{aligned} R^*(X,Y)Z &= \nabla^*_X\nabla^*_Y Z - \nabla^*_Y\nabla^*_X Z - \nabla^*_{[X,Y]}Z \\ &= \bar{R}(X,Y)Z + (\bar{\nabla}_X\pi)(Z)QY - (\bar{\nabla}_Y\pi)(Z)QX \\ &\quad +\pi(Z)\{(\bar{\nabla}_X Q)Y - (\bar{\nabla}_Y Q)X\} + \{(\bar{\nabla}_Y S)(X,Z) \\ &\quad -(\bar{\nabla}_X S)(Y,Z)\}\rho + S(X,Z)\bar{\nabla}_Y\rho - S(Y,Z)\bar{\nabla}_X\rho \\ &\quad +\pi(Z)\{\pi(QY)QX - \pi(QX)QY + S(Y,QX)X \\ &\quad -S(X,QY)Y\} + \pi(\rho)\{S(X,Z)QY - S(Y,Z)QX\} \\ &\quad +\{S(Y,Z)S(X,\rho) - S(X,Z)S(Y,\rho)\}\rho, \end{aligned} \tag{2.3}$$

where $\bar{R}(X,Y)Z = \bar{\nabla}_X\bar{\nabla}_Y Z - \bar{\nabla}_Y\bar{\nabla}_X Z - \bar{\nabla}_{[X,Y]}Z$ is the curvature tensor of the manifold with respect to the Levi-Civita connection $\bar{\nabla}$. The Riemannian curvature tensors of the connections ∇^* and $\bar{\nabla}$ are defined by

$$R^*(X,Y,Z,W) = g(R^*(X,Y,Z),W)$$

and

$$\bar{R}(X,Y,Z,W) = g(\bar{R}(X,Y,Z),W)$$

respectively.

3. submanifolds of a Riemannian manifold with a Ricci quarter-symmetric metric connection

Let M be an $n-$dimensional submanifold of a Riemannian manifold $\bar{M}$ with a Ricci quarter-symmetric metric connection. Decomposing the vector field ρ on M uniquely into their tangent and normal components ρ^T and $\rho^\perp$ respectively we have

$$\rho = \rho^T + \rho^\perp.$$

The Gauss formula for a submanifold M of a Riemannian manifold $\bar{M}$ with respect to the Riemannian connection $\bar{\nabla}$ is given by

$$\bar{\nabla}_X Y = \nabla_X Y + m(X, Y), \tag{3.1}$$

where X and Y are vector fields tangent to M and m is the second fundamental form of M in $\bar{M}$. If $m = 0$, then M is called totally geodesic with respect to the Riemannian connection. $H = \frac{1}{n} trace\, m$ is called the mean curvature vector of the submanifold. If $H = 0$, then M is called minimal. For the second fundamental form m, the covariant derivative of m is defined by

$$(\bar{\nabla}_X m)(Y, Z) = \nabla^\perp_X m(Y, Z) - m(\nabla_X Y, Z) - m(Y, \nabla_X Z).$$

for any vector field X tangent to M. $\bar{\nabla}$ is called the Van der Waerden-Bortolotti connection of M, that is, $\bar{\nabla}$ is the connection in $TM \oplus T^\perp M$ built with ∇ and $\nabla^\perp$[4].

Let $\dot{\nabla}$ be the induced connection from the Ricci quarter-symmetric metric connection. We define

$$\nabla^*_X Y = \dot{\nabla}_X Y + \dot{m}(X, Y), \tag{3.2}$$

where $\dot{m}$ is the induced second fundamental form.

The equation (3.2) is the Gauss equation with respect to the Ricci quarter-symmetric metric connection ∇^*

Using (2.1), from (3.1) and (3.2) we have

$$\begin{aligned} \dot{\nabla}_X Y + \dot{m}(X, Y) &= \nabla_X Y + m(X, Y) + \pi(Y)QX \\ &\quad -S(X, Y)\rho^T - S(X, Y)\rho^\perp. \end{aligned} \tag{3.3}$$

Now taking the tangential and normal parts we have

$$\dot{\nabla}_X Y = \nabla_X Y + \pi(Y)QX - S(X, Y)\rho^T \tag{3.4}$$

and

$$\dot{m}(X, Y) = m(X, Y) - S(X, Y)\rho^\perp. \tag{3.5}$$

If $\dot{m} = 0$, then M is called totally geodesic with respect to the Ricci quarter-symmetric connection.

From (3.4), we have

$$\dot{T}(X,Y) = \dot{\nabla}_X Y - \dot{\nabla}_Y X - [X,Y] = \pi(Y)QX - \pi(X)QY, \tag{3.6}$$

where $\dot{T}$ is the torsion tensor of M with respect to $\dot{\nabla}$ and X, Y are vector fields tangent to M.

Moreover using (3.4) we have

$$(\dot{\nabla}_X g)(Y,Z) = (\nabla_X g)(Y,Z). \tag{3.7}$$

In view of (2.1), (3.4), (3.6) and (3.7) we can state the following:

Theorem 3.1. *The induced connection on a submanifold of a Riemannian manifold with a Ricci quarter-symmetric metric connection is also a Ricci quarter-symmetric metric connection.*

Let $\{e_1, e_2, ..., e_n\}$ be an orthonormal basis of the tangent space of M. We define the mean curvature vector $\dot{H}$ of M with respect to the Ricci quarter-symmetric metric connection $\dot{\nabla}$ by

$$\dot{H} = \frac{1}{n}\sum_{i=1}^{n} \dot{m}(e_i, e_i).$$

So from (3.5), we find

$$\dot{H} = H - \frac{r}{n}\rho^{\perp}.$$

If $H = 0$, then M is called minimal with respect to the Ricci quarter-symmetric metric connection.

So we have the following result:

Theorem 3.2. *If M be an $n-$dimensional submanifold of an $m-$dimensional Riemannian manifold $\bar{M}$, then the mean curvature vector of M with respect to the Levi-Civita connection and the Ricci quarter-symmetric metric connection coincide if and only if the scalar curvature vanishes.*

Let N be a normal vector field on M. From (2.1), we have

$$\nabla^*_X N = \bar{\nabla}_X N + \pi(N)QX. \tag{3.8}$$

The usual Weingarten formula is given by

$$\bar{\nabla}_X N = -A_N X + \nabla^{\perp}_X N, \;\; N \in T^{\perp}(M) \tag{3.9}$$

where $-A_N X$ and $\nabla^{\perp}_X N$ are the tangential and normal parts of $\bar{\nabla}_X N$. From (3.8) and (3.9) we get

$$\nabla^*_X N = -\dot{A}_N X + \nabla^{\perp}_X N, \;\; \textit{where } \dot{A}_N = (A_N - \pi(N)Q)I, \tag{3.10}$$

which is the Weingarten formulae for a submanifold of a Riemannian manifold with respect to the Ricci quarter-symmetric metric connection.

Since A_N is symmetric, it is easy to see that

$$g(\dot{A}_N X, Y) = g(X, \dot{A}_N Y)$$

and

$$g([\dot{A}_N, \dot{A}_L]X, Y) = g(X, [A_N, A_L]Y), \tag{3.11}$$

where $[\dot{A}_N, \dot{A}_L] = \dot{A}_N \dot{A}_L - \dot{A}_L \dot{A}_N$, $[A_N, A_L] = A_N A_L - A_L A_N$, N and L are unit normal vector fields on M.

Theorem 3.3. *If M be an $n-$dimensional submanifold of an $m-$dimensional Riemannian manifold $\bar{M}$ admitting the Ricci quarter-symmetric metric connection, then the shape operators with respect to Levi-Civita connection are simultaneously diagonalizable if and only if the shape operators with respect to then Ricci quarter-symmetric metric connection are simultaneously diagonalizable.*

4. Gauss and codazzi equation with respect to the Ricci quarter-symmetric metric connection

We denote the curvature tensor of $\bar{M}$ with respect to the Ricci quarter-symmetric metric connection ∇^* by

$$R^*(X,Y)Z = \nabla^*_X \nabla^*_Y Z - \nabla^*_Y \nabla^*_X Z - \nabla^*_{[X,Y]} Z$$

and that of M with respect to the induced Ricci quarter-symmetric metric connection $\dot{\nabla}$ by

$$\dot{R}(X,Y)Z = \dot{\nabla}_X \dot{\nabla}_Y Z - \dot{\nabla}_Y \dot{\nabla}_X Z - \dot{\nabla}_{[X,Y]} Z.$$

We shall now find the equation of Gauss-Codazzi with respect to the Ricci quarter-symmetric metric connection.

$$\begin{aligned}
R^*(X,Y)Z &= \nabla^*_X \nabla^*_Y Z - \nabla^*_Y \nabla^*_X Z - \nabla^*_{[X,Y]} Z \\
&= \dot{\nabla}_X \dot{\nabla}_Y Z + \dot{m}(X, \dot{\nabla}_Y Z)) - \dot{A}_{\dot{m}(Y,Z)} X + \nabla^{\perp}_X \dot{m}(Y,Z) \\
&\quad - \dot{\nabla}_Y \dot{\nabla}_X Z - \dot{m}(Y, \dot{\nabla}_X Z)) + \dot{A}_{\dot{m}(X,Z)} Y - \nabla^{\perp}_Y \dot{m}(X,Z) \\
&\quad - \dot{\nabla}_{[X,Y]} Z - \dot{m}([X,Y],Z) \\
&= \dot{R}(X,Y)Z - \dot{A}_{\dot{m}(Y,Z)} X + \dot{A}_{\dot{m}(X,Z)} Y \\
&\quad + \dot{m}(X, \dot{\nabla}_Y Z) - \dot{m}(Y, \dot{\nabla}_X Z) - \dot{m}([X,Y],Z) \\
&\quad + \nabla^{\perp}_X \dot{m}(Y,Z) - \nabla^{\perp}_Y \dot{m}(X,Z).
\end{aligned} \tag{4.1}$$

Taking account of (3.10), we have

$$\begin{aligned} R^*(X,Y)Z &= \dot{R}(X,Y)Z - A_{\dot{m}(Y,Z)}X - \pi(\dot{m}(Y,Z))X + A_{\dot{m}(X,Z)}Y \\ &\quad +\pi(\dot{m}(X,Z))Y + \dot{m}(X,\dot{\nabla}_Y Z) - \dot{m}(Y,\dot{\nabla}_X Z) \\ &\quad -\dot{m}([X,Y],Z) + \nabla^{\perp}_X \dot{m}(Y,Z) - \nabla^{\perp}_Y \dot{m}(X,Z). \end{aligned} \tag{4.2}$$

Since $g(A_N X, Y) = g(h(X,Y),N)$, using (3.5) we obtain

$$\begin{aligned} R^*(X,Y,Z,W) &= \dot{R}(X,Y,Z,W) - g(A_{\dot{m}(Y,Z)}X,W) + g(A_{\dot{m}(X,Z)}Y,W) \\ &\quad +\pi(\dot{m}(Y,Z))g(QX,W) - \pi(\dot{m}(X,Z))g(QY,W) \\ &= \dot{R}(X,Y,Z,W) - g(m(Y,Z),m(X,W)) \\ &\quad +g(m(X,Z),m(Y,W)) + S(Y,Z)\pi(m(X,W)) \\ &\quad -S(X,Z)\pi(m(Y,W)) + S(X,W)\pi(m(Y,Z)) \\ &\quad -S(Y,W)\pi(m(X,Z)) + \pi(\rho^{\perp})[S(X,Z)S(Y,W) \\ &\quad -S(Y,Z)S(X,W)], \end{aligned} \tag{4.3}$$

where W is a tangent vector field on M.

From (4.2), the normal component of $R^*(X,Y)Z$ is given by

$$\begin{aligned} (R^*(X,Y)Z)^{\perp} &= \dot{m}(X,\dot{\nabla}_Y Z) - \dot{m}(Y,\dot{\nabla}_X Z) - \dot{m}([X,Y],Z) \\ &\quad +\nabla^{\perp}_X \dot{m}(Y,Z) - \nabla^{\perp}_Y \dot{m}(X,Z) \\ &= (\nabla^{\perp}_X \dot{m})(Y,Z) - (\nabla^{\perp}_Y \dot{m})(X,Z) \\ &\quad +\pi(Y)\dot{m}(QX,Z) - \pi(X)\dot{m}(QY,Z), \end{aligned} \tag{4.4}$$

where $(\nabla^{\perp}_X \dot{m})(Y,Z) = \nabla^{\perp}_X \dot{m}(Y,Z) - \dot{m}(\dot{\nabla}_X Y,Z) - \dot{m}(\dot{\nabla}_Y X,Z)$.

It is called the van der Waerden-Bortolotti connection with respect to the Ricci quarter-symmetric metric connection.

Also the equation (4.4) is the equation of Codazzi with respect to the Ricci quarter-symmetric metric connection.

From (3.2) and (3.10), we get

$$\begin{aligned} \nabla^*_X \nabla^*_Y N_1 &= -\dot{A}_{\nabla^{\perp}_Y N_1}X - \dot{m}(\dot{A}_{N_1}Y,X) \\ &\quad +\nabla^{\perp}_X \nabla^{\perp}_Y N_1 - \dot{\nabla}_X \dot{A}_{N_1}Y, \end{aligned} \tag{4.5}$$

$$\begin{aligned} \nabla^*_Y \nabla^*_X N_1 &= -\dot{A}_{\nabla^{\perp}_X N_1}Y - \dot{m}(\dot{A}_{N_1}X,Y) \\ &\quad +\nabla^{\perp}_Y \nabla^{\perp}_X N_1 - \dot{\nabla}_Y \dot{A}_{N_1}X \end{aligned} \tag{4.6}$$

and

$$\nabla^*_{[X,Y]}N_1 = -\dot{A}_{N_1}[X,Y] + \nabla^{\perp}_{[X,Y]}N_1. \tag{4.7}$$

So using (4.5)-(4.7), we have

$$\begin{aligned} R^*(X,Y,N_1,N_2) &= R^{\perp}(X,Y,N_1,N_2) - g(\dot{m}(\dot{A}_{N_1}Y,X),N_2) \\ &\quad + g(\dot{m}(\dot{A}_{N_1}X,Y),N_2), \end{aligned} \tag{4.8}$$

where N_1 and N_2 are normal vector fields on M. Hence in view of (3.5) and (3.10) the equation (4.8) turns into

$$\begin{aligned} R^*(X,Y,N_1,N_2) &= R^{\perp}(X,Y,N_1,N_2) - g(m(A_{N_1}Y,X),N_2) \\ &\quad + S(A_{N_1}Y,X)g(\rho^{\perp},N_2) + g(m(A_{N_1}X,Y),N_2) \\ &\quad - S(A_{N_1}X,Y)g(\rho^{\perp},N_2) \\ &= R^{\perp}(X,Y,N_1,N_2) + g([N_1,N_2]X,Y). \end{aligned} \tag{4.9}$$

The equation (4.9) is the equation of Ricci with respect to the Ricci quarter-symmetric metric connection.

If $\bar{M}$ is a space of constant curvature c with respect to the connection $\bar{\nabla}$, then the equation (2.2) reduce to

$$\begin{aligned} R^*(X,Y)Z &= c\{g(y,Z)X - g(X,Z)Y\} + (\bar{\nabla}_X\pi)(Z)QY - (\bar{\nabla}_Y\pi)(Z)QX \\ &\quad + \pi(Z)\{(\bar{\nabla}_X Q)Y - (\bar{\nabla}_Y Q)X\} + \{(\bar{\nabla}_Y S)(X,Z) \\ &\quad - (\bar{\nabla}_X S)(Y,Z)\}\rho + S(X,Z)\bar{\nabla}_Y\rho - S(Y,Z)\bar{\nabla}_X\rho \\ &\quad + \pi(Z)\{\pi(QY)QX - \pi(QX)QY + S(Y,QX)X \\ &\quad - S(X,QY)\} + \pi\{(S(X,Z)QY - S(Y,Z)QX\} \\ &\quad + \{S(Y,Z)S(X,\rho) - S(X,Z)S(Y,\rho)\}\rho. \end{aligned} \tag{4.10}$$

From (4.10)we have $R^*(X,Y,N_1,N_2) = 0$. Therefore using (3.11) and (4.9) we obtain

$$R^{\perp}(X,Y,N_1,N_2) = g([N_2,N_1]X,Y) = g([N_2^*,N_1^*]X,Y).$$

Hence we can state the following theorem:

Theorem 4.1. *If M be an $n-$dimensional submanifold of an $m-$dimensional space of constant curvature $\bar{M}(c)$ admitting Ricci quarter-symmetric metric connection, then the normal connection $\nabla^{\perp}$ is flat if and only if all second fundamental tensors with respect to the Ricci quarter-symmetric and the Levi-Civita connection are simultaneously diagonalizable.*

From the equation (4.3) we have

$$R^*(X,Y,Y,X) = \dot{R}(X,Y,Y,X) - g(m(X,X),m(Y,Y))$$

$$+g(m(X,Y),m(Y,X))+S(Y,Y)\pi(m(X,X))$$
$$-S(X,X)\pi(m(Y,Y))-S(X,Y)\pi(m(X,Y))$$
$$-S(Y,X)\pi(m(X,Y))+\pi(\rho^{\perp})[S(X,Y)S(Y,X)$$
$$-S(Y,Y)S(X,X)]. \tag{4.11}$$

Now if the sectional curvature of $\bar{M}$ and M at a point $p \in \bar{M}$ with respect to the Ricci quarter-symmetric metric connection is denoted by κ^* and $\dot{\kappa}$ respectively, then the equation (4.11) reduce to

$$\begin{aligned}\kappa^* &= \dot{\kappa} - g(m(X,X),m(Y,Y)) + g(m(X,Y),m(Y,X)) \\ &\quad +S(Y,Y)\pi(m(X,X)) - S(X,X)\pi(m(Y,Y)) \\ &\quad -S(X,Y)\pi(m(X,Y)) - S(Y,X)\pi(m(X,Y)) \\ &\quad +\pi(\rho^{\perp})[S(X,Y)S(Y,X) - S(Y,Y)S(X,X)].\end{aligned} \tag{4.12}$$

If we consider M is totally geodesic with respect to the Ricci quarter-symmetric metric connection, then from (3.5) we get $m(X,Y) = S(X,Y)\rho^{\perp}$ and using this result, the equation (4.12) becomes

$$\kappa^* = \dot{\kappa}.$$

Hence we have the following theorem:

Theorem 4.2. *Let M be an $n-$dimensional submanifold of an $m-$dimensional Riemannian manifold $\bar{M}$ admitting a Ricci quarter-symmetric metric connection. If M is totally geodesic with respect to the Ricci quarter-symmetric metric connection, then the sectional curvatures κ^* and $\dot{\kappa}$ of $\bar{M}$ and M (resp.) are identical.*

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