STUDY OF A HYPERBOLIC KAHLERIAN MANIFOLDS EQUIPPED WITH A QUARTER-SYMMETRIC METRIC CONNECTION

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Abstract. This paper contains the study of a hyperbolic Kaehlerian manifold with different approaches. We consider a hyperbolic Kaehlerian manifold with a quarter-symmetric metric connection and obtained expressions for holomorphic conharmonic curvature tensor, conformal curvature tensor with respect to a quarter-symmetric metric connection. We have also studied holomorphic conharmonic recurrent, conformal recurrent and Weyl projective recurrent with respect to a quarter-symmetric metric connection.

Keywords: Kaehlerian manifold; curvature tensor; Riemannian connection; Ricci tensor.

1. Introduction

Let \((M^n, g), (n > 2)\), be an even dimensional differentiable manifold with the structure \(F^h_i\). If \(F^h_i\) satisfies the relation

\[
F^i_j F^h_i = \delta^h_j, \tag{1.1}
\]

\[
F_{ij} = -F_{ji}, \quad (F_{ij} = g_{jk} F^k_i), \tag{1.2}
\]

and

\[
F^h_{i,j} = 0, \tag{1.3}
\]

then the manifold is called hyperbolic Kaehlerian (space) manifold i.e. in a hyperbolic Kaehlerian manifold, equations (1.1), (1.2) and (1.3) hold. There, \(F^h_i\) is a tensor field of type (1.1) and \(F^h_{i,j}\) is a covariant derivative of \(F^h_i\) with respect to Riemannian connection. Yano and Imai [3] considered a quarter-symmetric metric connection \(\nabla\) and Riemannian connection \(D\) with coefficients \(\Gamma^h_{ij}\) and \(\{^h_{ij}\}\), respectively. According to them, if the torsion tensor \(T\) of the connection \(V\) on \((M^n, g), (n > 2)\), satisfies

\[
T^i_{jk} = p_j A^i_k - p_k A^i_j, \tag{1.4}
\]

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the relation between the coefficients of quarter-symmetric metric connection \( \nabla \) and
Riemannian connection \( D \) is given by

\[
\Gamma^i_{jk} = \{^i_{jk}\} - p_k U^i_j + p_j V^i_k - p^i V_{jk},
\]

where

\[
U_{ij} = \frac{1}{2}(A_{ij} - A_{ji}), \quad V_{ij} = \frac{1}{2}(A_{ij} + A_{ji}),
\]

\( \forall g = 0 \) and \( p_i \) are the components of a 1-form. Also, \( A^i_j \) denotes the components of the tensor of the type (1,1). Equation (1.6) implies

\[
A_{ij} = U_{ij} + V_{ij}.
\]

In [4] a quarter-symmetric metric connection in a hyperbolic Kaehlerian manifold by taking \( V_{ij} = g_{ij} \) and \( U_{ij} = F_{ij} \) in (1.5) was constructed as the form

\[
\Gamma^i_{jk} = \{^i_{jk}\} - p_k F^i_j + p_j \delta^i_k - p^i g_{jk}.
\]

Also, it is shown [4] that the relation between the curvature tensor with respect to a quarter-symmetric metric connection and a Riemannian connection is given by

\[
\bar{R}_{ijkh} = R_{ijkh} + \frac{1}{n-2} \left( g_{kh} R_{ih} - g_{ih} R_{kh} \right).
\]

where

\[
p_{jk} = \nabla_j p_k - p_j p_k + p_k q_j + \frac{1}{2} p \cdot p^s g_{jk}.
\]

Additionally, the Ricci tensor and the scalar curvature are found as, [4], respectively

\[
\bar{R}_{jk} = R_{jk} - (n-2)p_{jk} - g_{jk} p_m^m - p_j q_k + p_k q_j - p^s p_s F_{jk},
\]

and

\[
\bar{R} = R - 2(n-1)p_m^m.
\]

We know that in a Kaehler manifold

\[
(a)p_j = p^h g_{jk}, \quad (b)q_i = F_{ij} p^j, \quad (c) \quad p^r = g^{jr} p_i.
\]

2. Holomorphic conharmonic curvature tensor

We know that the holomorphic conharmonic curvature tensor in a Riemannian manifold is defined as

\[
T_{ijkh} = R_{ijkh} + \frac{1}{n-2} \left( g_{kh} R_{ih} - g_{ih} R_{kh} \right).
\]
Therefore the holomorphic conharmonic curvature tensor with respect to a quart-
symmetric metric connection is given by

\begin{equation}
\overline{T}_{ijkh} = \overline{R}_{ijkh} + \frac{1}{n-2} (g_{ik} \overline{R}_{jkh} - g_{jk} \overline{R}_{ikh}),
\end{equation}

where \( \overline{R}_{ijkh} \) and \( \overline{R}_{ikh} \) denote the curvature tensor and the Ricci tensor with respect
to a quarter-symmetric metric connection, respectively.

Using (1.9), (1.11) in (2.2), we get

\begin{align}
\overline{T}_{ijkh} &= R_{ijkh} - g_{ih} p_{k j} + g_{ik} p_{h j} - g_{jk} p_{hi} + g_{h j} p_{ki} \\
&\quad + p_i p_h F_{ik} + p_j p_k F_{jh} - p_j p_k F_{ih} - p_i p_h F_{jk} \\
&\quad + \frac{1}{n-2} \left( g_{ik} (R_{jkh} - (n-2)p_{k j} - g_{jh} p^m_i - p_j q_h \\
&\quad + p_h q_j - p_k p^k F_{hi}) \\
&\quad - g_{jk} (R_{ikh} - (n-2)p_{hi} - g_{ih} p^m_i - p_i q_h \\
&\quad + p_h q_i - p_k p^k F_{hi}) \right).
\end{align}

Using \([1.13(a,b)]\) in (2.3), we find

\begin{align}
\overline{T}_{ijkh} &= R_{ijkh} + \frac{1}{n-2} (g_{ik} R_{jkh} - g_{jk} R_{ikh}) \\
&\quad - \frac{n}{n-2} (p_i p_k F_{ih} - p_i p_k F_{jh}) \\
&\quad + \frac{n-1}{n-2} (p_h p_j F_{ik} - p_i p_h F_{jk}) \\
&\quad - \frac{p^m_i}{n-2} (g_{ki} g_{h j} - g_{jk} g_{hi}) + (g_{h j} p_{ki} - g_{ikh} p_{ki}).
\end{align}

From (2.1) and (2.4), we obtain

\begin{align}
\overline{T}_{ijkh} &= T_{ijkh} - \frac{n}{n-2} (p_i p_k F_{ih} - p_i p_k F_{jh}) \\
&\quad + \frac{n-1}{n-2} (p_h p_j F_{ik} - p_i p_h F_{jk}) \\
&\quad - \frac{p^m_i}{n-2} (g_{ki} g_{h j} - g_{jk} g_{hi}) + (g_{h j} p_{ki} - g_{ikh} p_{ki}).
\end{align}

In this case, if

\begin{equation}
p_i F_{ih} = p_i F_{jh}, \quad p_i g_{h j} = p_k g_{h i}, \quad \text{and} \quad g_{k i} g_{h j} = g_{jk} g_{hi},
\end{equation}

then from (2.5), we get

\begin{equation}
\overline{T}_{ijkh} = T_{ijkh}.
\end{equation}

Thus, we conclude:
Theorem 2.1. In a hyperbolic Kaehlerian manifold equipped with a quarter-symmetric metric connection, the holomorphic conharmonic curvature tensor with respect to a quarter-symmetric metric connection will be equal to the holomorphic conharmonic curvature tensor with respect to a Riemannian connection if the following conditions hold:

1. \( p_j F_{ih} = p_i F_{jh} \)
2. \( p_k g_{ij} = p_{kj} g_{hi} \)
3. \( g_{ki} g_{hi} = g_{jk} g_{hi} \)

Now, we propose:

Theorem 2.2. In a hyperbolic Kaehlerian manifold equipped with a quarter-symmetric metric connection, the holomorphic conharmonic curvature tensor with respect to a quarter-symmetric metric connection satisfies the first Bianchi identity if

\[ p_j F_{ik} + p_i F_{kj} + p_k F_{ji} = 0 \]
and
\[ p_i g_{kh} = p_k g_{ih} \]

i.e.

\[ T_{ijkh} + T_{jkih} + T_{kijh} = 0, \]

if

\[ p_j F_{ik} + p_i F_{kj} + p_k F_{ji} = 0 \]
and
\[ p_i g_{kh} = p_k g_{ih}. \]

Proof. Interchanging \( i, j \) and \( k \) in a cyclic order in (2.5), we get

\[
\begin{align*}
\overline{T}_{ijkh} &= T_{ijkh} \\
&- \frac{n}{n-2} (p_j p_k F_{ih} - p_i p_k F_{jh}) \\
&+ \frac{n-1}{n-2} (p_h p_j F_{ik} - p_i p_h F_{jk}) \\
&- \frac{p_m^m}{n-2} (g_{ki} g_{hi} - g_{jk} g_{hi}) \\
&+ (p_{ki} g_{ij} - p_{kj} g_{hi}),
\end{align*}
\]

(2.8)

\[
\begin{align*}
\overline{T}_{jkih} &= T_{jkih} \\
&- \frac{n}{n-2} (p_k p_i F_{jh} - p_j p_i F_{kh}) \\
&+ \frac{n-1}{n-2} (p_h p_k F_{ji} - p_j p_h F_{ki}) \\
&- \frac{p_m^m}{n-2} (g_{ij} g_{hk} - g_{ki} g_{ih}) \\
&+ (p_{ij} g_{hk} - p_{ik} g_{hi}),
\end{align*}
\]

(2.9)

and

\[
\begin{align*}
\overline{T}_{kijh} &= T_{kijh} \\
&- \frac{n}{n-2} (p_i p_j F_{kh} - p_k p_j F_{ih}) \\
&+ \frac{n-1}{n-2} (p_h p_i F_{kj} - p_k p_h F_{ij}) \\
&- \frac{p_m^m}{n-2} (g_{jk} g_{hi} - g_{ij} g_{hk}) \\
&+ (p_{jk} g_{hi} - p_{ij} g_{hk}).
\end{align*}
\]

(2.10)
Adding equation (2.8), (2.9) and (2.10), we have
\[
\overline{T}_{ijkh} + \overline{T}_{jkih} + \overline{T}_{kijh} = T_{ijkh} + T_{jkih} + T_{kijh} + \frac{2(n-1)}{n-2} p_h (p_j F_{ik} + p_i F_{kj} + p_k F_{ji}) + (p_{ki} g_{hj} - p_{kj} g_{hi}) + (p_{ij} g_{hk} - p_{ik} g_{jh}) + (p_{jk} g_{hi} - p_{ji} g_{kh}).
\] (2.11)

Since in a Riemannian manifold the holomorphic conharmonic curvature tensor satisfies the first Bianchi identity, i.e.
\[
T_{ijkh} + T_{jkih} + T_{kijh} = 0.
\] (2.12)

Using (2.12) in (2.11), we have
\[
\overline{T}_{ijkh} + \overline{T}_{jkih} + \overline{T}_{kijh} = \frac{2(n-1)}{n-2} p_h (p_j F_{ik} + p_i F_{kj} + p_k F_{ji}) + (p_{ki} g_{hj} - p_{kj} g_{hi}) + (p_{ij} g_{hk} - p_{ik} g_{jh}) + (p_{jk} g_{hi} - p_{ji} g_{kh}).
\] (2.13)

Now, assuming that
\[
p_j F_{ik} + p_i F_{kj} + p_k F_{ji} = 0, \quad \text{and} \quad p_{ij} g_{hk} = p_{ik} g_{jh},
\] (2.14)
then from (2.13), we get
\[
\overline{T}_{ijkh} + \overline{T}_{jkih} + \overline{T}_{kijh} = 0.
\] (2.15)

This completes the proof. \(\square\)

Now, we propose:

**Theorem 2.3.** In a hyperbolic Kaehlerian manifold equipped with a quarter-symmetric metric connection, the holomorphic conharmonic curvature tensor with respect to a quarter-symmetric metric connection satisfies
\[
\overline{T}_{ijkh} = -\overline{T}_{jikh}.
\]

**Proof.** Interchanging \(i\) and \(j\) in (2.5), we have
\[
\overline{T}_{ijkh} = T_{jikh} - \frac{n}{n-2} (p_i p_k F_{jh} - p_j p_k F_{ih}) + \frac{1}{n-2} (p_h F_{ik} - p_j p_k F_{ih}) - \frac{p_{ik}^m}{n-2} (g_{kj} g_{hi} - g_{kj} g_{hi}) + (g_{hi} p_{kj} - g_{jk} p_{hi}).
\] (2.16)
Adding (2.5) and (2.16), we have

$$T_{ijkh} + T_{jikh} = T_{ijkh} + T_{jikh}. \quad (2.17)$$

Since in a Riemannian manifold the holomorphic conharmonic curvature tensor satisfies

$$T_{ijkh} + T_{jikh} = 0. \quad (2.18)$$

Then by using (2.18) in (2.17), we get

$$T_{ijkh} = -T_{jikh}. \quad (2.19)$$

## 3. Conformal curvature tensor

In the Riemannian manifold \((M^n, g)\), \((n > 2)\), the conformal curvature tensor of the type \((0,4)\) is defined as

$$C_{ijkh} = R_{ijkh} - \frac{1}{n-2}(R_{jk} g_{ih} - R_{ik} g_{jh} + R_{ih} g_{jk} - R_{jh} g_{ki})$$

$$+ \frac{R}{(n-1)(n-2)}(g_{ih} g_{jk} - g_{jh} g_{ki}). \quad (3.1)$$

The conformal curvature tensor with respect to a quarter-symmetric metric connection is given by

$$\overline{C}_{ijkh} = \overline{R}_{ijkh}$$

$$- \frac{1}{n-2}(\overline{R}_{jk} g_{ih} - \overline{R}_{ik} g_{jh} + \overline{R}_{ih} g_{jk} - \overline{R}_{jh} g_{ki})$$

$$+ \frac{R}{(n-1)(n-2)}(g_{ih} g_{jk} - g_{jh} g_{ki}). \quad (3.2)$$

Using (1.9), (1.11) and (1.12) in (3.2), we have

$$\overline{C}_{ijkh} = R_{ijkh}$$

$$- g_{ih}p_{kj} + g_{ik} p_{hj} - g_{jk} p_{hi} + g_{jh} p_{ki}$$

$$+ p_j p_k F_{ih} + p_i p_k F_{jh} + p_j p_k F_{ih} - p_i p_h F_{jk}$$

$$- \frac{1}{n-2}\left(g_{ih}(R_{jk} - (n-2)p_{kj} - g_{jk} p_{m}^m - p_j q_k + p_k q_j - p^p p_s F_{kj})\right)$$

$$- g_{jh}(R_{ik} - (n-2)p_{ki} - g_{ik} p_{m}^m - p_i q_k + p_k q_i - p^p p_s F_{ki})$$

$$+ g_{hj}(R_{ik} - (n-2)p_{ki} - g_{ik} p_{m}^m - p_i q_h + p_h q_i - p^p p_s F_{ki})$$

$$- g_{hk}(R_{ij} - (n-2)p_{ji} - g_{ij} p_{m}^m - p_i q_h + p_h q_j - p^p p_s F_{hi})$$

$$+ \frac{R - 2(n-1)p_{m}^m}{(n-1)(n-2)}(g_{ih} g_{jk} - g_{jh} g_{ki}). \quad (3.3)$$
Now using \([(1.13(a,b))]\) in (3.3), we have
\[
\overline{C}_{ijkh} = R_{ijkh} - \frac{1}{n-2} \left( \frac{R_{ijk} g_{jh} - R_{ijk} g_{jh} + R_{ijk} g_{jk} - R_{ijk} g_{kj}}{g_{ih} g_{jk} - g_{ih} g_{jk}} \right) \\
+ \frac{R}{(n-1)(n-2)} \left( g_{ih} g_{jk} - g_{ih} g_{jk} \right) \\
+ \frac{n+1}{n-2} \left( p_j p_h F_{ik} - p_j p_k F_{ih} \right) \\
+ \frac{n+1}{n-2} \left( p_j p_k F_{jh} - p_j p_h F_{jk} \right).
\]
(3.4)

Using (3.1) in (3.4), we have
\[
\overline{C}_{ijkh} = C_{ijkh} + \frac{n+1}{n-2} \left( p_j p_h F_{ik} - p_j p_k F_{ih} \right) \\
+ \frac{n+1}{n-2} \left( p_j p_k F_{jh} - p_j p_h F_{jk} \right).
\]
(3.5)

If we take \(p_h F_{ik} = p_k F_{ih}\) then (3.5) reduces to the form
\[
\overline{C}_{ijkh} = C_{ijkh}.
\]
(3.6)

Thus, we conclude:

**Theorem 3.1.** In a hyperbolic Kaehlerian manifold equipped with a quarter-symmetric metric connection, the conformal curvature tensor with respect to a quarter-symmetric metric connection will be equal to the conformal curvature tensor with respect to a Riemannian connection if and only if
\[
p_h F_{ik} = p_k F_{ih}.
\]
(3.7)

Now, we propose:

**Theorem 3.2.** In a hyperbolic Kaehlerian manifold equipped with a quarter-symmetric metric connection, the conformal curvature tensor with respect to a quarter-symmetric metric connection satisfies the first Bianchi identity if
\[
p_j F_{ik} + p_i F_{kj} + p_k F_{ji} = 0.
\]
(3.8)

**Proof.** Interchanging \(i, j\) and \(k\) in a cyclic order in (3.5), we find
\[
\overline{C}_{ijkh} = C_{ijkh} \\
+ \frac{n+1}{n-2} \left( p_j p_h F_{ik} - p_j p_k F_{ih} \right) \\
+ \frac{n+1}{n-2} \left( p_j p_k F_{jh} - p_j p_h F_{jk} \right).
\]
(3.9)
\[ \overline{C}_{jkih} = C_{jkih} + \frac{n+1}{n-2} (p_k p_h F_{ji} - p_k p_i F_{jh}) + \frac{n+1}{n-2} (p_j p_i F_{kh} - p_j p_k F_{ki}), \]  
(3.10)
and
\[ \overline{C}_{kijh} = C_{kijh} + \frac{n+1}{n-2} (p_i p_h F_{jk} - p_i p_k F_{jh}) + \frac{n+1}{n-2} (p_k p_j F_{ih} - p_k p_i F_{ij}). \]  
(3.11)
Adding (3.9), (3.10) and (3.11), we get
\[ C_{ijkh} + C_{jkih} + C_{kijh} = C_{ijkh} + C_{jkih} + C_{kijh} + 2 \frac{n+1}{n-2} p_k (p_j F_{ik} + p_i F_{kj} + p_k F_{ji}). \]  
(3.12)
Since in a Riemannian manifold the conformal curvature tensor satisfies the condition
\[ C_{ijkh} + C_{jkih} + C_{kijh} = 0, \]  
(3.13)
by using (3.13) in (3.12), we find
\[ \overline{C}_{ijkh} + \overline{C}_{jikh} + \overline{C}_{kijh} = 2 \frac{n+1}{n-2} p_k (p_j F_{ik} + p_i F_{kj} + p_k F_{ji}). \]  
(3.14)
If we take \( p_j F_{ik} + p_i F_{kj} + p_k F_{ji} = 0 \) then from (3.14), we get
\[ \overline{C}_{ijkh} + \overline{C}_{jikh} + \overline{C}_{kijh} = 0. \]  
(3.15)
Now, we propose:
Theorem 3.3. In a hyperbolic Kaehlerian manifold equipped with a quarter-symmetric metric connection, the conformal curvature tensor with respect to a quarter-symmetric metric connection satisfies the following properties:
1. \( \overline{C}_{ijkh} = -\overline{C}_{jikh}, \)
2. \( \overline{C}_{ijkh} = -\overline{C}_{ijkh}. \)
Proof. Interchanging \( i \) and \( j \) in (3.5), we find
\[ \overline{C}_{ijkh} = C_{jikh} + \frac{n+1}{n-2} (p_i p_h F_{jk} - p_i p_k F_{jh}) + \frac{n+1}{n-2} (p_j p_k F_{ih} - p_j p_h F_{ij}), \]  
(3.16)
Adding (3.5) and (3.16), we obtain

\[ C_{ijkh} + C_{jikh} = C_{ijkh} + C_{jikh}. \]  (3.17)

Since in a Riemannian manifold the conformal curvature tensor satisfies

\[ C_{ijkh} + C_{ijhk} = 0, \]  (3.18)

then by using (3.18) in (3.17), we get the expression (1). Now interchanging \( k \) and \( h \) in (3.5), we have

\[ C_{ijkh} = C_{ijhk} + \frac{n + 1}{n - 2} \left( p_j p_k F_{ih} - p_j p_h F_{jk} \right) \]

\[ + \frac{n + 1}{n - 2} \left( p_i p_h F_{jk} - p_i p_k F_{jh} \right). \]  (3.19)

Adding (3.5) and (3.19), we have

\[ C_{ijkh} + C_{ijhk} = C_{ijkh} + C_{ijhk}. \]  (3.20)

Since in a Riemannian manifold the conformal curvature tensor satisfies

\[ C_{ijkh} + C_{ijhk} = 0, \]  (3.21)

by using (3.21) in (3.20), we get expression (2).

Taking the covariant derivative of the conformal curvature tensor with respect to the Riemannian connection and quarter-symmetric metric connection, respectively, we get

\[ D_m C_{ijkh} = \partial_m C_{ijkh} - C_{rjkh} \left\{ m_i \right\} - C_{irkh} \left\{ m_j \right\} \]

\[ - C_{ijrh} \left\{ m_k \right\} - C_{ijkr} \left\{ m_h \right\}. \]  (3.22)

and

\[ \nabla_m C_{ijkh} = \partial_m C_{ijkh} - C_{rjkh} \Gamma^r_{m1} - C_{irkh} \Gamma^r_{mj} \]

\[ - C_{ijrh} \Gamma^r_{mk} - C_{ijkr} \Gamma^r_{mh}. \]  (3.23)

Subtracting (3.22) from (3.23), we get

\[ \nabla_m C_{ijkh} - D_m C_{ijkh} = C_{rjkh} \left( \left\{ m_i \right\} - \Gamma^r_{m1} \right) \]

\[ + C_{irkh} \left( \left\{ m_j \right\} - \Gamma^r_{mj} \right) \]

\[ + C_{ijrh} \left( \left\{ m_k \right\} - \Gamma^r_{mk} \right) \]

\[ + C_{ijkr} \left( \left\{ m_h \right\} - \Gamma^r_{mh} \right). \]  (3.24)
Now using (1.8) in (3.24), we find

\[
\nabla_m C_{ijkh} - D_m C_{ijkh} = C_{rjkh} (p_i F'_m - p_m \delta^r_i + p^r g_{mi}) + C_{irkh} (p_j F'_m - p_m \delta^r_j + p^r g_{mj}) + C_{ijrh} (p_k F'_m - p_m \delta^r_k + p^r g_{mk}) + C_{ijkr} (p_h F'_m - p_m \delta^r_h + p^r g_{mh}).
\]

Using [1.13(c)] in (3.25), we get

\[

abla_m C_{ijkh} - D_m C_{ijkh} = (C_{rjkh} p_i + C_{irkh} p_j + C_{ijrh} p_k + C_{ijkr} p_h) F'_m.
\]

(3.26)

Considering that the expression
\[
C_{rjkh} p_i + C_{irkh} p_j + C_{ijrh} p_k + C_{ijkr} p_h = 0
\]

(3.27)

is satisfied, then it is finally obtained that

\[
\nabla_m C_{ijkh} = D_m C_{ijkh}.
\]

(3.28)

Thus, we conclude:

**Theorem 3.4.** In a hyperbolic Kaehlerian manifold equipped with a quarter-symmetric metric connection, if the conformal curvature tensor with respect to a Riemannian connection is recurrent with respect to the Riemannian connection then it is also recurrent with respect to the quarter-symmetric metric connection if and only if

\[
C_{rjkh} p_i + C_{irkh} p_j + C_{ijrh} p_k + C_{ijkr} p_h = 0.
\]

Taking the covariant derivative of the holomorphic conharmonic curvature tensor with respect to the Riemannian connection and quarter-symmetric metric connection, respectively, we have

\[
D_m T_{ijkh} = \partial_m T_{ijkh} - T_{rjkh} \left\{^r_m\right\} - T_{irkh} \left\{^r_m\right\} - T_{ijrh} \left\{^r_m\right\} - T_{ijkr} \left\{^r_m\right\},
\]

(3.29)

and

\[
\nabla_m T_{ijkh} = \partial_m T_{ijkh} - T_{rjkh} \Gamma'_m - T_{irkh} \Gamma'_m - T_{ijrh} \Gamma'_m - T_{ijkr} \Gamma'_m.
\]

(3.30)

Subtracting (3.29) from (3.30), we get

\[
\nabla_m T_{ijkh} - D_m T_{ijkh} = T_{rjkh} \left\{^{r}_m\right\} - \Gamma'_m + T_{irkh} \left\{^{r}_m\right\} - \Gamma'_m + T_{ijrh} \left\{^{r}_m\right\} - \Gamma'_m + T_{ijkr} \left\{^{r}_m\right\} - \Gamma'_m.
\]

(3.31)
Now, using (1.8) in (3.31), we find
\[
\nabla_m T_{ijkh} - D_m T_{ijkh} = T_{rijhk}(p_i F_m^r - p_m \delta_i^r + p^r g_{mi})
+ T_{irjkh}(p_j F_m^r - p_m \delta_j^r + p^r g_{mj})
+ T_{ijrkh}(p_k F_m^r - p_m \delta_k^r + p^r g_{mk})
+ T_{ijkrh}(p_h F_m^r - p_m \delta_h^r + p^r g_{mh}).
\]
(3.32)

Using [1.13(c)] in (3.32), we obtain
\[
\nabla_m T_{ijkh} - D_m T_{ijkh} = (T_{rikjh} p_i + T_{irkjh} p_j
+ T_{ijrkh} p_k + T_{ijkrh} p_h) F_m^r.
\]
(3.33)

If we take the condition
\[
T_{rikjh} p_i + T_{irkjh} p_j + T_{ijrkh} p_k + T_{ijkrh} p_h = 0,
\]
then
\[
\nabla_m T_{ijkh} = D_m T_{ijkh}.
\]
(3.35)

Thus, we conclude:

**Theorem 3.5.** In a hyperbolic Kahlerian manifold equipped with a quarter-symmetric metric connection, if the holomorphic conharmonic curvature tensor with respect to a Riemannian connection is recurrent with respect to the Riemannian connection then it is also recurrent with respect to the quarter-symmetric metric connection if and only if the following condition holds
\[
T_{rikjh} p_i + T_{irkjh} p_j + T_{ijrkh} p_k + T_{ijkrh} p_h = 0.
\]

Taking the covariant derivative of the Weyl projective curvature tensor with respect to a Riemannian connection and quarter-symmetric metric connection, respectively, we can write
\[
D_m W_{ijkh} = \partial_m W_{ijkh} - W_{rijk} \Gamma_{mi}^r - W_{rikh} \Gamma_{mj}^r
- W_{ijrh} \Gamma_{mk}^r - W_{ijkr} \Gamma_{mh}^r.
\]
(3.36)

and
\[
\nabla_m W_{ijkh} = \partial_m W_{ijkh} - W_{rijk} \Gamma_{mi}^r - W_{rikh} \Gamma_{mj}^r
- W_{ijrh} \Gamma_{mk}^r - W_{ijkr} \Gamma_{mh}^r.
\]
(3.37)

Subtracting (3.36) from (3.37), we get
\[
\nabla_m W_{ijkh} - D_m W_{ijkh} = W_{rijhk} \Gamma_{mi}^r + W_{rikj} \Gamma_{mj}^r
- \Gamma_{mi}^r + W_{ijrk} \Gamma_{mk}^r - \Gamma_{mh}^r.
\]
(3.38)
Now, using (1.8) in (3.38), we find
\[
\nabla_m W_{ijkh} - D_m W_{ijkh} = \begin{align*}
W_{r jkh} & (p_i F^r_m - p_m \delta^r_i + p^r g_{mi}), \\
W_{rkh} & (p_j F^r_m - p_m \delta^r_j + p^r g_{mj}), \\
W_{ijk} & (p_k F^r_m - p_m \delta^r_k + p^r g_{mk}), \\
W_{ijkh} & (p_h F^r_m - p_m \delta^r_h + p^r g_{mh}).
\end{align*}
\]
(3.39)

Using [1.13(c)] in (3.39), it is obtained that
\[
\nabla_m W_{ijkh} - D_m W_{ijkh} = (W_{r jkh} p_i + W_{rkh} p_j + W_{ijk} p_k + W_{ijkh} p_h) F^r_m.
\]
(3.40)

If we consider that the expression
\[
W_{r jkh} p_i + W_{rkh} p_j + W_{ijk} p_k + W_{ijkh} p_h = 0
\]
(3.41)
holds, then we find
\[
\nabla_m W_{ijkh} = D_m W_{ijkh}.
\]
(3.42)

Thus, we conclude:

**Theorem 3.6.** In a hyperbolic Kaehlerian manifold equipped with a quarter-symmetric metric connection, if the Weyl projective curvature tensor with respect to a Riemannian connection is recurrent with respect to the Riemannian connection then it is also recurrent with respect to a quarter-symmetric metric connection if and only if
\[
W_{r jkh} p_i + W_{rkh} p_j + W_{ijk} p_k + W_{ijkh} p_h = 0.
\]

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