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SOME EQUIVALENT QUASINORMS ON $L_{\phi,E}$

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Abstract. In this paper we define a new operator ideal $L_{\phi,E}$ by using block sequence spaces and symmetric norming function. Also we define different quasi-norms on this class and deal with equivalence of these quasi-norms.

Keywords: Operator ideal; sequence spaces; norming function; quasi-norm.

1. Introduction

The operator ideal theory has a special importance in functional analysis. One of the most important methods of constructing operator ideals is using $s-$ numbers. Pietsch defined the approximation numbers of a bounded linear operator in Banach spaces, in 1963 [14]. Later on, the other examples of $s-$numbers, namely Kolmogorov numbers, Weyl numbers, etc. are introduced to the Banach space setting.

In this paper, we denote the set of all natural numbers and nonnegative real numbers by $\mathbb{N}$ and $\mathbb{R}^+$, respectively.

A finite rank operator is defined as a bounded linear operator whose dimension of the range space is finite [10].

Let ω be the set of all real valued sequences. A sequence space is any vector subspace of ω.

Maddox defined the linear space $l\,(p)$ as follows in [8]:

$$l\,(p) = \left\{ x \in \omega : \sum_{n=1}^{\infty} |x_n|^{p_n} < \infty \right\},$$

where (p_n) is a bounded sequence of strictly positive real numbers.

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The set of all sequences whose generalized weighted mean transforms are in the space $l(p)$ is the sequence space $l(u,v;p)$ which is is introduced by Altay and Başar in [1] as follows:

$$l(u,v;p)=\left\{x\in\omega:\sum_{n=1}^{\infty}\left|u_n\sum_{k=1}^{n}v_kx_k\right|^{p_n}<\infty\right\},$$

where $u_n,\ v_k\neq 0$ for all $n,\ k\in\mathbb{N}$.

If $p_n=p$ for all $n\in\mathbb{N}$, $l(u,v;p)=Z(u,v;l_p)$ which is defined by Malkowsky and Savaş [12] as follows:

$$Z(u,v;l_p)=\left\{x\in\omega:\sum_{n=1}^{\infty}\left|u_n\sum_{k=1}^{n}v_kx_k\right|^{p}<\infty\right\},$$

where $1<p<\infty$.

The Cesaro sequence space ces_p is defined as

$$ces_p=\left\{x=(x_k)\in\omega:\sum_{n=1}^{\infty}\left(\frac{1}{n}\sum_{k=1}^{n}|x_k|\right)^{p}<\infty\right\},$$

where $1<p<\infty$ ([18], [21], [22]). Afterwards, Mursaleen and Khan defined the Cesaro vector-valued sequence space by

$$Ces(X,p,q)=\left\{x=(x_k):\sum_{k=1}^{\infty}\left(\frac{1}{Q_k}\sum_{n=1}^{k}|x_n|\right)^{p_k}<\infty\right\},$$

where $p=(p_k)$ and $q=(q_k)$ are bounded sequences of positive real numbers and $Q_n=\sum_{k=0}^{n}q_k$, $(n\in\mathbb{N})$ [13]. Here if $q_k=1$ for each k then $Ces(X,p,q)$ is reduced to ces_p.

Let E and F be real or complex Banach spaces. $\mathcal{L}(E,F)$ and $\mathcal{L}$ denotes the space of all bounded linear operators from E to F and the space of all bounded linear operators between any two arbitrary Banach spaces, respectively.

A map $s=(s_n):\mathcal{L}\to\mathbb{R}^+$ assigning to every operator $T\in\mathcal{L}$ a non-negative scalar sequence $(s_n(T))_{n\in\mathbb{N}}$ is called an $s-$number sequence if the following conditions are satisfied for all Banach spaces E,F,E_0 and F_0:

$(S1)$ $\|T\|=s_1(T)\geq s_2(T)\geq\ldots\geq 0$ for every $T\in\mathcal{L}(E,F)$,

$(S2)$ $s_{m+n-1}(S+T)\leq s_m(S)+s_n(T)$ for every $S,T\in\mathcal{L}(E,F)$ and $m,n\in\mathbb{N}$,

$(S3)$ $s_n(RST)\leq\|R\|\,s_n(S)\,\|T\|$ for some $R\in\mathcal{L}(F,F_0)$, $S\in\mathcal{L}(E,F)$ and $T\in\mathcal{L}(E_0,E)$, where E_0,F_0 are arbitrary Banach spaces,

$(S4)$ If $rank(T)\leq n$, then $s_n(T)=0$,

$(S5)$ $s_n(I:l_2^n\to l_2^n)=1$, where I denotes the identity operator on the $n-$dimensional Hilbert space l_2^n, where $s_n(T)$ denotes the $n-th$ $s-$number of the operator T [2].

One of the example of s-number sequence is the approximation number, which is defined by Pietsch. The $n-$th approximation number, denoted by $a_n(T)$, is defined as

$$a_n(T) = \inf\{\|T - A\| : A \in \mathcal{L}(E,F), rank(A) < n\},$$

where $T \in \mathcal{L}(E,F)$ and $n \in \mathbb{N}$ [14].

Pietsch [14] defined an operator $T \in \mathcal{L}(E,F)$ to be l_p type operator if $\sum\limits_{n=1}^{\infty}(a_n(T))^p < \infty$ for $0 < p < \infty$. Then, in [3] Constantin defined the class of $ces-p$ type operators by using the Cesaro sequence spaces, where an operator $T \in \mathcal{L}(E,F)$ is called $ces-p$ type if $\sum\limits_{n=1}^{\infty}\left(\frac{1}{n}\sum\limits_{k=1}^{n}a_k(T)\right)^p < \infty,\ \ 1 < p < \infty$. Afterwards Tita in [24] proved that the class of l_p type operators and the class of $ces-p$ type operators are coincides.

As a generalization of l_p type operators, $A-p$ type operators were examined in [5]. Also in [9], [10], [11] Maji and Srivastava studied the class $A^{(s)}-p$ of $s-$type ces_p operators using $s-$number sequence and Cesaro sequence spaces and they introduced a new class $A^{(s)}_{p,q}$ of $s-$type $ces(p,q)$ operators by using weighted Cesaro sequence space for $1 < p < \infty$. Recently, the class of $s-$type $Z(u,v;l_p)$ operators have been defined and studied on some properties of this class in [4].

The idea of quasi-normed operator ideals is developed by the fact that, some important operator ideals which do not possess a natural norm should also be covered. There exists a lot of different quasi-norms on every operator ideal. In addition to this, the nice quasi-norms are determined by the completeness of the corresponding topology [15].

Now give the definitions of operator ideal and quasi-norm:

Let E' be the dual of E, which is composed of continuous linear functionals on E. Let $x' \in E'$ and $y \in F$, then the map $x' \otimes y : E \to F$ is defined by

$$(x' \otimes y)(x) = x'(x)\,y,\ \ x \in E.$$

A subcollection $\mathfrak{F}$ of $\mathcal{L}$ is called an operator ideal if each component $\mathfrak{F}(E,F) = \mathfrak{F} \cap \mathcal{L}(E,F)$ satisfies the following conditions:

$(OI-1)$ if $x' \in E'$, $y \in F$, then $x' \otimes y \in \mathfrak{F}(E,F)$,

$(OI-2)$ if $S,T \in \mathfrak{F}(E,F)$, then $S+T \in \mathfrak{F}(E,F)$,

$(OI-3)$ if $S \in \mathfrak{F}(E,F)$, $T \in \mathcal{L}(E_0,E)$ and $R \in \mathcal{L}(F,F_0)$, then $RST \in \mathfrak{F}(E_0,F_0)$[15].

A function $\alpha : \mathfrak{F} \to \mathbb{R}^+$ is said to be a quasi-norm on the operator ideal $\mathfrak{F}$ if the following conditions hold:

$(QN-1)$ If $x' \in E'$, $y \in F$, then $\alpha(x' \otimes y) = \|x'\|\,\|y\|$;

$(QN-2)$ there exists a constant $C \geq 1$ such that $\alpha(S+T) \leq C[\alpha(S) + \alpha(T)]$;

$(QN-3)$ if $S \in \mathfrak{F}(E,F)$, $T \in \mathcal{L}(E_0,E)$ and $R \in \mathcal{L}(F,F_0)$, then $\alpha(RST) \leq \|R\|\,\alpha(S)\,\|T\|$[15].

In particular if $C=1$ then α becomes a norm on the operator ideal $\mathfrak{F}$.

An ideal $\mathfrak{F}$ with a quasi-norm α,which is denoted by $[\mathfrak{F},\alpha]$, is said to be a quasi-Banach operator ideal if each component $\mathfrak{F}(E,F)$ is complete under the quasi-norm α.

Let ℓ_∞ be the space of all bounded real sequences and $K\subset\ell_\infty$ be the set of all sequences x such that $card\{i\in\mathbb{N},x_i\neq 0\}<n$ and $x_1\geq x_2\geq\ldots\geq 0$.

A function $\phi:K\to\mathbb{R}$ is called symmetric norming function, if the following conditions satisfied

(ϕ1) $\phi(x)>0$ for every $x\in K$,

(ϕ2) $\phi(\alpha x)=\alpha\phi(x)$ for every $x\in K$ and $\alpha\geq 0$,

(ϕ3) $\phi(x+y)\leq\phi(x)+\phi(y)$ for every $x,y\in K$

(ϕ4) $\phi(1,0,0,\ldots)=1$

(ϕ5) if the inequality $\sum_1^k x_i\leq\sum_1^k y_i$ holds for $k=1,2,\ldots$, then $\phi(x)\leq\phi(y)$ holds [28].

It's given that ([27], [19]) for all symmetric norming functions ϕ, the function $\phi_{(p)}$ defined as

$$\phi_{(p)}:(x_i)\in K\to\left(\phi\left(\{x_i^p\}\right)\right)^{\frac{1}{p}},1\leq p\leq\infty$$

is also a symmetric norming function. For more details on symmetric norming functions we refer to ([7], [20], [23], [25]-[27], [30], [31]).

By using the properties of symmetric norming function and the sequence $(a_n(T))$, the class $\mathcal{L}_\phi(E,F)$ is defined in [25] and [29] as follows

$$\mathcal{L}_\phi(E,F)=\{T\in\mathcal{L}(E,F):\phi(\{a_n(T)\})<\infty\}.$$

Let $E=(E_n)$ be a partition of finite subsets of the positive integer such that

$$\max E_n<\min E_{n+1}$$

for $n=1,2,\ldots$. In [6] Foroutannia defined the sequence space $l_p(E)$ as

$$l_p(E)=\left\{x=(x_n)\in\omega:\sum_{n=1}^{\infty}\left|\sum_{j\in E_n}x_j\right|^p<\infty\right\},(1\leq p<\infty)$$

with the seminorm $|||\cdot|||_{p,E}$, which is defined in the following way:

$$|||x|||_{p,E}=\left(\sum_{n=1}^{\infty}\left|\sum_{j\in E_n}x_j\right|^p\right)^{\frac{1}{p}}.$$

For example, if $E_n=\{2n-1,2n\}$ for all n, then $x=(x_n)\in l_p(E)$ if and only if $\sum_{n=1}^{\infty}|x_{2n-1}+x_{2n}|^p<\infty$. It is obvious that $|||\cdot|||_{p,E}$ cannot be a norm, since we have

$\||x\||_{p,E} = 0$ while $x = (1, -1, 0, 0, \ldots) \neq \theta$ and $E_n = \{2n-1, 2n\}$ for all n. In the special case $E_n = \{n\}$ for $n = 1, 2, \ldots,$ we have $l_p(E) = l_p$ and $\||x\||_{p,E} = \|x\|_p$.

For more information about block sequence spaces, we refer to [16],[17].

In [32], the class of $l_p(E)$ type operators, which is denoted by $L_{p,E}(E, F)$, is given and it is shown that this class is a quasi-Banach operator ideal by the quasinorm

$$\|T\|_{p,E} = \left(\sum_{n=1}^{\infty} \left(\sum_{j \in E_n} s_j(T) \right)^p \right)^{\frac{1}{p}}.$$

Also a new class of operators $L_{\phi_{(p)},E}$ is defined. Further it is proved that by quasinorm

$$\|T\|_{\phi_{(p)},E} = \phi_{(p)} \left(\left\{ \sum_{j \in E_i} s_j(T) \right\} \right)$$

this class is a quasi-Banach operator ideal.

2. Main Results

Now we define a new class $L_{\phi,E}(E, F)$ including the class $L_\phi(E, F)$ as

$$L_{\phi,E}(E, F) = \left\{ T \in \mathcal{L}(E, F) : \phi \left(\left\{ \sum_{j \in E_n} s_j(T) \right\} \right) < \infty \right\}.$$

For example if we take $E_n = \{n\}$ for $n = 1, 2, \ldots,$ we have $L_{\phi,E}(E, F) = L_\phi(E, F)$. Also if we take $E_n = \{2n-1, 2n\}$ for all n, we get $\phi(\{s_{2n-1}(T) + s_{2n}(T)\}) < \infty$.

In this section we show some equivalent quasinorms on operator ideal $L_{\phi,E}(E, F)$.

Theorem 2.1. $\|T\|_{\phi,E} = \phi \left(\left\{ \sum_{j \in E_n} s_j(T) \right\} \right)$ *is a quasinorm on operator ideal* $L_{\phi,E}(E, F)$.

Proof. If $x' \in E$ and $y \in F$, then the equality

$$\phi \left(\left\{ \sum_{j \in E_n} s_j(x' \otimes y) \right\} \right) = \phi(\{s_1(x' \otimes y)\}) = \|x' \otimes y\| = \|x'\| \|y\| < \infty$$

holds since $x' \otimes y$ is a rank one operator, $s_n(x' \otimes y) = 0$ for $n \geq 2$. Therefore $\|x' \otimes y\|_{\phi,E} = \|x'\| \|y\|$ and $x' \otimes y \in L_{\phi,E}$.

Let $S, T \in L_{\phi,E}$. Then we have that

$$\begin{aligned}\sum_{n=1}^{\infty}\left(\sum_{j\in E_n} s_j\left(S+T\right)\right) &\leq \sum_{n=1}^{\infty}\left(\sum_{j\in E_n} s_{2j-1}\left(S+T\right)+\sum_{j\in E_n} s_{2j}\left(S+T\right)\right)\\ &\leq \sum_{n=1}^{\infty}\left(2\sum_{j\in E_n} s_{2j-1}\left(S+T\right)\right)\\ &\leq \sum_{n=1}^{\infty}\left(2\sum_{j\in E_n} s_j\left(S\right)+s_j\left(T\right)\right).\end{aligned}$$

By using ($\phi 5$) we can get

$$\begin{aligned}\phi\left(\left\{\sum_{j\in E_n} s_j\left(S+T\right)\right\}\right) &\leq \phi\left(\left\{2\left(\sum_{j\in E_n} s_j\left(S\right)+\sum_{j\in E_n} s_j\left(T\right)\right)\right\}\right)\\ &\leq 2\left[\phi\left(\left\{\left(\sum_{j\in E_n} s_j\left(S\right)\right)\right\}\right)+\phi\left(\left\{\left(\sum_{j\in E_n} s_j\left(T\right)\right)\right\}\right)\right]\\ &< \infty.\end{aligned}$$

It follows that

$$\left\|S+T\right\|_{\phi,E} \leq 2\left(\left\|S\right\|_{\phi,E}+\left\|T\right\|_{\phi,E}\right)$$

and also $S+T\in L_{\phi,E}$.

We have that

$$\begin{aligned}\sum_{n=1}^{\infty}\left(\sum_{j\in E_n} s_j\left(RST\right)\right) &\leq \sum_{n=1}^{\infty}\left(\sum_{j\in E_n}\left\|R\right\|\left\|T\right\| s_j\left(S\right)\right)\\ &\leq \left\|R\right\|\left\|T\right\|\sum_{n=1}^{\infty}\left(\sum_{j\in E_n} s_j\left(S\right)\right).\end{aligned}$$

By using the properties of ϕ function, we obtain

$$\phi\left(\left\{\sum_{j\in E_n} s_j\left(RST\right)\right\}\right) \leq \left\|R\right\|\left\|T\right\|\phi\left(\left\{\sum_{j\in E_n} s_j\left(S\right)\right\}\right) < \infty$$

and also the inequality

$$\left\|RST\right\|_{\phi,E} \leq \left\|R\right\|\left\|T\right\|\left\|S\right\|_{\phi,E}$$

holds.

Hence, $\left\|T\right\|_{\phi,E}=\phi\left(\left\{\sum_{j\in E_n} s_j\left(T\right)\right\}\right)$ is a quasinorm on operator ideal $L_{\phi,E}\left(E,F\right)$. □

Proposition 2.1. *The quasinorm* $\|T\|^{+}_{\phi,E} = \phi\left(\left\{\sum_{j\in E_n} s_{2j-1}(T)\right\}\right)$ *is equivalent with* $\|T\|_{\phi,E}$.

Proof. The equivalence can be easily seen from the fact that

$$\sum_{n=1}^{k}\sum_{j\in E_n} s_{2j-1}(T) \le \sum_{n=1}^{k}\sum_{j\in E_n} s_j(T) \le 2\sum_{n=1}^{k}\sum_{j\in E_n} s_{2j-1}(T).$$

□

Remark 2.1. For the particular case if $E_n = \{n\}$ for $n = 1, 2, \ldots,$ we get Proposition 1.1 in [28].

Proposition 2.2. *The quasinorm* $\|T\|_{\phi_{(p)},E}$ *is equivalent with*

$$\|T\|^{\nabla}_{\phi_{(p)},E} = \phi_{(p)}\left(\left\{\frac{1}{n}\sum_{i=1}^{n}\sum_{j\in E_i} s_j(T)\right\}\right), 1 < p < \infty$$

where

$$E_n = \{nN - N + 1, nN - N + 2, \ldots, nN\}\, for\, all\ n.$$

Proof. This is a consequence of Hardy's inequality.

$$\sum_{n=1}^{k}\left(\sum_{j\in E_n} s_j(T)\right)^p \le \sum_{n=1}^{k}\left(\frac{1}{n}\sum_{i=1}^{n}\sum_{j\in E_i} s_j(T)\right)^p \le \left(\frac{p}{p-1}\right)^p\sum_{n=1}^{k}\left(\sum_{j\in E_n} s_j(T)\right)^p.$$

□

Remark 2.2. In particular case if we take $N = 1$ we get Proposition 1.2 in [28].

Theorem 2.2. *If* (α_n) *is a nonincreasing positive sequence and* $\lim \alpha_{Nn} \neq 0$, *then the quasinorm* $\|T\|_{\phi_{(p)},E}$ *is equivalent with the quasinorm*

$$\|T\|^{\circ}_{\phi_{(p)},E} = \phi_{(p)}\left(\left\{\frac{1}{\alpha_1 + \ldots + \alpha_n}\sum_{i=1}^{n}\sum_{j\in E_i} s_j(T)\right\}\right), 1 < p < \infty \text{ where}$$

$$E_n = \{nN - N + 1, nN - N + 2, \ldots, nN\}\, for\ all\ n\,.$$

Proof. We know that the sequences (α_n) and $(s_n(T))$ are decreasing, so we can write that

$$\frac{1}{n\alpha_1} n\alpha_{Nn}\sum_{j\in E_i} s_j(T) = \frac{\alpha_{Nn}}{\alpha_1}\sum_{j\in E_i} s_j(T) \le \frac{1}{\alpha_1 + \ldots + \alpha_n}\sum_{i=1}^{n}\sum_{j\in E_i}\alpha_j s_j(T)$$
$$\le \frac{1}{n\alpha_{Nn}}\alpha_1\sum_{i=1}^{n}\sum_{j\in E_i} s_j(T).$$

If $\lim \alpha_{Nn} = \alpha \neq 0$, we get

$$\frac{\alpha}{\alpha_1} \sum_{j\in E_i} s_j(T) \leq \frac{1}{\alpha_1 + \ldots + \alpha_n} \sum_{i=1}^{n} \sum_{j\in E_i} \alpha_j s_j(T) \leq \frac{\alpha_1}{\alpha} \left(\frac{1}{n} \sum_{i=1}^{n} \sum_{j\in E_i} s_j(T) \right).$$

By using Hardy's inequality we obtain

$$\begin{aligned} \sum_{i=1}^{n} \left(\frac{\alpha}{\alpha_1} \sum_{j\in E_i} s_j(T) \right)^p &\leq \sum_{i=1}^{n} \left(\frac{1}{\alpha_1 + \ldots + \alpha_n} \sum_{i=1}^{n} \sum_{j\in E_i} \alpha_j s_j(T) \right)^p \\ &\leq \sum_{i=1}^{n} \left(\frac{\alpha_1}{\alpha} \right)^p \left(\frac{1}{n} \sum_{i=1}^{n} \sum_{j\in E_i} s_j(T) \right)^p \\ &\leq \left(\frac{\alpha_1}{\alpha} \right)^p \left(\frac{p}{p-1} \right)^p \sum_{i=1}^{n} \sum_{j\in E_i} (s_j(T))^p, \ 1 < p < \infty. \end{aligned}$$

By the property ($\phi 5$) it results

$$\frac{\alpha_1}{\alpha} \|T\|_{\phi_{(p)},E} \leq \|T\|^{\circ}_{\phi_{(p)},E} \leq \frac{\alpha_1}{\alpha} \frac{p}{p-1} \|T\|_{\phi_{(p)},E}.$$

Hence $\|T\|_{\phi_{(p)},E}$ is equivalent with $\|T\|^{\circ}_{\phi_{(p)},E}$. □

Remark 2.3. In particular case if we take $N = 1$ then we get Theorem 1.4 in [28].

Theorem 2.3. *Let (u_n) and (w_n) are sequences of non-negative real numbers such that $u_1 \geq u_2 \geq ... \geq u_n \geq \ldots$ and $w_1 \leq w_2 \leq ... \leq w_n \leq ...$ and $w_n \leq n \leq \frac{w_n}{u_n}$. Let $\lim_{n\to\infty} u_{Nn} \neq 0$, then the quasinorm $\|T\|_{\phi_{(p)},E}$ is equivalent to*

$$\|T\|^{\gamma}_{\phi_{(p)},E} = \phi_{(p)} \left(\left\{ \frac{1}{w_n} \sum_{i=1}^{n} \sum_{j\in E_i} u_j s_j(T) \right\} \right) \textit{for } 1 \leq p < \infty.$$

where

$$E_n = \{nN - N + 1, nN - N + 2, \ldots, nN\} \textit{ for all } n.$$

Proof. Since the sequences (u_n) and $(a_n(T))$ are decreasing, we can write

$$\frac{1}{n} n u_{Nn} \sum_{j\in E_i} s_j(T) \leq \frac{1}{w_n} \sum_{i=1}^{n} \sum_{j\in E_i} u_j s_j(T) \leq \frac{1}{n u_{Nn}} u_1 \sum_{i=1}^{n} \sum_{j\in E_i} s_j(T).$$

Summing from $n=1$ to k, we get

$$\sum_{n=1}^{k}\left(u_{Nn}\sum_{j\in E_i}s_j\left(T\right)\right)^p \leq \sum_{n=1}^{k}\left(\frac{1}{w_n}\sum_{i=1}^{n}\sum_{j\in E_i}u_js_j\left(T\right)\right)^p$$
$$\leq \sum_{n=1}^{k}\left(\frac{u_1}{nu_{Nn}}\sum_{i=1}^{n}\sum_{j\in E_i}s_j\left(T\right)\right)^p.$$

If $\lim_{n\to\infty}u_{Nn}=u\neq 0$, then we obtain

$$u^p\sum_{n=1}^{k}\left(\sum_{j\in E_i}s_j\left(T\right)\right)^p \leq \sum_{n=1}^{k}\left(\frac{1}{w_n}\sum_{i=1}^{n}\sum_{j\in E_i}u_js_j\left(T\right)\right)^p$$
$$\leq \left(\frac{u_1}{u}\right)^p\sum_{n=1}^{k}\left(\frac{1}{n}\sum_{i=1}^{n}\sum_{j\in E_i}s_j\left(T\right)\right)^p$$

for every $k\in\mathbb{N}$. By using Hardy's inequality, we get

$$u^p\sum_{n=1}^{k}\left(\sum_{j\in E_i}s_j\left(T\right)\right)^p \leq \sum_{n=1}^{k}\left(\frac{1}{w_n}\sum_{i=1}^{n}\sum_{j\in E_i}u_js_j\left(T\right)\right)^p$$
$$\leq \left(\frac{u_1}{u}\right)^p\left(\frac{p}{p-1}\right)^p\sum_{i=1}^{n}\left(\sum_{j\in E_i}s_j\left(T\right)\right)^p$$

for every $k\in\mathbb{N}$. From the properties of the function ϕ, we obtain that

$$u\left\|T\right\|_{\phi_{(p)},E}\leq \left\|T\right\|^{\gamma}_{\phi_{(p)},E}\leq \left(\frac{u_1}{u}\right)\left(\frac{p}{p-1}\right)\left\|T\right\|_{\phi_{(p)},E}.$$

□

Remark 2.4. For the particular case, if we choose $N=1$, we get Theorem 2.2 in [30]. And also if we take $u_i=\alpha_i$ and $w_n=\alpha_1+\alpha_2+...+\alpha_n$ in Theorem 3, where $N=1$ then we obtain Theorem 1.4 in [28], where $\alpha_1\leq 1$. If we take $u_i=1$ and $w_n=n$ in Theorem 3, then we obtain Proposition 1.2 in [28].

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