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SOME NEW RESULTS ON PARA-SASAKIAN MANIFOLD WITH A QUATER-SYMMETRIC METRIC CONNECTION

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Abstract. The objective of the present paper is to study some new results on para-Sasakian manifold with a quarter-symmetric metric connection. We classify the para-Sasakian manifold with respect to the quarter-symmetric metric connection satisfying the conditions $\bar{P}.\bar{S} = 0$, $\bar{R}.\bar{S} = 0$ and $\bar{S}.\bar{R} = 0$. Also, we obtain the conditions for the manifold with a quarter-symmetric metric connection to be ξ-projectively flat and ξ-conformally flat.
Keywords: Para-Sasakian manifold, metric connection, curvature tenor.

1. Introduction

In 1924, the idea of semi-symmetric linear connection on a differentiable manifold was introduced by A. Friedmann and J. A. Schouten [2]. In 1930, E. Bartolotti [5] gave a geometrical meaning of such a connection. Further, H. A. Hayden [6] introduced the idea of metric connection with the torsion on a Riemannian manifold. K. Yano studied some curvature conditions for semi-symmetric connection in Riemannian manifold [12] and this was further studied by various authors such as Kalpana and P. Srivastava [11], Venkatesha, K. T. P. Kumar and C. S. Bagewadi ([13],[17]), I. E. Hirică and L. Nicolescu ([9], [10]) and many others.

In 1975, S. Golab defined and studied the quarter symmetric connection in a differentiable manifold with affine connection [14]. In 1977, T. Adati and K. Matsumoto defined para-Sasakian and special para-Sasakian manifold [15], which are special classes of an almost paracontact manifold introduced by I. Satō ([7], [8]). Recently, semi-symmetric non-metric connection on P-Sasakian manifolds has been studied by A. Barman [1]. P-Sasakian manifolds satisfying various curvature conditions have been also studied by C. Özgür [3], D. Tarafdar and U. C. De [4], U. C. De and G. Pathak [16] and others.

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A linear connection $\bar{\nabla}$ in an n-dimensional differentiable manifold is said to be a quarter-symmetric connection [6] if its torsion tensor T is of the form

$$T(X,Y) = \bar{\nabla}_X Y - \bar{\nabla}_Y X - [X,Y] \tag{1.1}$$

satisfies

$$T(X,Y) = \eta(Y)\phi X - \eta(X)\phi Y, \tag{1.2}$$

where η is a 1-form and ϕ is a $(1,1)$ tensor field. In particular, if $\phi X = X$ and $\phi Y = Y$, then the quarter-symmetric connection reduces to the semi-symmetric connection [2]. Thus, the notion of quarter-symmetric connection generalizes the idea of semi-symmetric connection. And if quarter-symmetric linear connection $\bar{\nabla}$ satisfies the condition

$$(\bar{\nabla}_X g)(Y,Z) = 0 \tag{1.3}$$

for all $X, Y, Z \in \chi(M)$, where $\chi(M)$ is the Lie algebra of vector fields on the manifold M, then $\bar{\nabla}$ is said to be a quarter-symmetric metric connection.
A relation between the quarter-symmetric metric connection $\bar{\nabla}$ and the Levi-Civita connection ∇ in an n-dimensional para-Sasakian manifold M is given by[13]

$$\bar{\nabla}_X Y = \nabla_X Y + \eta(Y)\phi X - g(\phi X, Y)\xi. \tag{1.4}$$

Definition 1.1. An almost paracontact Riemannian manifold M^n is called ξ-projectively flat (resp., ξ-conformally flat) if the condition $P(X,Y)\xi = 0$ (resp., $C(X,Y)\xi = 0$) holds on M^n, where the projective curvature tenor (resp., conformal curvature tensor) is defined by (1.5) (*resp.*, (1.6))

$$P(X,Y)Z = R(X,Y)Z - \frac{1}{(n-1)}[S(Y,Z)X - S(X,Z)Y] \tag{1.5}$$

for any vector fields $X, Y, Z \in \chi(M)$.

$$C(X,Y)Z = R(X,Y)Z - \frac{1}{(n-2)}[S(Y,Z)X - S(X,Z)Y \tag{1.6}$$

$$+g(Y,Z)QX - g(X,Z)QY] + \frac{r}{(n-1)(n-2)}[g(Y,Z)X - g(X,Z)Y]$$

for any vector fields $X, Y, Z \in \chi(M)$, where R, S and r are the curvature tensor, the Ricci tensor and the scalar curvature respectively on M with respect to the Levi-Civita connection and Q is the Ricci operator with respect to the Levi-Civita connection and is related to $g(QX,Y) = S(X,Y)$.

Motivated by the above studies, in this paper we study some curvatures conditions in a para-Sasakian manifold with respect to the quarter-symmetric metric connection. The paper is organized as follows : In section 2, we give a brief introduction to the para-Sasakian manifold and define the quarter-symmetric metric connection. In section 3, we discuss the curvature tensor $\bar{R}$, the Ricci tensor $\bar{S}$ and the scalar curvature $\bar{r}$ with respect to the quarter-symmetric metric connection. In sections 4, 5 and 6, we classify the manifold with respect to the quarter-symmetric metric connection satisfying the conditions $\bar{P}.\bar{S} = 0$, $\bar{R}.\bar{S} = 0$ and $\bar{S}.\bar{R} = 0$, where $\bar{P}$ is the projective curvature tensor with respect to the quarter-symmetric metric connection. In section 7, we consider Weyl conformal curvature tensor with respect to the quarter-symmetric metric connection and discuss its characteristic properties. In section 8, we investigate the conditions for the manifold with respect to the quarter-symmetric metric connection to be Weyl conformally flat and Weyl ξ-conformally flat.

2. Preliminaries

An n-dimensional differentiable manifold M is said to admit an almost paracontact Riemannian structure (ϕ, ξ, η, g), where ϕ is a $(1,1)$ tensor field, ξ is a vector field, η is a 1-form and g is a Riemannian metric on M such that

(2.1) $$\phi\xi = 0, \ \ \eta(\phi X) = 0, \ \ \eta(\xi) = 1, \ \ \eta(X) = g(X, \xi),$$

(2.2) $$\phi^2 X = X - \eta(X)\xi, \ \ g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y)$$

for all vector fields $X, Y \in \chi(M)$. The equation $\eta(\xi) = 1$ is equivalent to $|\eta| \equiv 1$ and then ξ is just the metric dual of η, where g is the Riemannian metric on M. If (ϕ, ξ, η, g) satisfy the following equations:

(2.3) $$d\eta = 0, \quad \nabla_X \xi = \phi X,$$

(2.4) $$(\nabla_X \phi)Y = -g(X, Y)\xi - \eta(Y)X + 2\eta(X)\eta(Y)\xi,$$

then M is called a para-Sasakian manifold or briefly a P-Sasakian manifold [14]. Especially, a P-Sasakian manifold M is called a special para-Sasakian manifold or briefly a SP-Sasakian manifold if M admits a 1-form η satisfying

(2.5) $$(\nabla_X \eta)Y = -g(X, Y) + \eta(X)\eta(Y).$$

Moreover, the curvature tensor R, the Ricci tensor S and the Ricci operator Q in a para-Sasakian manifold M with respect to the Levi-Civita connection ∇ satisfies [2]

(2.6) $$\eta(R(X, Y)Z) = \eta(Y)g(X, Z) - \eta(X)g(Y, Z),$$

(2.7) $$R(X, Y)\xi = \eta(X)Y - \eta(Y)X,$$

(2.8) $$R(\xi, X)Y = \eta(Y)X - g(X, Y)\xi,$$

(2.9) $$S(X, \xi) = -(n-1)\eta(X), \quad Q\xi = -(n-1)\xi,$$

where $g(QX, Y) = S(X, Y)$. It yields to

(2.10) $$S(\phi X, \phi Y) = S(X, Y) + (n-1)\eta(X)\eta(Y)$$

for any vector fields $X, Y, Z \in \chi(M)$.

An almost paracontact Riemannian manifold M is said to be an η-Einstein manifold if the Ricci tensor S of type (0,2) is of the form

(2.11) $$S(X, Y) = ag(X, Y) + b\eta(X)\eta(Y),$$

where a and b are smooth functions on M. In particular, if $b = 0$, then an η-Einstein manifold is an Einstein manifold.

3. Curvature tensor on a para-Sasakian manifold with respect to the quarter-symmetric metric connection

If R and $\bar{R}$, respectively, are the curvature tensors of Levi-Civita connection ∇ and quarter-symmetric metric connection $\bar{\nabla}$ in a para-Sasakian manifold M. Then we have

(3.1) $$\bar{R}(X, Y)Z = R(X, Y)Z + 3g(\phi X, Z)\phi Y - 3g(\phi Y, Z)\phi X$$
$$+[\eta(X)Y - \eta(Y)X]\eta(Z) - [\eta(X)g(Y, Z) - \eta(Y)g(X, Z)]\xi,$$

where $X, Y, Z \in \chi(M)$.

Contracting X in (3.1), we have

(3.2) $$\bar{S}(Y, Z) = S(Y, Z) + 2g(Y, Z) - (n+1)\eta(Y)\eta(Z) - 3\, g(\phi Y, Z)\, \psi,$$

where $\bar{S}$ and S are the Ricci tensors of the connections $\bar{\nabla}$ and ∇, respectively on M and $\psi = trace\, \phi$.

This gives

(3.3) $$\bar{Q}Y = QY + 2Y - (n+1)\eta(Y)\xi - 3\, \phi Y\, \psi.$$

Contracting again Y and Z in (3.2), it follows that

(3.4) $$\bar{r} = r + n - 1 - 3\, \psi^2,$$

where $\bar{r}$ and r are the scalar curvatures of the connections $\bar{\nabla}$ and ∇, respectively on M.

Now, from (2.1), (2.7)-(2.9) and (3.1)-(3.3), we have

$$\bar{R}(X, Y)\xi = 2[\eta(X)Y - \eta(Y)X], \tag{3.5}$$

$$\bar{R}(X, \xi)Y = -\bar{R}(\xi, X)Y = 2[g(X, Y)\xi - \eta(Y)X], \tag{3.6}$$

$$\bar{S}(Y, \xi) = -2(n-1)\eta(Y), \tag{3.7}$$

$$\bar{Q}\xi = -2(n-1)\xi, \tag{3.8}$$

where $X, Y \in \chi(M)$.

4. Para-Sasakian manifold with respect to the quarter-symmetric metric connection satisfying $\bar{P}.\bar{S} = 0$

The projective curvature tensor $\bar{P}$ on a para-Sasakian manifold with respect to the quarter-symmetric metric connection $\bar{\nabla}$ is given by

$$\bar{P}(X, Y)Z = \bar{R}(X, Y)Z - \frac{1}{(n-1)}[\bar{S}(Y, Z)X - \bar{S}(X, Z)Y] \tag{4.1}$$

for any vector fields $X, Y, Z \in \chi(M)$. The manifold is said to be projectively flat with respect to the quarter-symmetric metric connection if $\bar{P}$ vanishes identically on M.

In this section we study para-Sasakian manifold with a quarter-symmetric metric connection $\bar{\nabla}$ satisfying the condition

$$\bar{P}(X, Y).\bar{S} = 0. \tag{4.2}$$

Then we have

$$\bar{S}(\bar{P}(X, Y)U, V) + \bar{S}(U, \bar{P}(X, Y)V) = 0, \tag{4.3}$$

for any vector fields $X, Y, Z, U, V \in \chi(M)$.

Putting $U = Y = \xi$ in (4.3) and using the fact $\bar{P}(X, \xi)\xi = 0$, it follows that

$$\bar{S}(\xi, \bar{P}(X, \xi)V) = 0. \tag{4.4}$$

This implies that

$$g[\bar{R}(X, \xi)V - \frac{1}{(n-1)}(\bar{S}(\xi, V)X - \bar{S}(X, V)\xi), \xi] = 0.$$

Using (2.1), (3.6) and (3.7), we get

$$\bar{S}(X, V) = -2(n-1)g(X, V). \tag{4.5}$$

Thus (4.5) is of the form $\bar{S}(X, V) = ag(X, V) + b\eta(X)\eta(V)$, where $a = -2(n-1)$ and $b = 0$.

This result shows that the manifold under the consideration is an Einstein manifold. Hence we can state the following theorem:

Theorem 4.1. *If a para-Sasakian manifold with respect to the quarter-symmetric metric connection $\bar{\nabla}$ satisfies $\bar{P}.\bar{S} = 0$, then the manifold is an Einstein manifold with respect to the quarter symmetric metric connection.*

Now using (3.1) and (3.2) in (4.1), we have

$$\begin{aligned} \bar{P}(X, Y)Z &= P(X, Y)Z + 3g(\phi X, Z)\phi Y - 3g(\phi Y, Z)\phi X \\ &+[\eta(X)Y - \eta(Y)X]\eta(Z) - [\eta(X)g(Y, Z) - \eta(Y)g(X, Z)]\xi \\ &-\frac{1}{(n-1)}[2g(Y, Z)X - 2g(X, Z)Y - (n+1)\eta(Y)\eta(Z)X \\ &+(n+1)\eta(X)\eta(Z)Y - 3g(\phi Y, Z)\psi + 3g(\phi X, Z)\psi], \end{aligned} \tag{4.6}$$

where

$$P(X, Y)Z = R(X, Y)Z - \frac{1}{(n-1)}[S(Y, Z)X - S(X, Z)Y] \tag{4.7}$$

is the projective curvature tensor with respect to the Levi-Civita connection. Putting $Z = \xi$ in (4.6) and using (2.1), we get

$$\bar{P}(X, Y)\xi = P(X, Y)\xi. \tag{4.8}$$

Thus we have

Theorem 4.2. *An n-dimensional para-Sasakian manifold is ξ-projectively flat with respect to the quarter-symmetric metric connection if and only if the manifold is also ξ-projectively flat with respect to the Levi-Civita connection.*

5. Para-Sasakian manifold with quarter-symmetric metric connection satisfying $\bar{R}.\bar{S} = 0$

Now we consider a para-Sasakian manifold with respect to the quarter-symmetric metric connection $\bar{\nabla}$ satisfying the condition

$$\bar{R}(X, Y).\bar{S} = 0. \tag{5.1}$$

Then we have

$$\bar{S}(\bar{R}(X, Y)U, V) + \bar{S}(U, \bar{R}(X, Y)V) = 0 \tag{5.2}$$

for any vector fields $X, Y, Z, U, V \in \chi(M)$.

Putting $X = \xi$ in (5.2), it follows that

$$\bar{S}(\bar{R}(\xi, Y)U, V) + \bar{S}(U, \bar{R}(\xi, Y)V) = 0. \tag{5.3}$$

In view of (3.6), we have

$$\eta(U)\bar{S}(Y, V) - g(Y, U)\bar{S}(\xi, V) + \eta(V)\bar{S}(U, Y) - g(Y, V)\bar{S}(U, \xi) = 0. \tag{5.4}$$

By setting $U = \xi$ in (5.4) and using (2.1) and (3.7), we get

$$\bar{S}(Y, V) = -2(n-1)g(Y, V). \tag{5.5}$$

Thus (5.5) is of the form $\bar{S}(Y, V) = ag(Y, V) + b\eta(Y)\eta(V)$, where $a = -2(n-1)$ and $b = 0$.

This result shows that the manifold under the consideration is an Einstein manifold. Hence we can state the following theorem:

Theorem 5.1. *If a para-Sasakian manifold with respect to the quarter-symmetric metric connection $\bar{\nabla}$ satisfies $\bar{R}.\bar{S} = 0$, then the manifold is an Einstein manifold with respect to the quarter symmetric metric connection.*

6. Para-Sasakian manifold with respect to the quarter-symmetric metric connection satisfying $\bar{S}.\bar{R} = 0$

In this section we consider a para-Sasakian manifold with respect to the quarter-symmetric metric connection $\bar{\nabla}$ satisfying the condition

$$(\bar{S}(X, Y).\bar{R})(U, V)Z = 0 \tag{6.1}$$

for any vector fields $X, Y, Z, U, V \in \chi(M)$.

This implies that

$$(X \wedge_{\bar{S}} Y)\bar{R}(U, V)Z + \bar{R}((X \wedge_{\bar{S}} Y)U, V)Z + \bar{R}(U, (X \wedge_{\bar{S}} Y)V)Z \tag{6.2}$$

$$+ \bar{R}(U, V)(X \wedge_{\bar{S}} Y)Z = 0,$$

where the endomorphism $X \wedge_{\bar{S}} Y$ is defined by

$$(X \wedge_{\bar{S}} Y)Z = \bar{S}(Y, Z)X - \bar{S}(X, Z)Y. \tag{6.3}$$

Taking $Y = \xi$ in (6.2), we have

$$(X \wedge_{\bar{S}} \xi)\bar{R}(U, V)Z + \bar{R}((X \wedge_{\bar{S}} \xi)U, V)Z + \bar{R}(U, (X \wedge_{\bar{S}} \xi)V)Z \tag{6.4}$$

$$+ \bar{R}(U, V)(X \wedge_{\bar{S}} \xi)Z = 0.$$

From (6.3), (6.4) and (3.7), we have

$$(6.5)\qquad -2(n-1)[\eta(\bar{R}(U,V)Z)X + \eta(U)\bar{R}(X,V)Z + \eta(V)\bar{R}(U,X)Z$$
$$+\eta(Z)\bar{R}(U,V)X] - \bar{S}(X,\bar{R}(U,V)Z)\xi - \bar{S}(X,U)\bar{R}(\xi,V)Z$$
$$-\bar{S}(X,V)\bar{R}(U,\xi)Z - \bar{S}(X,Z)\bar{R}(U,V)\xi = 0.$$

Taking inner product of (6.5) with ξ, we get

$$(6.6)\qquad -2(n-1)[\eta(\bar{R}(U,V)Z)\eta(X) + \eta(U)\eta(\bar{R}(X,V)Z) + \eta(V)\eta(\bar{R}(U,X)Z)$$
$$+\eta(Z)\eta(\bar{R}(U,V)X)] - \bar{S}(X,\bar{R}(U,V)Z) - \bar{S}(X,U)\eta(\bar{R}(\xi,V)Z)$$
$$-\bar{S}(X,V)\eta(\bar{R}(U,\xi)Z) - \bar{S}(X,Z)\eta(\bar{R}(U,V)\xi) = 0.$$

By setting $U = Z = \xi$ in the last equation and using (3.5), (3.6) and (3.8), we get

$$(6.7)\qquad \bar{S}(X,V) = 2(n-1)g(X,V) - 4(n-1)\eta(X)\eta(V).$$

Thus (6.7) is of the form $\bar{S}(X,V) = ag(X,V) + b\eta(X)\eta(V)$, where $a = 2(n-1)$ and $b = -4(n-1)$.

This result shows that the manifold under the consideration is an η-Einstein manifold. Hence we can state the following theorem:

Theorem 6.1. *If a para-Sasakian manifold with respect to the quarter-symmetric metric connection $\bar{\nabla}$ satisfies $\bar{S}.\bar{R} = 0$, then the manifold is an η-Einstein manifold with respect to the quarter symmetric metric connection.*

7. Weyl conformal curvature tensor on para-Sasakian manifold with respect to the quarter-symmetric metric connection

The Weyl conformal curvature tensor $\bar{C}$ on a para-Sasakian M with respect to the quarter-symmetric metric connection $\bar{\nabla}$ is defined by

$$(7.1)\qquad \bar{C}(X,Y)Z = \bar{R}(X,Y)Z - \frac{1}{(n-2)}[\bar{S}(Y,Z)X - \bar{S}(X,Z)Y$$
$$+g(Y,Z)\bar{Q}X - g(X,Z)\bar{Q}Y] + \frac{\bar{r}}{(n-1)(n-2)}[g(Y,Z)X - g(X,Z)Y],$$

where $\bar{Q}$ is the Ricci operator with respect to the quarter-symmetric metric connection and is related to $g(\bar{Q}X,Y) = \bar{S}(X,Y)$ and $\bar{r}$ is the scalar curvature with respect

to the quarter-symmetric metric connection.
By taking inner product of (7.1) with U and using (3.1)-(3.4), we have

$$
\begin{aligned}
g(\bar{C}(X,Y)Z,U) = g[&R(X,Y)Z + 3g(\phi X,Z)\phi Y - 3g(\phi Y,Z)\phi X\\
&+ \eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X - \eta(X)g(Y,Z)\xi + \eta(Y)g(X,Z)\xi, U]\\
&- \frac{1}{(n-2)}g[S(Y,Z)X + 2g(Y,Z)X - (n+1)\eta(Y)\eta(Z)X\\
&- 3g(\phi Y,Z)X\psi - S(X,Z)Y - 2g(X,Z)Y\\
&+ (n+1)\eta(X)\eta(Z)Y + 3g(\phi X,Z)Y\psi\\
&+ g(Y,Z)(QX + 2X - (n+1)\eta(X)\xi - 3\phi X\psi)\\
&- g(X,Z)(QY + 2Y - (n+1)\eta(Y)\xi - 3\phi Y\psi), U]\\
&+ \frac{r+n-1-3\psi^2}{(n-1)(n-2)}g[g(Y,Z)X - g(X,Z)Y, U].
\end{aligned} \tag{7.2}
$$

From which we have

$$
\bar{C}(X,Y,Z,U) = C(X,Y,Z,U) + F(X,Y,Z,U), \tag{7.3}
$$

where $g(C(X,Y)Z,U) = C(X,Y,Z,U)$ and $g(\bar{C}(X,Y)Z,U) = \bar{C}(X,Y,Z,U)$ are the Weyl conformal curvature tensor with respect to the Levi-Civita connection and quarter-symmetric metric connection, respectively on M and

$$
\begin{aligned}
F(X,Y,Z,U) &= 3[g(\phi X,Z)g(\phi Y,U) - g(\phi Y,Z)g(\phi X,U)]\\
&+\frac{1}{(n-2)}[-3\eta(X)\eta(Z)g(Y,U) + 3\eta(Y)\eta(Z)g(X,U) + 3\eta(X)\eta(U)g(Y,Z)\\
&-3\eta(Y)\eta(U)g(X,Z) - 4g(Y,Z)g(X,U) + 4g(X,Z)g(Y,U) + 3g(\phi Y,Z)g(X,U)\psi\\
&-3g(\phi X,Z)g(Y,U)\psi + 3g(Y,Z)g(\phi X,U)\psi - 3g(X,Z)g(\phi Y,U)\psi]\\
&+\frac{n-1-3\psi^2}{(n-1)(n-2)}[g(Y,Z)g(X,U) - g(X,Z)g(Y,U)].
\end{aligned} \tag{7.4}
$$

Thus the Weyl conformal curvature tensor on a para-Sasakian manifold with respect to the quarter-symmetric metric connection satisfies the following properties:

$$
\bar{C}(X,Y,Z,U) + \bar{C}(Y,X,Z,U) = 0, \tag{7.5}
$$

$$
\bar{C}(X,Y,Z,U) + \bar{C}(Y,Z,X,U) + \bar{C}(Z,X,Y,U) = 0, \tag{7.6}
$$

where $X, Y, Z, U \in \chi(M)$.

8. Para-Sasakian manifold with Weyl conformally flat and Weyl ξ-conformally flat conditions with respect to the quarter-symmetric metric connection

Let us assume that the manifold M with respect to the quarter-symmetric metric connection is Weyl conformally flat, that is, $\bar{C} = 0$. Then from (7.1), it follows that

$$\bar{R}(X,Y)Z = \frac{1}{(n-2)}[\bar{S}(Y,Z)X - \bar{S}(X,Z)Y + g(Y,Z)\bar{Q}X - g(X,Z)\bar{Q}Y] - \frac{\bar{r}}{(n-1)(n-2)}[g(Y,Z)X - g(X,Z)Y]. \tag{8.1}$$

Taking inner product of (8.1) with ξ and using (2.1), we have

$$\eta(\bar{R}(X,Y)Z) = \frac{1}{(n-2)}[\bar{S}(Y,Z)\eta(X) - \bar{S}(X,Z)\eta(Y) + g(Y,Z)\eta(\bar{Q}X) - g(X,Z)\eta(\bar{Q}Y)] - \frac{\bar{r}}{(n-1)(n-2)}[g(Y,Z)\eta(X) - g(X,Z)\eta(Y)]. \tag{8.2}$$

Putting $X = \xi$ in (8.2) and using (2.1) and (3.6)-(3.8), (8.2) reduces to

$$\bar{S}(Y,Z) = (2 + \frac{\bar{r}}{n-1})g(Y,Z) - (2n + \frac{\bar{r}}{n-1})\eta(Y)\eta(Z). \tag{8.3}$$

Thus (8.3) is of the form $\bar{S}(Y,Z) = ag(Y,Z) + b\eta(Y)\eta(Z)$, where $a = (2 + \frac{\bar{r}}{n-1})$ and $b = -(2n + \frac{\bar{r}}{n-1})$.

This result shows that the manifold under the consideration is an η-Einstein manifold. Hence we can state the following theorem:

Theorem 8.1. *An n-dimensional Weyl conformally flat para-Sasakian manifold with respect to the quarter-symmetric metric connection $\bar{\nabla}$ is an η-Einstein manifold.*

Now from (7.1) and (3.1)-(3.4), we have

$$\bar{C}(X,Y)Z = C(X,Y)Z + 3g(\phi X,Z)\phi Y - 3g(\phi Y,Z)\phi X + \frac{1}{(n-2)}[-3\eta(X)\eta(Z)Y + 3\eta(Y)\eta(Z)X + 3\eta(X)g(Y,Z)\xi - 3\eta(Y)g(X,Z)\xi - 4g(Y,Z)X + 4g(X,Z)Y + 3g(\phi Y,Z)X\psi - 3g(\phi X,Z)Y\psi + 3g(Y,Z)\phi X\psi - 3g(X,Z)\phi Y\psi] \tag{8.4}$$

$$+\frac{n-1-3\psi^2}{(n-1)(n-2)}[g(Y,Z)X-g(X,Z)Y],$$

where

$$C(X,Y)Z = R(X,Y)Z - \frac{1}{(n-2)}[S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX$$

$$-g(X,Z)QY] + \frac{r}{(n-1)(n-2)}[g(Y,Z)X - g(X,Z)Y]$$

is the Weyl conformal curvature tensor with respect to the Levi-Civita connection. By putting $Z = \xi$ in (8.4) and using (2.1), we obtain

$$\bar{C}(X,Y)\xi = C(X,Y)\xi + \frac{1}{(n-2)}[\eta(X)Y - \eta(Y)X + 3\eta(Y)\phi X\psi \tag{8.5}$$

$$-3\eta(X)\phi Y\psi] + \frac{n-1-3\psi^2}{(n-1)(n-2)}[\eta(Y)X - \eta(X)Y].$$

Hence we can state the following theorem:

Theorem 8.2. *An n-dimensional para-Sasakian manifold is Weyl ξ-conformally flat with respect to the quarter-symmetric metric connection if and only if the manifold is also Weyl ξ-conformally flat with respect to the Levi-Civita connection provided the trace of ϕ is zero.*

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