ON COMMUTATIVE HYPER BE-ALGEBRAS

Akbar Rezaei, Akefe Radfar and Arsham Borumand Saeid

Abstract. In this paper, we introduce commutative hyper BE-algebra and study it in detail. We show that every commutative (row diagonal, column row, very thin) hyper BE-algebra is a BE-algebra.

1. Introduction and Preliminaries

H. S. Kim and Y. H. Kim introduced the notion of a BE-algebra as a generalization of a dual BCK-algebra [4]. S. S. Ahn and et al. introduced the notions of terminal sections of a BE-algebras and gave some characterization of commutative BE-algebras in terms of lattices order relations and terminal sections [1]. A. Rezaei and et al. show that a commutative implicative BE-algebra is equivalent to the commutative self-distributive BE-algebra. Also, they proved that every Hilbert algebra is a self-distributive BE-algebra and commutative self-distributive BE-algebra is a Hilbert algebra and show that cannot remove the conditions commutativity and self-distributivity [7].

The hyper algebraic structure theory was introduced in 1934, by F. Marty at the 8th congress of Scandinavian Mathematicians [5]. Hyper-structures have many applications to several sectors of both pure and applied sciences. Y. B. Jun and et al. applied the hyper-structures to BCK-algebras and introduced the notion of a hyper BCK-algebra which is a generalization of BCK-algebra and investigated some related properties [3].

Recently, A. Radfar and et al. introduced the notion of hyper BE-algebra and defined some types of hyper-filters in hyper BE-algebras. They showed that under special condition hyper BE-algebras are equivalent to dual hyper K-algebras [6].

In this paper we characterize the relation between dual hyper K-algebras and commutative hyper BE-algebras and some types of commutative hyper BE-algebras and characterization of RD/CR/V-hypercommutative BE-algebras are state. We show that every commutative RD-hyper BE-algebra of order 3 is a commutative BE-algebra.
Definition 1.1. [2] Let $H$ be a nonempty set and $\circ : H \times H \to P(H)$ be a hyper-operation. Then $(H; \circ, 0)$ is called a hyper $K$-algebra, if it satisfies the following axioms:

$(HK_1)$ $(x \circ z) \circ (y \circ z) < x \circ y,$
$(HK_2)$ $(x \circ y) \circ z = (x \circ z) \circ y,$
$(HK_3)$ $x < x,$
$(HK_4)$ $x < y$ and $y < x$ imply that $x = y,$
$(HK_5)$ $0 < x,$ for all $x, y, z \in H.$

Where $x < y$ is defined by $0 \in x \circ y.$

Theorem 1.1. [2] Let $H$ be a hyper $K$-algebra. Then

(i) $x \in x \circ 0,$
(ii) $0 \in 0 \circ x,$ for all $x \in H.$

Definition 1.2. [6] Let $H$ be a nonempty set and $\circ : H \times H \to P(H)$ be a hyper-operation. Then $(H; \circ, 1)$ is called a hyper $BE$-algebra, if it satisfies the following axioms:

$(HBE_1)$ $x < 1$ and $x < x,$
$(HBE_2)$ $x \circ (y \circ z) = y \circ (x \circ z),$  
$(HBE_3)$ $x \in 1 \circ x,$
$(HBE_4)$ $1 < x$ implies $x = 1,$ for all $x, y, z \in H.$

A hyper-$BE$-algebra is said to be

(i) row hyper $BE$-algebra (for short, $R$-hyper $BE$-algebra), if $1 \circ x = \{x\},$ for all $x \in H,$
(ii) column hyper $BE$-algebra (for short, $C$-hyper $BE$-algebra), if $x \circ 1 = \{1\},$ for all $x \in H,$
(iii) diagonal hyper $BE$-algebra (for short, $D$-hyper $BE$-algebra), if $x \circ x = \{1\},$ for all $x \in H,$
(iv) thin hyper $BE$-algebra (for short, $T$-hyper $BE$-algebra), if that is $RC$-hyper $BE$-algebra,
(v) very thin hyper $BE$-algebra (for short, $V$-hyper $BE$-algebra), if that is $RCD$-hyper $BE$-algebra,
(H; ∘, 1) is called a dual hyper K-algebra if satisfies (HBE₁), (HBE₂) and the following axioms:

\[(DHK₁) \quad x \circ y < (y \circ z) \circ (x \circ z),\]

\[(DHK₄) \quad x < y \text{ and } y < x \text{ imply that } x = y, \text{ for all } x, y, z \in H.\]

Where the relation “<” is defined by \(x < y \iff 1 \in x \circ y\). For any two nonempty subsets \(A\) and \(B\) of \(H\), we define \(A < B\) if and only if there exist \(a \in A\) and \(b \in B\) such that \(a < b\) and

\[A \circ B = \bigcup_{a \in A, b \in B} a \circ b.\]

**Theorem 1.2.** [6] Let \(H\) be a hyper BE-algebra. Then

(i) \(A \circ (B \circ C) = B \circ (A \circ C),\)

(ii) \(A < A,\)

(iii) \(1 < A \text{ implies } 1 \in A,\)

(iv) \(x < y \circ x,\)

(v) \(x < y \circ z \text{ implies } y < x \circ z,\)

(vi) \(x < (x \circ y) \circ y,\)

(vii) \(z \in x \circ y \text{ implies } x < z \circ y,\)

(viii) \(y \in 1 \circ x \text{ implies } y < x, \text{ for all } x, y, z \in H \text{ and } A, B, C \subseteq H.\)

**Corollary 1.1.** [6] Every dual hyper K-algebra is a hyper BE-algebra.

**Theorem 1.3.** [6] Let \(H\) be a CD-hyper BE-algebra. Then

(i) \(x \circ (y \circ x) = \{1\},\)

(ii) \(z \in x \circ y \text{ implies } y \circ z = \{1\},\)

for all \(x, y, z \in H.\)

**Theorem 1.4.** [8] Let \(X\) be a commutative BE-algebra. Then \(x \leq y \text{ and } y \leq x \text{ implies } x = y, \text{ for all } x, y \in X.\)
2. On commutative hyper $BE$-algebras

**Definition 2.1.** A hyper $BE$-algebra (dual hyper $K$-algebra) $H$ is said to be commutative if $(x \circ y) \circ z = (y \circ x) \circ z$, for all $x, y \in H$.

**Example 2.1.** (i). Let $H = \{1, a, b\}$. Define the hyper-operations “$\circ_1$” as follows:

<table>
<thead>
<tr>
<th>$\circ_1$</th>
<th>1</th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>{1}</td>
<td>{a}</td>
<td>{b}</td>
</tr>
<tr>
<td>a</td>
<td>{1, b}</td>
<td>{1, a, b}</td>
<td>{1, a}</td>
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<tr>
<td>b</td>
<td>{1, a, b}</td>
<td>{a}</td>
<td>{1, a, b}</td>
</tr>
</tbody>
</table>

Then $(H, \circ_1)$ is a commutative hyper $BE$-algebra.

(ii). Define the hyper operation “$\circ$” on $\mathbb{R}$ as follows:

\[ x \circ y = \begin{cases} 
{y} & \text{if } x = 1 \\
\mathbb{R} & \text{otherwise}
\end{cases} \]

Then $(\mathbb{R}, \circ, 1)$ is a commutative hyper $BE$-algebra.

**Lemma 2.1.** Let $H$ be a commutative hyper $BE$-algebra. Then $H$ satisfies in $(DHK_1)$.

**Proof.** Let $H$ be a commutative hyper $BE$-algebra and $x, y, z \in H$. Then

\begin{align*}
(x \circ y) \circ ((y \circ z) \circ (x \circ z)) &= (x \circ y) \circ ((y \circ (z \circ y) \circ y)) \\
&= (x \circ y) \circ ((z \circ y) \circ (x \circ y)) \\
&= (z \circ y) \circ ((x \circ y) \circ (x \circ y)).
\end{align*}

Now, using $(HBE_1)$, we have $1 \in (x \circ y) \circ (x \circ y)$.

Also, by $(HBE_2)$,

\[ 1 \in (z \circ y) \circ ((x \circ y) \circ (x \circ y)). \]

Thus

\[ 1 \in (x \circ y) \circ ((y \circ z) \circ (x \circ z)). \]

Therefore $(DHK_1)$ holds.

**Theorem 2.1.** $H$ is a commutative dual hyper $K$-algebra if and only if $H$ is a commutative hyper $BE$-algebra and satisfies in $(DHK_2)$.

**Proof.** Let $H$ be a commutative dual hyper $K$-algebra. Using Corollary 1.1, $H$ is a commutative hyper $BE$-algebra.

Conversely, let $H$ be a commutative hyper $BE$-algebra, satisfies in condition $(DHK_2)$ and $x, y, z \in H$. Then by Lemma 2.1, $(DHK_1)$ holds. Therefore $H$ is a dual hyper $K$-algebra.
The following example shows that condition (DHK$_2$) in Theorem 2.1, is necessary.

**Example 2.2.** Let $H = \{1, a, b\}$. Define the hyper-operation “$\circ$” as follows:

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>a</th>
<th>b</th>
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<tbody>
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<td>[b]</td>
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<td>[1]</td>
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<td>b</td>
<td>[1, a, b]</td>
<td>[1, b]</td>
<td>[1, a, b]</td>
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</table>

Then $H$ is a commutative hyper $BE$-algebra. Since $a < b$ and $b < a$, $H$ is not a hyper $K$-algebra.

**Proposition 2.1.** Let $H$ be a commutative $R$-hyper $BE$-algebra. Then $x \circ y = y \circ x = \{1\}$ implies $x = y$.

**Proof.** Let $x \circ y = y \circ x = \{1\}$. Since $H$ is a commutative $R$-hyper $BE$-algebra,

$$\{x\} = 1 \circ x = (y \circ x) \circ x = (x \circ y) \circ y = 1 \circ y = \{y\}.$$ 

Therefore $x = y$. □

**Lemma 2.2.** Let $H$ be a commutative $C$-hyper $BE$-algebra. Then

(i) $H$ is a commutative $CD$-hyper $BE$-algebra,

(ii) $y \in 1 \circ x$ implies $1 \circ x \subseteq 1 \circ y$,

(iii) $y \in 1 \circ x$ if and only if $x \in 1 \circ y$,

for all $x, y \in H$.

**Proof.** (i). Let $x \in H$. Then by commutativity, $x \circ x \subseteq (1 \circ x) \circ x = (x \circ 1) \circ 1 = 1 \circ 1 = \{1\}$. Thus $H$ is a commutative $D$-hyper $BE$-algebra.

(ii). Let $y \in 1 \circ x$. Using Theorem 1.2 (viii), $y < x$ and so $1 \in y \circ x$. Since $y \in 1 \circ x$, by Theorem 1.3(ii), $x \circ y = \{1\}$ and by commutativity,

$$1 \circ x \subseteq (y \circ x) \circ x = (x \circ y) \circ y = 1 \circ y.$$ 

Hence $1 \circ x \subseteq 1 \circ y$.

(iii). Let $y \in 1 \circ x$. By (ii), $1 \circ x \subseteq 1 \circ y$. By (HBE$_3$), $x \in 1 \circ x \subseteq 1 \circ y$. Thus $x \in 1 \circ y$. Similarly, $x \in 1 \circ y$ implies $y \in 1 \circ x$. □

**Corollary 2.1.** $H$ is a commutative $C$-hyper $BE$-algebra if and only if $H$ is a commutative $CD$-hyper $BE$-algebra.

**Lemma 2.3.** Let $H$ be a commutative $CR$-hyper $BE$-algebra. Then

(i) $H$ is a commutative $V$-hyper $BE$-algebra,
Let $H$ be a hyper BE-algebra. Then

(iii) If $x \circ y = \{1\}$, then $y \circ x = \{y\}$.

Proof. (i). By Corollary 2.1, $H$ is a commutative CRD-hyper BE-algebra and so it is a commutative V-hyper BE-algebra.

(ii). If $x \circ y = \{1\}$, then $(y \circ) \circ x = (x \circ) \circ y = \{1\} \circ y = \{y\}$.

(iii). Let $x \circ y = \{1\}$. At first we prove that $y \circ x$ is a singleton set. Let $a, b \in y \circ x$. By (ii), $a \circ x \subseteq (y \circ) \circ x = \{y\}$. Thus $a \circ x = \{y\}$. By a similar way, $b \circ x = \{y\}$. By (i), $H$ is a V-hyper BE-algebra and so it is a CD-hyper BE-algebra. Now, using Theorem 1.3 (ii) and since $a \in y \circ x$, we have $x \circ a = x \circ b = \{1\}$. By (ii), $(a \circ x) \circ x = \{a\}$. Thus

\[\{a, b\} \subseteq y \circ x = (a \circ x) \circ x = \{a\}.\]

Hence $a = b$ and $y \circ x$ is a singleton set. Now, let $y \circ x = \{z\}$. By using (ii), $y \circ z = y \circ (y \circ x) = \{y\}$. $\blacklozenge$

**Theorem 2.2.** Every commutative CR-hyper BE-algebra is a commutative BE-algebra.

Proof. Let $H$ be a commutative CR-hyper BE-algebra. Let $a, b \in x \circ y$. By Lemma 2.3 (i), $H$ is a V-hyper BE-algebra and so is a CD-hyper BE-algebra. By Theorem 1.3 (ii) and since $a, b \in x \circ y$, we can see that $y \circ a = y \circ b = \{1\}$. Now, using Lemma 2.3 (iii),

\[a \circ y = \{c\}, \quad c \circ y = \{a\}, \quad b \circ y = \{d\}, \quad \text{and} \quad d \circ y = \{b\} \quad \text{for some} \quad c, d \in H.\]

Since $d = b \circ y \subseteq (x \circ y) \circ y = (y \circ x) \circ x$, there is $t_1 \in x \circ y$, such that $d \in t_1 \circ x$. By Theorem 1.3 (ii), we imply that $x \circ d = \{1\}$. In a similar way, $c \in x \circ y \subseteq (x \circ y) \circ y = (y \circ x) \circ x$, there is $t_2 \in y \circ x$, such that $c \in t_2 \circ x$. By Theorem 1.3 (ii), we get $x \circ c = \{1\}$. By (HBE$_2$),

\[b \circ a \subseteq b \circ (x \circ y) = x \circ (b \circ y) = x \circ d = \{1\}.
\]

Thus $a \circ b = b \circ a = \{1\}$. Now, using Proposition 2.1, $a = b$. Therefore $x \circ y$ is a singleton set for every $x, y \in H$ and so $H$ is a commutative BE-algebra. $\blacklozenge$

3. **Characterization of commutative hyper BE-algebra of order 3**

From now on, $H$ is a commutative hyper BE-algebra of order 3.

**Lemma 3.1.** Let $H$ be a hyper BE-algebra. Then

(i) $1 \neq 1 \circ x$, for all $1 \neq x \in H$,
Three concepts - commutative C-hyper BE-algebra, commutative D-hyper BE-algebra and commutative CD-hyper BE-algebra - coincide.

Proof. (i) Let \( x \neq 1 \) and \( x \in H \). Then \( 1 \leq 1 \circ x \) implies \( 1 < x \). By \((HBE_4)\), \( x = 1 \), which is a contradiction.

(ii) The proof is clear by using \((HBE_1)\). \( \square \)

**Theorem 3.1.** Let \( H \) be a commutative D-hyper BE-algebra of order 3. Then \( H \) is a commutative CD-hyper BE-algebra of order 3.

Proof. Let \( H = \{1, a, b\} \) be a commutative D-hyper BE-algebra and it is not a C-hyper BE-algebra. Then \( a \circ 1 \neq \{1\} \) or \( b \circ 1 \neq \{1\} \). Without loss of generality, let \( a \circ 1 \neq \{1\} \).

By \((HBE_4)\), \( 1 \in a \circ 1 \). Thus \( a \circ 1 = \{1, a\}, \{1, b\} \) or \( \{1, a, b\} \) and so we have three cases:

Case 1: If \( a \circ 1 = \{1, a\} \), then
\[
1 \circ a \subseteq 1 \circ (a \circ 1) = a \circ (1 \circ 1) = a \circ 1 = \{1, a\}.
\]

By Lemma 3.1, \( 1 \notin 1 \circ a \) and so \( 1 \circ a = \{a\} \). Now, by commutativity,
\[
\{1\} = a \circ a = (1 \circ a) \circ a = (a \circ 1) \circ 1 = \{1, a\} \circ 1 = \{1, a\},
\]
which is a contradiction. Thus \( a \circ 1 \neq \{1, a\} \).

Case 2: If \( a \circ 1 = \{1, b\} \), then by Lemma 3.1(ii) and \((HBE_2)\),
\[
\{1, b\} = a \circ 1 \subseteq a \circ (b \circ 1) = b \circ (a \circ 1) = b \circ \{1, b\} = b \circ 1 \cup b \circ b = 1 \cup b \circ 1.
\]

Hence \( b \circ 1 = \{1, b\} \). Which is a contradiction.

Case 3: If \( a \circ 1 = \{1, a, b\} \), then by commutativity,
\[
(1 \circ a) \circ a = (a \circ 1) \circ 1 = \{1, a\} \circ 1 \supseteq a \circ 1 = \{1, a, b\}.
\]

Now, by Lemma 3.1, \( 1 \circ a = \{a\} \) or \( \{a, b\} \). Since \( 1 \circ a = \{a\} \), we have
\[
(1 \circ a) \circ a = a \circ a = \{1\}.
\]

Thus \( 1 \circ a \neq \{a\} \) and so \( 1 \circ a = \{a, b\} \). Also,
\[
\{1, a, b\} = (1 \circ a) \circ a = \{a, b\} \circ a = a \circ a \cup b \circ a = 1 \cup b \circ a.
\]

Thus \( \{a, b\} \subseteq b \circ a \). Also, since \( 1 \circ a = \{a, b\} \), by Theorem 1.2 (viii), \( b < a \) and so \( 1 \in b \circ a \). Hence \( b \circ a = \{1, a, b\} \). Also, by \( 1 \circ a = \{a, b\} \) and Lemma 2.2(iii), we imply that \( a \in 1 \circ b \) and so \( 1 \circ b = \{a, b\} \). Now, since \( H \) is a D---hyper BE---algebra and by \((HBE_2)\),
\[
b \circ a \subseteq b \circ (1 \circ b) = 1 \circ (b \circ b) = 1 \circ 1 = \{1\}.
\]

Thus \( b \circ a = \{1\} \), which is a contradiction. \( \square \)

**Corollary 3.1.** Three concepts - commutative C-hyper BE-algebra, commutative D-hyper BE-algebra and commutative CD-hyper BE-algebra - coincide.
Corollary 3.2. Every commutative RD-hyper BE-algebra of order 3 is a commutative BE-algebra.

Theorem 3.2. There exist two commutative $V$-hyper BE-algebra of order 3 up to isomorphism.

Proof. Let $H = \{1, a, b\}$ be a commutative $V$-hyper BE-algebra. Then by Theorem 2.2, $H$ is a BE-algebra and so,

$$1 \circ 1 = \{1\}, \quad 1 \circ a = \{a\}, \quad 1 \circ b = \{b\}$$

and

$$a \circ 1 = b \circ 1 = a \circ a = b \circ b = \{1\}.\]$$

By Theorem 1.4, $a$ and $b$ are comparable, hence $a < b$ or $b < a$. Case $a < b$ and case $b < a$ are isomorphic. So, without lose of generality let $a < b$. Then $a \circ b = \{1\}$. By commutativity, we have,

$$(b \circ a) \circ a = (a \circ b) \circ b = 1 \circ b = \{b\}.$$

If $b \circ a = \{1\}$, then $(b \circ a) \circ a = 1 \circ a = \{a\}$, which is a contradiction.

If $b \circ a = \{a\}$, then $(b \circ a) \circ a = a \circ a = \{1\}$, which is a contradiction.

Since $H$ is a BE-algebra, $b \circ a = \{b\}$. Therefore $H$ is a commutative $V$-hyper BE-algebra.

$$
\begin{array}{c|ccc}
\circ & 1 & a & b \\
\hline
1 & \{1\} & \{a\} & \{b\} \\
\hline
a & \{1\} & \{1\} & \{1\} \\
b & \{1\} & \{b\} & \{1\} \\
\end{array}
$$

Now, if $a$ and $b$ are comparable, then $a \circ b = \{a\}$ or $\{b\}$. If $a \circ b = \{a\}$, then by Theorem 1.3(ii), $a \circ a = \{1\}$ and so $b < a$, which is a contradiction. Hence $a \circ b = \{b\}$. By a similar way $b \circ a = \{a\}$. So, $H$ is a commutative $V$-algebra.

$$
\begin{array}{c|ccc}
\circ & 1 & a & b \\
\hline
1 & \{1\} & \{a\} & \{b\} \\
\hline
a & \{1\} & \{1\} & \{b\} \\
b & \{1\} & \{a\} & \{1\} \\
\end{array}
$$

□

Theorem 3.3. There exist three commutative $D$-hyper BE-algebra of order 3 up to isomorphism.
Proof. By Theorem 3.1, \( H \) is a CD-hyper BE-algebra and so,

\[
1 \circ 1 = 1 \circ a = a \circ a = b \circ 1 = b \circ b = \{1\}.
\]

By Lemma 3.1(i), \( 1 \notin 1 \circ a \). Also, by \( (HBE_3) \), \( a \in 1 \circ a \). Thus \( 1 \circ a = \{a\} \) or \( \{a, b\} \). By a similar way, \( 1 \circ b = \{b\} \) or \( \{a, b\} \). If \( 1 \circ a = \{a\} \), then \( 1 \circ b = \{b\} \) (Since \( 1 \circ b = \{a, b\} \) we have \( b \in 1 \circ a \), which is a contradiction). Thus \( H \) is a commutative \( V \)-hyper BE-algebra. By Theorem 3.2, two commutative \( V \)-hyper BE-algebra exist.

Now, if \( 1 \circ a = \{a, b\} \), then by Lemma 2.2(iii), \( a \in 1 \circ b \) and so \( 1 \circ b = \{a, b\} \). By Theorem 1.3 (ii), \( 1 \circ a = \{a, b\} \) implies \( b \circ a = \{1\} \) and \( 1 \circ b = \{a, b\} \) implies \( a \circ b = \{1\} \). Therefore \( H \) is a commutative hyper BE-algebra in the following table.

<table>
<thead>
<tr>
<th>( \circ )</th>
<th>1</th>
<th>( a )</th>
<th>( b )</th>
</tr>
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<tr>
<td>( b )</td>
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</tbody>
</table>

\( \square \)

Theorem 3.4. (i). There exist 889 commutative hyper BE-algebras of order 3 up to isomorphism.

(ii). There exist 68 commutative R-hyper BE-algebras of order 3 up to isomorphism.

4. Conclusion and future work

Now, in the following table we summarize the results of this paper and show that the number of all kinds of commutative hyper BE-algebras of order 3. We note that by Corollary 3.1, the three concepts - commutative \( D \)-hyper BE-algebra, commutative \( C \)-hyper BE-algebra and commutative \( CD \)-hyper BE-algebra - coincide. Also, by Theorem 2.2, Corollaries 3.1 and 3.2, every commutative \( RD/CR/V \)-hyper BE-algebra is a BE-algebra.

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<table>
<thead>
<tr>
<th>Type</th>
<th>Condition</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>commutative hyper BE-algebra</td>
<td>$a &lt; b$ and $b \nleq a$</td>
<td>325</td>
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<tr>
<td>commutative hyper BE-algebra</td>
<td>$a &lt; b$ and $b &lt; a$</td>
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<td>commutative hyper BE-algebra</td>
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<tr>
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<td>(commutative RD-hyper)commutative BE-algebra</td>
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In the future work we will try to get some results on another type of commutative hyper BE-algebras and state some properties on this structure and investigate some relationships between them.

Acknowledgement. The authors would like to express their gratitude to the anonymous referees for their comments and suggestions which improved the paper.

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