ON PRESERVING INTUITIONISTIC FUZZY gpr-CLOSED SETS

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Abstract. In this paper we introduce the concepts of intuitionistic fuzzy apr-closed and intuitionistic fuzzy apr-continuous mappings in intuitionistic fuzzy topological spaces and obtain several results concerning the preservation of intuitionistic fuzzy gpr-closed sets. Furthermore, we characterize intuitionistic fuzzy pre-regular $T_{\frac{1}{2}}$-spaces due to Thakur and Bajpai[13] in terms of intuitionistic fuzzy apr-continuous and intuitionistic fuzzy apr-closed mappings and obtain some of the basic properties and characterization of these mappings.

1. Introduction

After the introduction of fuzzy sets by Zadeh [15] in 1965 and fuzzy topology by Chang [4] in 1968, research was conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. The concept of intuitionistic fuzzy sets was introduced by Atanassov [1, 2] as a generalization of fuzzy sets. In 2008 Thakur and Chaturvedi extended the concepts of fuzzy g-closed sets[9] and fuzzy g-continuity [7] in intuitionistic fuzzy topological spaces. Recently many generalizations of intuitionistic fuzzy g-closed sets[9] like intuitionistic fuzzy rg-closed sets [8], intuitionistic fuzzy sg-closed sets [12], intuitionistic fuzzy w-closed sets[10], intuitionistic fuzzy rw-closed sets [11], intuitionistic fuzzy gpr-closed sets[13] have appeared in the literature. In this paper we introduce the concepts of intuitionistic fuzzy apr-closed and intuitionistic fuzzy apr-continuous mappings using intuitionistic fuzzy gpr-closed sets. These definitions enable us to obtain conditions under which maps and inverse maps preserve intuitionistic fuzzy gpr-closed sets [13]. We also characterize intuitionistic fuzzy pre-regular $T_{\frac{1}{2}}$-spaces in terms of intuitionistic fuzzy apr-continuous and intuitionistic fuzzy apr-closed mappings. Finally some of basic properties of intuitionistic fuzzy apr-continuous and intuitionistic fuzzy apr-closed mappings are investigated.

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2. Preliminaries

Since we shall require the following known definitions, notations and some properties, we recall them in this section.

Definition 2.1. [1] Let $X$ be a nonempty fixed set. An intuitionistic fuzzy set $A$ is an object having the form

$$A = \{< x, \mu_A(x), \gamma_A(x) > : x \in X \}$$

Where the functions $\mu_A : X \to I$ and $\gamma_A : X \to I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\gamma_A(x)$) of each element $x \in X$ to the set $A$, respectively, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each element $x \in X$.

Definition 2.2. [1] Let $X$ be a nonempty set and the intuitionistic fuzzy sets $A$ and intuitionistic fuzzy set $B$ be in the form

$$A = \{< x, \mu_A(x), \gamma_A(x) > : x \in X \},$$

$$B = \{< x, \mu_B(x), \gamma_B(x) > : x \in X \}$$

and let $\{A_i : i \in J\}$ be an arbitrary family of intuitionistic fuzzy sets in $X$.

Then:
(a) $A \subseteq B$ if $\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$.
(b) $A = B$ if $A \subseteq B$ and $B \subseteq A$
(c) $A^c = \{< x, \gamma_A(x), \mu_A(x) > : x \in X \}$
(d) $\bigcap A_i = \{< x, \wedge \mu_A(x), \vee \gamma_A(x) > : x \in X \}$
(e) $\bigcup A_i = \{< x, \vee \mu_A(x), \wedge \gamma_A(x) > : x \in X \}$
(f) $\tilde{0} = \{< x, 0, 1 > : x \in X \}$ and $\tilde{1} = \{< x, 1, 0 > : x \in X \}$

Definition 2.3. [5] An intuitionistic fuzzy topology on a nonempty set $X$ is a family $\tau$ of intuitionistic fuzzy sets in $X$, satisfying the following axioms:

$(T_1)$ $\tilde{0}$ and $\tilde{1} \in \tau$
$(T_2)$ $G_1 \cap G_2 \in \tau$
$(T_3)$ $G_1 \cup G_2 \in \tau$

In this case the pair $(X, \tau)$ is called an intuitionistic fuzzy topological space and each intuitionistic fuzzy set in $\tau$ is known as an intuitionistic fuzzy open set in $X$. The complement $A^c$ of an intuitionistic fuzzy open set $A$ is called an intuitionistic fuzzy closed set in $X$.

Definition 2.4. [5] Let $(X, \tau)$ be an intuitionistic fuzzy topological space and $A = \{< x, \mu_A(x), \gamma_A(x) > : x \in X \}$ be an intuitionistic fuzzy set in $X$. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure of $A$ are defined by:

$$\text{int}(A) = \bigcup \{K : K \text{ is an intuitionistic fuzzy open set such that } K \subseteq A \}$$

$$\text{cl}(A) = \bigcap \{K : K \text{ is an intuitionistic fuzzy closed set such that } A \subseteq K \}$$
Definition 2.5. [6] An intuitionistic fuzzy set $A$ of an intuitionistic fuzzy topological space $(X, \tau)$ is called:

(a) intuitionistic fuzzy pre-open if $A \subseteq \text{int}(\text{cl}(A))$ and intuitionistic fuzzy pre-closed if $\text{cl}(\text{int}(A)) \subseteq A$

(b) intuitionistic fuzzy regular open if $A = \text{int}(\text{cl}(A))$ and intuitionistic fuzzy regular closed if $A = \text{cl}(\text{int}(A))$.

Definition 2.6. [6] If $A$ is an intuitionistic fuzzy set in intuitionistic fuzzy topological space $(X, \tau)$ then $\text{pcl}(A) = \cap\{K: K$ is an intuitionistic fuzzy pre-closed set such that $A \subseteq K\}$.

Definition 2.7. [13] An intuitionistic fuzzy set $A$ of an intuitionistic fuzzy topological space $(X, \tau)$ is called:

(a) intuitionistic fuzzy $\text{gpr}$-closed if $\text{pcl}(A) \subseteq O$ whenever $A \subseteq O$ and $O$ is intuitionistic fuzzy regular open.

(b) intuitionistic fuzzy $\text{gpr}$-open if and only if $A^c$ is intuitionistic fuzzy $\text{gpr}$-closed.

Definition 2.8. [13] An intuitionistic fuzzy topological space $(X, \tau)$ is said to be intuitionistic fuzzy pre-regular $T_2$-space if every intuitionistic fuzzy $\text{gpr}$-closed set in $X$ is intuitionistic fuzzy pre-closed in $X$.

Remark 2.1. [13] Every intuitionistic fuzzy regular closed set is intuitionistic fuzzy $\text{gpr}$-closed but its converse may not be true.

Remark 2.2. [13] Every intuitionistic fuzzy pre-closed set is intuitionistic fuzzy $\text{gpr}$-closed but its converse may not be true.

Theorem 2.1. [13] An intuitionistic fuzzy set $A$ of an intuitionistic fuzzy topological space is intuitionistic fuzzy $\text{gpr}$-open if and only if $F \subseteq \text{pint}(A)$ whenever $F$ is intuitionistic fuzzy regular closed and $F \subseteq A$.

Theorem 2.2. [13] Let $(X, \tau)$ be an intuitionistic fuzzy topological space and $\text{IFPC}$ (resp. $\text{IFRO}(X)$) be the family of all intuitionistic fuzzy pre-closed (resp. intuitionistic fuzzy regular open) sets of $X$. Then $\text{IFPC}(X) = \text{IFRO}(X)$ if and only if every intuitionistic fuzzy set of $X$ is intuitionistic fuzzy $\text{gpr}$-closed.

Definition 2.9. [5] Let $X$ and $Y$ be two nonempty sets and $f : X \to Y$ be a mapping. Then:

(a) If $B = \{< (y, \mu_B(y), \gamma_B(y)) : y \in Y\}$ is an intuitionistic fuzzy set in $Y$, then the pre-image of $B$ under $f$ denoted by $f^{-1}(B)$ is the intuitionistic fuzzy set in $X$ defined by $f^{-1}(B) = \{< x, f^{-1}(\mu_B)(x), f^{-1}(\gamma_B)(x) : x \in X\}$.

(b) If $A = \{< x, \lambda_A(x), \nu_A(x) : x \in X\}$ is an intuitionistic fuzzy set in $X$, then the image of $A$ under $f$ denoted by $f(A)$ is the intuitionistic fuzzy set in $Y$ defined by $f(A) = \{< y, f(\lambda_A)(y), f(\nu_A)(y) : y \in Y\}$ where $f(\nu_A) = 1 - f(1 - \nu_A)$. 
Definition 2.10. Let \((X, \tau)\) and \((Y, \sigma)\) be two intuitionistic fuzzy topological spaces and let \(f : X \to Y\) be a mapping. Then \(f\) is said to be:

(a). Intuitionistic fuzzy continuous [6] if the pre-image of each intuitionistic fuzzy open set if \(Y\) is an intuitionistic fuzzy open set in \(X\).
(b). Intuitionistic fuzzy \(gpr\)-continuous [13] if the pre image of every intuitionistic fuzzy closed set in \(Y\) is an intuitionistic fuzzy \(gpr\)-closed set in \(X\).
(c). Intuitionistic fuzzy irresolute [6] if the pre-image of every intuitionistic fuzzy semi-closed set in \(Y\) is an intuitionistic fuzzy semi-closed set in \(X\).
(d). Intuitionistic fuzzy \(gpr\)-irresolute [15] if the pre-image of every intuitionistic fuzzy \(gpr\)-closed set in \(Y\) is an intuitionistic fuzzy \(gpr\)-closed set in \(X\).
(e). Intuitionistic fuzzy pre-closed [6] if the image of each intuitionistic fuzzy closed set in \(X\) is an intuitionistic fuzzy pre-closed set in \(Y\).
(f). Intuitionistic fuzzy pre-regular closed [8] if the image of each intuitionistic fuzzy regular closed set in \(X\) is an intuitionistic fuzzy regular closed set in \(Y\).
(g). Intuitionistic fuzzy \(R\) mapping [8] if the pre-image of each intuitionistic fuzzy regular open set of \(Y\) is an intuitionistic fuzzy regular open set in \(X\).

Remark 2.3. [13] Every intuitionistic fuzzy continuous mapping is intuitionistic fuzzy \(gpr\)-continuous, but the converse may not be true.

Remark 2.4. [13] Every intuitionistic fuzzy \(gpr\)-irresolute mapping is intuitionistic fuzzy \(gpr\) -continuous, but the converse may not be true. The concepts of intuitionistic fuzzy \(gpr\)-irresolute and intuitionistic fuzzy continuous mapping are independent.

3. Intuitionistic Fuzzy apr-Closed and Intuitionistic fuzzy apr-continuous mappings

Definition 3.1. A mapping \(f : (X, \tau) \to (Y, \sigma)\) is said to be intuitionistic fuzzy \(apr\)-closed provided that \(f(F) \subseteq \text{pint}(A)\) whenever \(F\) is intuitionistic fuzzy regular closed set in \(X\), \(A\) is an intuitionistic fuzzy \(gpr\)-open set in \(Y\) and \(f(F) \subseteq A\).

Theorem 3.1. Every intuitionistic fuzzy pre-regular closed mapping is intuitionistic fuzzy \(apr\)-closed.

Proof. Let \(f : (X, \tau) \to (Y, \sigma)\) be an intuitionistic fuzzy pre-regular closed mapping. Let \(F\) be intuitionistic fuzzy regular closed set in \(X\) and \(A\) is an intuitionistic fuzzy \(gpr\)-open set in \(Y\) such that \(f(F) \subseteq A\). Since \(f\) is intuitionistic fuzzy pre-regular closed mapping, \(f(a)\) is intuitionistic fuzzy regular closed set in \(Y\). Now \(A\) is intuitionistic fuzzy \(gpr\)-open and \(f(F) \subseteq A \Rightarrow f(F) \subseteq \text{pint}(A)\). Hence \(f\) is intuitionistic fuzzy \(apr\)-closed.

Remark 3.1. The converse of Theorem 3.1 may not be true.
Example 3.1. Let $X = \{a, b\}$ and $U = \{< a, 0.6, 0.3 >, < b, 0.3, 0.6 >\}$ be an intuitionistic fuzzy set on $X$. Let $\tau = \{0, X, 1\}$ be intuitionistic fuzzy topology on $X$. Then the mapping $f : (X, \tau) \to (X, \tau)$ defined by $f(a) = b$ and $f(b) = a$ is intuitionistic fuzzy $apr$-closed but it is not intuitionistic fuzzy pre-regular closed.

Definition 3.2. A mapping $f : (X, \tau) \to (Y, \sigma)$ is said to be intuitionistic fuzzy $apr$-continuous provided that $pcl(F) \subseteq f^{-1}(O)$ whenever $F$ is intuitionistic fuzzy $gpr$-closed set in $X$, $O$ is an intuitionistic fuzzy regular open set in $Y$ and $F \subseteq f^{-1}(O)$.

Theorem 3.2. Every intuitionistic fuzzy $R$-mapping is intuitionistic fuzzy $apr$-continuous.

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be an intuitionistic fuzzy $R$- mapping. Let $O$ be an intuitionistic fuzzy regular open set of $Y$ and $F$ is an intuitionistic fuzzy $gpr$-closed set of $X$ such that $F \subseteq f^{-1}(O)$. Now since $f$ is intuitionistic fuzzy $R$-mapping, $f^{-1}(O)$ is intuitionistic fuzzy regular open set in $X$. Since $F$ is intuitionistic fuzzy $gpr$-closed and $F \subseteq f^{-1}(O) \Rightarrow pcl(F) \subseteq f^{-1}(O)$. Hence $f$ is intuitionistic fuzzy $apr$-continuous. □

Remark 3.2. The converse of Theorem 3.2 may not be true.

Example 3.2. Let $X = \{a, b\}$ and $U = \{< a, 0.3, 0.7 >, < b, 0.4, 0.6 >\}$ be an intuitionistic fuzzy set on $X$. Let $\tau = \{0, X, 1\}$ be intuitionistic fuzzy topology on $X$. Then the mapping $f : (X, \tau) \to (X, \tau)$ defined by $f(a) = b$ and $f(b) = a$ is intuitionistic fuzzy $apr$-continuous but it is not intuitionistic fuzzy $R$-mapping.

Theorem 3.3. If $f : (X, \tau) \to (Y, \sigma)$ is a bijection, then $f$ is intuitionistic fuzzy $apr$-closed if and only if $f^{-1}$ is intuitionistic fuzzy $apr$-continuous.

Proof. Obvious. □

4. Preserving Intuitionistic Fuzzy $gpr$-closed sets

In this section the concepts of intuitionistic fuzzy $apr$-continuous and intuitionistic fuzzy $apr$-closed mappings are used to obtain some results on preservation of intuitionistic fuzzy $gpr$-closed sets.

Theorem 4.1. If a mapping $f : (X, \tau) \to (Y, \sigma)$ is intuitionistic fuzzy $gpr$-continuous and intuitionistic fuzzy $apr$-closed then $f^{-1}(A)$ is intuitionistic fuzzy $gpr$-closed set in $X$ whenever $A$ is intuitionistic fuzzy $gpr$-closed set in $Y$.

Proof. Suppose that $f : (X, \tau) \to (Y, \sigma)$ is intuitionistic fuzzy $gpr$-continuous and intuitionistic fuzzy $apr$-closed. Let $A$ be an intuitionistic fuzzy $gpr$-closed set in $Y$ such that $f^{-1}(A) \subseteq O$, where $O$ be an intuitionistic fuzzy regular open set in $X$. Then $O^c \subseteq f^{-1}(A)^c$ which implies that $f(O^c) \subseteq int(A^c) = (cl(A))^c$. Hence $f^{-1}(cl(A)) \subseteq O$. Since $f$ is intuitionistic fuzzy $gpr$-continuous and $f^{-1}(cl(A))$ is intuitionistic fuzzy $gpr$-closed in $X$. Therefore $pcl(f^{-1}(cl(A))) \subseteq O$ which implies that $pcl(f^{-1}(A)) \subseteq O$. Hence $f^{-1}(A)$ is intuitionistic fuzzy $gpr$-closed set in $X$. □
Corollary 4.1. If a mapping \( f : (X, \tau) \to (Y, \sigma) \) is intuitionistic fuzzy continuous and intuitionistic fuzzy apr-closed then \( f^{-1}(A) \) is intuitionistic fuzzy gpr-closed set in \( X \) whenever \( A \) is intuitionistic fuzzy gpr-closed set in \( Y \).

Theorem 4.2. If a mapping \( f : (X, \tau) \to (Y, \sigma) \) is intuitionistic fuzzy gpr-continuous and intuitionistic fuzzy apr-closed then \( f^{-1}(A) \) is intuitionistic fuzzy gpr-open set in \( X \) whenever \( A \) is intuitionistic fuzzy gpr-open set in \( Y \).

Proof. Suppose that \( f : (X, \tau) \to (Y, \sigma) \) is intuitionistic fuzzy gpr-continuous and intuitionistic fuzzy apr-closed mapping. Let \( A \) is intuitionistic fuzzy gpr-open in \( Y \). Then by definition 2.7 \( A^c \) is intuitionistic fuzzy gpr-closed in \( Y \). Hence by theorem 4.1 \( f^{-1}(A^c) \) is intuitionistic fuzzy gpr-closed in \( X \). Since \( f^{-1}(A^c) = (f^{-1}(A))^c \) for every intuitionistic fuzzy set \( A \) of \( Y \). Hence \((f^{-1}(A))^c \) is intuitionistic fuzzy gpr-closed set in \( X \). Therefore \( f^{-1}(A) \) is intuitionistic fuzzy gpr-open set in \( X \). \( \Box \)

Corollary 4.2. If a mapping \( f : (X, \tau) \to (Y, \sigma) \) is intuitionistic fuzzy continuous and intuitionistic fuzzy apr-closed then \( f^{-1}(A) \) is intuitionistic fuzzy gpr-open set in \( X \) whenever \( A \) is intuitionistic fuzzy apr-closed set in \( Y \).

Theorem 4.3. If \( f : (X, \tau) \to (Y, \sigma) \) is intuitionistic fuzzy apr-continuous and intuitionistic fuzzy apr-closed mapping then the image of every intuitionistic fuzzy gpr-closed set of \( X \) is intuitionistic fuzzy gpr-closed in \( Y \).

Proof. Let \( B \) be an intuitionistic fuzzy gpr-closed set of \( X \), and \( f(B) \subseteq O \). where \( O \) is intuitionistic fuzzy regular open set in \( Y \). Then \( B \subseteq f^{-1}(O) \) and since \( f \) is intuitionistic fuzzy apr-continuous, \( pcl(B) \subseteq f^{-1}(O) \) which implies that \( f(pcl(B)) \subseteq O \). Since \( f \) is intuitionistic fuzzy apr-closed mapping and \( pcl(B) \) is intuitionistic fuzzy pre-closed in \( X \), \( f(pcl(B)) \) is intuitionistic fuzzy pre-closed in \( Y \). Hence we have \( pcl(f(B)) \subseteq pcl(f(pcl(B))) = f(pcl(B)) \subseteq O \). Hence \( f(B) \) is intuitionistic fuzzy gpr-closed in \( Y \). \( \Box \)

5. A Characterization of Intuitionistic Fuzzy pre regular \( T_{1\frac{1}{2}} \) - spaces

In the following theorems we give a characterization of a class of intuitionistic fuzzy pre-regular \( T_{1\frac{1}{2}} \) - spaces by using the concepts of intuitionistic fuzzy apr-closed and intuitionistic fuzzy apr-continuous mapping.

Theorem 5.1. An intuitionistic fuzzy topological space \( (X, \tau) \) is intuitionistic fuzzy pre-regular \( T_{1\frac{1}{2}} \) - space if and only if every mapping \( f : (X, \tau) \to (Y, \sigma) \) is intuitionistic fuzzy apr-continuous.

Proof. Necessity: Let \( f : (X, \tau) \to (Y, \sigma) \) is intuitionistic fuzzy mapping. Let \( A \) is intuitionistic fuzzy gpr-closed set of \( X \) and \( A \subseteq f^{-1}(O) \) where \( O \) is intuitionistic fuzzy regular open set of \( Y \). Since \( X \) is intuitionistic fuzzy pre-regular \( T_{1\frac{1}{2}} \) - space, \( A \)
is intuitionistic fuzzy pre-closed set in $X$. Therefore $pcl(A) = A \subseteq f^{-1}(O)$. Hence $A$ is intuitionistic fuzzy apr-continuous.

Sufficiency: Let $A$ be a nonempty intuitionistic fuzzy gpr-closed set in $X$ and let $Y$ be intuitionistic fuzzy topological space with the intuitionistic fuzzy topology $\sigma = \{ \emptyset, A, 1 \}$. Finally let $f : (X, \tau) \to (Y, \sigma)$ be identity mapping. By assumption $f$ is intuitionistic fuzzy apr-continuous. Since $A$ is intuitionistic fuzzy gpr-closed in $X$ and intuitionistic fuzzy open in $Y$ and $A \subseteq f^{-1}(A)$, it follows that $pcl(A) \subseteq f^{-1}(A) = A$, because $f$ is identity mapping. Hence $A$ is intuitionistic fuzzy pre-closed in $X$ and therefore $X$ is intuitionistic fuzzy pre-regular $T_{1/2}$-space.

An analogous argument proves the following result for intuitionistic fuzzy apr-closed mapping.

**Theorem 5.2.** An intuitionistic fuzzy topological space $(X, \tau)$ is intuitionistic fuzzy pre-regular $T_{1/2}$-space if and only if every mapping $f : (X, \tau) \to (Y, \sigma)$ is intuitionistic fuzzy apr-closed.

### 6. Properties of Intuitionistic Fuzzy apr-closed and Intuitionistic Fuzzy apr-continuous mappings

In this section we investigate some of the properties of intuitionistic fuzzy apr-closed and intuitionistic fuzzy apr-continuous mappings.

**Theorem 6.1.** Every intuitionistic fuzzy gpr-continuous and intuitionistic fuzzy apr-closed mapping is intuitionistic fuzzy gpr-irresolute.

**Proof.** Suppose that $f : (X, \tau) \to (Y, \sigma)$ is intuitionistic fuzzy gpr-continuous and intuitionistic fuzzy apr-closed mapping and $A$ is intuitionistic fuzzy gpr-closed set in $Y$. Let $f^{-1}(A) \subseteq O$ where $O$ be an intuitionistic fuzzy regular open set in $X$. Then $O^c \subseteq f^{-1}(A)$ which implies that $f(O^c) \subseteq int(A^c) = (cl(A))^c$. Hence $f^{-1}(cl(A)) \subseteq O$. Since $f$ is intuitionistic fuzzy gpr-continuous $f^{-1}(cl(A))$ is intuitionistic fuzzy gpr-closed in $X$. Therefore $pcl(f^{-1}(cl(A))) \subseteq O$ which implies that $pcl(f^{-1}(A)) \subseteq O$. Hence $f^{-1}(A)$ is intuitionistic fuzzy gpr-closed set in $X$. Therefore $f$ is intuitionistic fuzzy gpr-irresolute.

**Theorem 6.2.** Every intuitionistic fuzzy continuous and intuitionistic fuzzy apr-closed mapping is intuitionistic fuzzy gpr-irresolute.

**Proof.** It follows from Remark 2.3 and Theorem 6.1.

**Theorem 6.3.** If $f : (X, \tau) \to (Y, \sigma)$ is intuitionistic fuzzy mapping for which $f(F)$ is intuitionistic fuzzy-pre-open set in $Y$ for every intuitionistic fuzzy regular closed set $F$ of $X$ then $f$ is intuitionistic fuzzy apr-closed mapping.
Proof. Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) be intuitionistic fuzzy mapping, \( F \) intuitionistic fuzzy regular closed in \( X \), \( A \) intuitionistic fuzzy \( gpr \)-open in \( Y \) and \( f(F) \subseteq A \). By hypothesis \( f(F) \) is intuitionistic fuzzy \( apr \)-open in \( X \). Therefore \( f(F) = pint(f(F)) \subseteq pint(A) \). Hence \( f \) is intuitionistic fuzzy \( apr \)-closed. \( \Box \)

**Theorem 6.4.** If \( f : (X, \tau) \rightarrow (Y, \sigma) \) is intuitionistic fuzzy mapping for which \( f^{-1}(V) \) is intuitionistic fuzzy \( apr \)-closed in \( X \) for every intuitionistic fuzzy regular open set \( V \) of \( Y \), then \( f \) is intuitionistic fuzzy \( apr \)-continuous mapping.

Proof. Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) be intuitionistic fuzzy mapping. Let \( F \) be intuitionistic fuzzy \( gpr \)-closed set in \( X \) and \( V \) intuitionistic fuzzy regular open set of \( Y \) such that \( F \subseteq f^{-1}(V) \). By hypothesis \( f^{-1}(V) \) is intuitionistic fuzzy \( apr \)-closed in \( X \). Hence \( pcl(f^{-1}(V)) = f^{-1}(V) \). Therefore \( pcl(F) \subseteq pcl(f^{-1}(V)) = f^{-1}(V) \). Hence \( f \) is intuitionistic fuzzy \( apr \)-continuous. \( \Box \)

**Remark 6.1.** Since the identity mapping on any intuitionistic fuzzy topological space is both intuitionistic fuzzy \( apr \)-continuous and intuitionistic fuzzy \( apr \)-closed, it is clear that the converse of Theorem 6.3 and Theorem 6.4 do not hold.

**Theorem 6.5.** If \( \text{IFRO}(Y) = \text{IFPC}(Y) \) where \( \text{IFRO}(Y) \) (resp. \( \text{IFPC}(Y) \)) denotes the family of all intuitionistic fuzzy regular open (resp. intuitionistic fuzzy \( apr \)-open) sets of \( Y \), then the mapping \( f : (X, \tau) \rightarrow (Y, \sigma) \) is intuitionistic fuzzy \( apr \)-closed if and only if \( f(F) \) is intuitionistic fuzzy \( apr \)-open set in \( Y \), for every intuitionistic fuzzy regular closed set \( F \) of \( X \).

Proof. Necessity: Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) is intuitionistic fuzzy \( apr \)-closed mapping. By Theorem 2.2 [13] every intuitionistic fuzzy set of \( Y \) is intuitionistic fuzzy \( gpr \)-closed and hence all are intuitionistic fuzzy \( gpr \)-open. Thus for any intuitionistic fuzzy regular closed set \( F \) of \( X \), \( f(F) \) is intuitionistic fuzzy \( apr \)-open in \( Y \). Since \( f \) is intuitionistic fuzzy \( apr \)-closed, \( f(F) \subseteq pint(f(F)) \) and then \( f(F) = pint(f(F)) \). Hence \( f(F) \) is intuitionistic fuzzy \( apr \)-open.

Sufficiency: Let \( F \) be an intuitionistic fuzzy regular closed set of \( X \) and \( A \) be an intuitionistic fuzzy \( gpr \)-open set of \( Y \) and \( f(F) \subseteq A \). By hypothesis \( f(F) \) is intuitionistic fuzzy \( apr \)-open in \( Y \) and \( f(F) = pint(f(F)) \subseteq pint(A) \). Hence \( f \) is intuitionistic fuzzy \( apr \)-closed. \( \Box \)

**Theorem 6.6.** If \( \text{IFRO}(Y) = \text{IFPC}(Y) \) where \( \text{IFRO}(Y) \) (resp. \( \text{IFPC}(Y) \)) denotes the family of all intuitionistic fuzzy regular open (resp. intuitionistic fuzzy \( apr \)-closed) sets of \( Y \) then the mapping \( f : (X, \tau) \rightarrow (Y, \sigma) \) is intuitionistic fuzzy \( apr \)-closed if and only if \( f \) is intuitionistic fuzzy \( apr \)-regular closed.

Proof. Necessity: Let \( O \) be an intuitionistic fuzzy regular closed set of \( X \). Then by theorem 6.5 \( f(O) \) is intuitionistic fuzzy \( apr \)-open in \( Y \). Since every intuitionistic fuzzy \( apr \)-open set is intuitionistic fuzzy regular open, therefore \( f(O) \) is intuitionistic fuzzy regular open in \( Y \) and hence by hypothesis \( f(O) \) is intuitionistic fuzzy \( apr \)-closed.

\[ \Box \]
pre-closed in Y and therefore \( f(O) \) is intuitionistic fuzzy regular closed in Y. Hence \( f \) is intuitionistic fuzzy pre-regular closed.

Sufficiency: Let \( F \) be an intuitionistic fuzzy regular closed set of \( X \) and \( A \) be an intuitionistic gpr-open set of \( Y \) and \( f(F) \subseteq A \). Since \( f \) is intuitionistic fuzzy pre regular closed, \( f(F) \) is intuitionistic fuzzy regular closed in \( Y \) and therefore \( (f(F))^c \) is intuitionistic fuzzy regular open in \( Y \). By hypothesis \( (f(F))^c \) is intuitionistic fuzzy pre-closed in \( Y \) and hence \( f(F) \) is intuitionistic fuzzy pre-open in \( Y \) which implies that \( f(F) = \text{pint}(f(F)) \subseteq \text{pint}(A) \). Hence \( f \) is intuitionistic fuzzy apr-closed.

**Theorem 6.7.** If \( \text{IFRO}(X) = \text{IFPC}(X) \) where \( \text{IFRO}(X) \) (resp. \( \text{IFPC}(X) \)) denotes the family of all intuitionistic fuzzy regular open (resp. intuitionistic fuzzy pre-closed) sets of \( X \), then the mapping \( f : (X, \tau) \to (Y, \sigma) \) is intuitionistic fuzzy apr-continuous if and only if \( f^{-1}(O) \) is intuitionistic fuzzy apr-continuous in \( X \) for every intuitionistic fuzzy regular open set \( O \) of \( Y \).

**Proof.** Necessity: Let \( f : (X, \tau) \to (Y, \sigma) \) be intuitionistic fuzzy apr-continuous mapping. By Theorem 2.2 [13] every intuitionistic fuzzy set of \( X \) is intuitionistic fuzzy gpr-closed and hence all are intuitionistic fuzzy gpr-open. Thus for any intuitionistic fuzzy regular open set \( O \) of \( Y \), \( f^{-1}(O) \) is intuitionistic fuzzy gpr-closed in \( X \). Since \( f^{-1}(O) \subseteq f^{-1}(O) \) and \( f \) is intuitionistic fuzzy apr-continuous then \( \text{pcl}(f^{-1}(O)) \subseteq f^{-1}(O) \). Hence \( f^{-1}(O) \) is intuitionistic fuzzy pre-closed set in \( X \).

Sufficiency: Let \( O \) be an intuitionistic fuzzy regular open set of \( Y \) and \( A \) be an intuitionistic fuzzy gpr-closed set of \( X \) such that \( A \subseteq f^{-1}(O) \) then \( \text{pcl}(A) \subseteq \text{pcl}(f^{-1}(O)) = f^{-1}(O) \) because by hypothesis \( f^{-1}(O) \) is intuitionistic fuzzy pre-closed in \( X \). Hence \( f \) is intuitionistic fuzzy apr-continuous.

**Theorem 6.8.** If \( \text{IFRO}(X) = \text{IFPC}(X) \) where \( \text{IFRO}(X) \) (resp. \( \text{IFPC}(X) \)) denotes the family of all intuitionistic fuzzy regular open (resp. intuitionistic fuzzy pre-closed) sets of \( X \), then the mapping \( f : (X, \tau) \to (Y, \sigma) \) is intuitionistic fuzzy apr-continuous if and only if it is intuitionistic fuzzy R-mapping.

**Proof.** Necessity: Let \( f : (X, \tau) \to (Y, \sigma) \) be intuitionistic fuzzy apr-continuous mapping. Let \( O \) is an intuitionistic fuzzy regular open set of \( Y \), then by Theorem 6.7 \( f^{-1}(O) \) is intuitionistic fuzzy pre-closed in \( X \) and so by hypothesis \( f^{-1}(O) \) is intuitionistic fuzzy regular open in \( X \). Hence \( f \) is an intuitionistic fuzzy R-mapping.

Sufficiency: Let \( O \) be an intuitionistic fuzzy regular open set of \( Y \) and \( A \) be an intuitionistic fuzzy gpr-closed set of \( X \) such that \( A \subseteq f^{-1}(O) \). Since \( f \) is intuitionistic fuzzy R-mapping, \( f^{-1}(O) \) is intuitionistic fuzzy regular open in \( X \) and thus by hypothesis \( f^{-1}(O) \) is intuitionistic fuzzy pre-closed in \( X \) which implies that \( \text{pcl}(A) \subseteq \text{pcl}(f^{-1}(O)) \). Hence \( f \) is intuitionistic fuzzy apr-continuous.

**Theorem 6.9.** If \( f : (X, \tau) \to (Y, \sigma) \) is intuitionistic fuzzy pre-regular closed and \( g : (Y, \sigma) \to (Z, \phi) \) is intuitionistic fuzzy apr-closed mapping, then \( gf : (X, \tau) \to (Z, \phi) \) is intuitionistic fuzzy apr-closed.
Proof. Let $F$ be an intuitionistic fuzzy regular closed set of $X$ and $A$ is intuitionistic fuzzy $gpr$-open set of $Z$ for which $gof(F) \subseteq A$ since $f : (X, \tau) \to (Y, \sigma)$ is intuitionistic fuzzy pre-regular closed mapping, $f(F)$ is intuitionistic fuzzy regular closed set of $Y$. Now $g : (Y, \sigma) \to (Z, \phi)$ is intuitionistic fuzzy $apr$-closed mapping, then $g(f(F)) \subseteq \text{pint}(A)$. Hence $gof : (X, \tau) \to (Z, \phi)$ is intuitionistic fuzzy $apr$-closed mapping. 

**Theorem 6.10.** If $f : (X, \tau) \to (Y, \sigma)$ is intuitionistic fuzzy $apr$-closed and $g : (Y, \sigma) \to (Z, \phi)$ is intuitionistic fuzzy open and intuitionistic fuzzy $apr$-irresolute then $gof : (X, \tau) \to (Z, \phi)$ is intuitionistic fuzzy $apr$-closed.

**Proof.** Let $F$ be an intuitionistic fuzzy regular closed set of $X$ and $A$ is intuitionistic fuzzy $gpr$-open set of $Z$ for which $gof(F) \subseteq A$. Then $f(F) \subseteq g^{-1}(A)$. Since $g$ is $apr$-irresolute, $g^{-1}(A)$ is intuitionistic fuzzy $gpr$-open in $X$ and $f : (X, \tau) \to (Y, \sigma)$ is intuitionistic fuzzy $apr$-closed mapping. It follows that $f(F) \subseteq \text{pint}(g^{-1}(A))$. Thus $(gof)(F) = g(f(F)) \subseteq g(\text{pint}(g^{-1}(A)) \subseteq \text{pint}(g(g^{-1}(A))) \subseteq \text{pint}(A)$. Hence $gof : (X, \tau) \to (Z, \phi)$ is intuitionistic fuzzy $apr$-closed.

**Theorem 6.11.** If $f : (X, \tau) \to (Y, \sigma)$ is intuitionistic fuzzy $apr$-continuous and $g : (Y, \sigma) \to (Z, \phi)$ is intuitionistic fuzzy $R$-mapping then $gof : (X, \tau) \to (Z, \phi)$ is intuitionistic fuzzy $apr$-continuous.

**Proof.** Let $A$ be an intuitionistic fuzzy $gpr$-closed set of $X$ and $V$ is intuitionistic fuzzy regular open set of $Z$ for which $A \subseteq (gof)^{-1}(V)$. Now since $g : (Y, \sigma) \to (Z, \phi)$ is intuitionistic fuzzy $R$-mapping, $g^{-1}(V)$ is intuitionistic fuzzy regular open set of $Y$. Since $f : (X, \tau) \to (Y, \sigma)$ is intuitionistic fuzzy $apr$-continuous, $\text{pcl}(A) \subseteq f^{-1}(g^{-1}(V)) = (gof)^{-1}(V)$. Hence $gof : (X, \tau) \to (Z, \phi)$ is intuitionistic fuzzy $apr$-continuous mapping.

**REFERENCES**


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