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FIXED POINTS FOR TWO PAIRS OF ABSORBING MAPPINGS IN WEAK PARTIAL METRIC SPACES

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Abstract. In this paper, a general fixed point theorem for two pairs of absorbing mappings in weak partial metric space, using implicit relations, has been proved.

Keywords: weak partial metric space; fixed point; pointwise absorbing mappings; implicit relation.

1. Introduction

In 1994, Matthews [13] introduced the concept of partial metric space as a part of the study of denotational semantics of dataflow networks and proved the Banach contraction principle in such spaces. The notion of partial metric spaces plays an important role in the constructing models in theory of computation.

Many authors studied the fixed points for mappings satisfying some contractive conditions in [1], [3], [11] and in other papers. In [11], some fixed point theorems for particular pairs of mappings are proved, generalizing some results from [1] and [3].

In 1999, Heckmann [10] introduced the notion of weak partial metric spaces, which is a generalization of partial metric spaces. Some results for mappings in weak partial metric spaces have been recently obtained by[2] and [4].

The notion of absorbing mappings have been introduced and studied in [5] - [7] as well as in other papers. Some fixed point theorems for two pairs of absorbing mappings in metric spaces have been proved in [12], [14], [15].

Several classical fixed point theorems and common fixed point theorems have been unified considering a general condition by an implicit relation in [16] - [18] and in other papers. Recently, the method has been used in the studies of fixed points in metric spaces, symmetric spaces, quasi - metric spaces, b - metric spaces, Hilbert

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spaces, ultra - metric spaces, convex metric spaces, compact metric spaces, in two and three metric spaces, for single valued mappings, hybrid pairs of mappings and set-valued mappings.

Some fixed point theorems for pairs of mappings satisfying implicit relations in partial metric spaces have been proved in [8], [9], [19] - [21].

Some results for pointwise absorbing mappings satisfying implicit relations have been obtained in [15].

The purpose of this paper is to prove a general fixed point theorem for two pairs of pointwise absorbing mappings in weak partial metric spaces using an implicit relation.

2. Preliminaries

Definition 2.1. ([13]) A partial metric on a nonempty set X is a function $p : X \times X \to \mathbb{R}_+$ such that for all $x, y, z \in X$:

$(P_1) : x = y$ if and only if $p(x,x) = p(y,y) = p(x,y)$,

$(P_2) : p(x,x) \leq p(x,y)$,

$(P_3) : p(x,y) = p(y,x)$,

$(P_4) : p(x,z) \leq p(x,y) + p(y,z) - p(y,y)$.

The pair (X,p) is called a partial metric space.

If $p(x,y) = 0$, then $x = y$, but the converse does not always hold true.

Each partial metric p on X generates a T_0 - topology τ_p on X which has as base the family of open p - balls $\{B_p(x,\varepsilon) : x \in X, \varepsilon > 0\}$, where $B_p(x,\varepsilon) = \{y \in X : p(x,y) \leq p(x,x) + \varepsilon\}$ for all $x \in X$ and $\varepsilon > 0$.

If p is a partial metric on X, then

$$d_w(x,y) = p(x,y) - \min\{p(x,x), p(y,y)\}$$

is a ordinary metric on X.

A sequence $\{x_n\}$ in a partial metric space (X,p) converges with respect to τ_p to a point $x \in X$, denoted $x_n \to x$, if and only if

$$p(x,x) = \lim_{n\to\infty} p(x_n, x).$$

Remark 2.1. Let $\{x_n\}$ be a sequence in a partial metric (X,p) and $x \in X$. Then $\lim_{n\to\infty} d_w(x_n, x) = 0$ if and only if

$$p(x,x) = \lim_{n\to\infty} p(x_n, x) = \lim_{n,m\to\infty} p(x_n, x_m). \tag{2.1}$$

Definition 2.2. ([13])

a) A sequence $\{x_n\}$ in a partial metric space (X,p) is called a Cauchy sequence if $\lim_{n,m\to\infty} p(x_n,x_m)$ exists and is finite.

b) A partial metric space (X,p) is said to be complete if every Cauchy sequence $\{x_n\}$ in X converges with respect to τ_p to a point $x\in X$ such that

$$p(x,x)=\lim_{n,m\to\infty} p(x_n,x_m).$$

Definition 2.3. ([10]) A weak partial metric on a nonempty set X is a function $p: X\times X\to\mathbb{R}_+$ such that for all $x,y,z\in X$:

$(wP_1): x=y$ if and only if $p(x,x)=p(y,y)=p(x,y)$,

$(wP_2): p(x,y)=p(y,x)$,

$(wP_3): p(x,z)\le p(x,y)+p(y,z)-p(y,y)$.

The pair (X,p) is called a weak partial metric space.

Obviously, every partial metric space is a weak partial metric space, but the converse is not true.

For example, let $X=[0,\infty)$ and $p(x,y)=\frac{x+y}{2}$, then (X,p) is a weak partial metric space and is not a partial metric space.

Theorem 2.1. *([2]) Let (X,p) be a weak partial metric space. Then $d_w(x,y): X\times X\to\mathbb{R}_+$ is a metric on X.*

Remark 2.2. In a weak partial metric space, the convergence of sequences, Cauchy sequences and completeness are defined as in partial metric space.

Theorem 2.2. *([2]) Let (X,p) be a weak partial metric space.*

a) $\{x_n\}$ is a Cauchy sequence in (X,p) if and only if $\{x_n\}$ is a Cauchy sequence in metric space (X,d_w).

b) (X,p) is complete if and only if (X,d_w) is complete.

Lemma 2.1. *Let (X,p) be a weak partial metric space and $\{x_n\}$ is a sequence in X. If $\lim_{n\to\infty} x_n=x$ and $p(x,x)=0$ then*

$$\lim_{n\to\infty} p(x_n,y)=p(x,y),\forall y\in X.$$

Proof. By (wP_3),

$$p(x,y)\le p(x,x_n)+p(x_n,y),$$

hence

$$p(x,y)-p(x,x_n)\le p(x_n,y)\le p(x_n,x)+p(x,y).$$

Letting n tend to infinity we obtain

$$\lim_{n\to\infty} p(x_n,y)=p(x,y).$$

□

Remark 2.3. Remark 2.1 is still true for weak partial metric spaces.

Definition 2.4. ([6]) Let (X, d) be a metric space and f, g be self mappings on X.

1) f is called g - absorbing if there exists $R > 0$ such that $d(gx, gfx) \leq Rd(fx, gx)$ for all $x \in X$.

Similarly, g is f - absorbing.

2) f is called pointwise g - absorbing if for given $x \in X$ there exists $R > 0$ such that $d(gx, gfx) \leq Rd(fx, gx)$.

Similarly, g is pointwise f - absorbing.

Remark 2.4. 1) If (X, p) is a weak partial metric space we have a similar definition to Definition 2.4 with p instead d.

2) If g is the identity mapping on X, then f is trivially absorbing.

3. Implicit relations

Definition 3.1. Let $\mathfrak{F}_W$ be the set of all lower semi - continuous functions $F : \mathbb{R}_+^5 \to \mathbb{R}$ satisfying the following conditions:

(F_1) : F is nonincreasing in variable t_5,

(F_2) : For all $u, v \geq 0$, there exists $h \in [0, 1)$ such that

(F_{2a}) : $F(u, v, v, u, u + v) \leq 0$ and

(F_{2b}) : $F(u, v, u, v, u + v) \leq 0$,

implies $u \leq hv$.

(F_3) : $F(t, t, 0, 0, 2t) > 0, \forall t > 0$.

Example 3.1. $F(t_1, ..., t_5) = t_1 - k \max\left\{t_2, t_3, t_4, \frac{t_5}{2}\right\}$, where $k \in [0, 1)$.

(F_1) : Obviously.

(F_2) : Let $u, v \geq 0$ be and $F(u, v, v, u, u + v) = \max\left\{u, v, \frac{u+v}{2}\right\} \leq 0$. If $u > v$ then $u(1 - k) \leq 0$, a contradiction. Hence $u \leq v$ which implies $u \leq hv$, where $0 \leq h = k < 1$.

Similarly, $F(u, v, u, v, u + v) \leq 0$ implies $u \leq hv$.

(F_3) : $F(t, t, 0, 0, 2t) = t(1 - k) > 0, \forall t > 0$.

The proofs for the following examples are similar to the proof of Example 3.1.

Example 3.2. $F(t_1, ..., t_5) = t_1 - k \max\{t_2, t_3, t_4, t_5\}$, where $k \in \left[0, \frac{1}{2}\right)$.

Example 3.3. $F(t_1, ..., t_5) = t_1 - k \max\left\{\frac{t_2+t_3+t_4}{3}, \frac{t_5}{2}\right\}$, where $k \in [0, 1)$.

Example 3.4. $F(t_1, ..., t_5) = t_1^2 - k \max\left\{t_2, \frac{t_3+t_4}{2}, \frac{t_5}{2}\right\}$, where $k \in [0, 1)$.

Example 3.5. $F(t_1, ..., t_5) = t_1^2 - a\max\left\{t_2^2, t_3^2, t_4^2\right\} - bt_5^2$, where $a, b \geq 0$ and $a + 4b < 1$.

Example 3.6. $F(t_1, ..., t_5) = t_1^2 - at_2t_3 - bt_3t_4 - ct_5^2$, where $a, b, c \geq 0$ and $a + b + 4c < 1$.

Example 3.7. $F(t_1, ..., t_5) = t_1^2 + \frac{t_1}{1+t_5} - \left(at_2^2 + bt_3^2 + ct_4^2\right)$, where $a, b, c \geq 0$ and $a+b+c < 1$.

Example 3.8. $F(t_1, ..., t_5) = t_1 - at_2 - bt_3 - c\max\{2t_4, t_5\}$, where $a, b, c, d \geq 0$ and $a + b + 2c < 1$.

Example 3.9. $F(t_1, ..., t_5) = t_1 - \frac{at_3t_4}{1+t_2} - bt_2 - c(t_3 + t_4) - dt_5$, where $a, b, c, d \geq 0$ and $a + b + 2c + 2d < 1$.

Example 3.10. $F(t_1, ..., t_5) = t_1^2 - t_1(at_2 + bt_3 + ct_4) - dt_5^2$, where $a, b, c, d \geq 0$ and $a + b + c + 4d < 1$.

4. Main results

Theorem 4.1. *Let (X, p) be a weak partial metric space and A, B, S and T be self mappings on X such that*

1) $T(X) \subset A(X)$ and $S(X) \subset B(X)$,

2) for all $x, y \in X$

$$F\left(\begin{array}{c} p(Sx, Ty), p(Ax, By), p(Sx, Ax), \\ p(Ty, By), p(Sx, By) + p(Ax, Ty) \end{array}\right) \leq 0. \tag{4.1}$$

If one of $A(X), B(X), S(X), T(X)$ is a closed subset of X, then

3) $\mathcal{C}(A, S) \neq \emptyset$,

4) $\mathcal{C}(B, T) \neq \emptyset$.

Moreover, if S is pointwise A - absorbing and T is pointwise B - absorbing, then A, B, S and T have a unique common fixed point z with $p(z, z) = 0$.

Proof. Let x_0 be an arbitrary point of X. Since $S(X) \subset B(X)$, there exists $x_1 \in X$ such that $y_0 = Sx_0 = Bx_1$. Since $T(X) \subset A(X)$, there exists $x_2 \in X$ such that $y_1 = Tx_1 = Ax_2$. Continuing this process we construct two sequences $\{x_n\}$ and $\{y_n\}$ in X by

$$y_{2n} = Sx_{2n} = Bx_{2n+1},\ y_{2n+1} = Tx_{2n+1} = Ax_{2n+2},\ n \in \mathbb{N}. \tag{4.2}$$

First we prove that $\{y_n\}$ is a Cauchy sequence in (X, p).

By (4.1) for $x = x_{2n}$ and $y = x_{2n+1}$ we have

$$F\left(\begin{array}{c} p(Sx_{2n}, Tx_{2n+1}), p(Ax_{2n}, Bx_{2n+1}), p(Sx_{2n}, Ax_{2n}), \\ p(Tx_{2n+1}, Bx_{2n+1}), p(Sx_{2n}, Bx_{2n+1}) + p(Ax_{2n}, Tx_{2n+1}) \end{array}\right) \leq 0.$$

By (4.2) we obtain

$$F\left(\begin{array}{c} p\left(y_{2n},y_{2n+1}\right),p\left(y_{2n-1},y_{2n}\right),p\left(y_{2n-1},y_{2n}\right), \\ p\left(y_{2n},y_{2n+1}\right),p\left(y_{2n},y_{2n}\right)+p\left(y_{2n-1},y_{2n+1}\right)\end{array}\right)\leq 0. \tag{4.3}$$

By (wP_3) we have

$$p\left(y_{2n-1},y_{2n+1}\right)\leq p\left(y_{2n-1},y_{2n}\right)+p\left(y_{2n},y_{2n+1}\right)-p\left(y_{2n},y_{2n}\right).$$

By (4.3) and (F_1) we obtain

$$F\left(\begin{array}{c} p\left(y_{2n},y_{2n+1}\right),p\left(y_{2n-1},y_{2n}\right),p\left(y_{2n-1},y_{2n}\right), \\ p\left(y_{2n},y_{2n+1}\right),p\left(y_{2n-1},y_{2n}\right)+p\left(y_{2n},y_{2n+1}\right)\end{array}\right)\leq 0.$$

By (F_{2a}) we obtain

$$p\left(y_{2n+1},y_{2n}\right)\leq hp\left(y_{2n},y_{2n-1}\right).$$

By (4.1) for $x=x_{2n}$ and $y=x_{2n-1}$ we obtain

$$F\left(\begin{array}{c} p\left(Sx_{2n},Tx_{2n-1}\right),p\left(Ax_{2n},Bx_{2n-1}\right),p\left(Sx_{2n},Ax_{2n}\right), \\ p\left(Tx_{2n-1},Bx_{2n-1}\right),p\left(Sx_{2n},Bx_{2n-1}\right)+p\left(Ax_{2n},Tx_{2n-1}\right)\end{array}\right)\leq 0.$$

By (4.1) we obtain

$$F\left(\begin{array}{c} p\left(y_{2n},y_{2n-1}\right),p\left(y_{2n-1},y_{2n-2}\right),p\left(y_{2n},y_{2n-1}\right), \\ p\left(y_{2n-1},y_{2n-2}\right),p\left(y_{2n},y_{2n-2}\right)+p\left(y_{2n-1},y_{2n-1}\right)\end{array}\right)\leq 0. \tag{4.4}$$

By (wP_3),

$$p\left(y_{2n-2},y_{2n}\right)\leq p\left(y_{2n-2},y_{2n-1}\right)+p\left(y_{2n-1},y_{2n}\right)-p\left(y_{2n-1},y_{2n-1}\right).$$

By (4.4) and (F_1) we obtain

$$F\left(\begin{array}{c} p\left(y_{2n},y_{2n-1}\right),p\left(y_{2n-1},y_{2n-2}\right),p\left(y_{2n},y_{2n-1}\right), \\ p\left(y_{2n-1},y_{2n-2}\right),p\left(y_{2n-2},y_{2n-1}\right)+p\left(y_{2n-1},y_{2n}\right)\end{array}\right)\leq 0.$$

By (F_{2b}),

$$p\left(y_{2n},y_{2n-1}\right)\leq hp\left(y_{2n-1},y_{2n-2}\right).$$

Hence,

$$p\left(y_n,y_{n+1}\right)\leq hp\left(y_{n-1},y_{n-2}\right)\leq ...\leq h^n p\left(y_0,y_1\right).$$

For $n,m\in\mathbb{N}$, $m>n$, repeating (wP_3) we obtain

$$\begin{array}{rcl} p\left(y_n,y_m\right) & \leq & p\left(y_n,y_{n+1}\right)+p\left(y_{n+1},y_{n+2}\right)+...+p\left(y_{m-1},y_m\right) \\ & \leq & h^n\left(1+h+...+h^{m-1}\right)p\left(y_0,y_1\right) \\ & \leq & \dfrac{h^n}{1-h}p\left(y_0,y_1\right). \end{array}$$

Then,

$$p(y_n, y_m) \leq \frac{h^n}{1-h} p(y_0, y_1) \to 0 \ as \ n, m \to \infty. \tag{4.5}$$

This shows that $\{y_n\}$ is a Cauchy sequence in (X, p). By Theorem 2.2 (a), $\{y_n\}$ is a Cauchy sequence in (X, d_w). Since (X, p) is complete, by Theorem 2.2 (b), (X, d_w) is a complete metric space. Since $\{y_n\}$ is Cauchy in (X, d_w), it follows that $\{y_n\}$ converges to a point z in (X, d_w). Hence,

$$\lim_{n \to \infty} d_w(y_n, z) = 0.$$

By Remark 2.3, (2.1) and (4.5) we obtain

$$p(z, z) = \lim_{n \to \infty} p(y_n, z) = \lim_{n, m \to \infty} p(y_n, y_m) = 0 \tag{4.6}$$

Also, by Theorem 2.2, $Sx_{2n} \to z$, $Tx_{2n+1} \to z$, $Bx_{2n+1} \to z$, $Ax_{2n+2} \to z$. Suppose that $T(X)$ is a closed subset in (X, p). Then

$$\lim_{n \to \infty} Tx_{2n+1} = z \in T(X).$$

Since $T(X) \subset A(X)$, there exists $u \in X$ such that $z = Au$.

By (4.1) for $x = u$ and $y = x_{2n+1}$ we obtain

$$F\left(\begin{array}{c} p(Su, Tx_{2n+1}), p(Au, Bx_{2n+1}), p(Su, Au), \\ p(Tx_{2n+1}, Bx_{2n+1}), p(Su, Bx_{2n+1}) + p(Au, Tx_{2n+1}) \end{array}\right) \leq 0,$$

$$F\left(\begin{array}{c} p(Su, y_{2n+1}), p(Au, y_{2n-1}), p(Su, Au), \\ p(y_{2n+1}, y_{2n}), p(Su, y_{2n}) + p(Au, y_{2n+1}) \end{array}\right) \leq 0.$$

Letting n tend to infinity, by Lemma 2.1, and (4.6) we have

$$F(p(Su, z), 0, p(Su, z), 0, p(Su, z)) \leq 0,$$

which implies by (F_{2b}) that $p(Su, z) = 0$, i.e. $z = Su$. Hence, $z = Au = Su$ and $\mathcal{C}(A, S) \neq \emptyset$.

Since $z \in S(X) \subset B(X)$, then, there exists $v \in X$ such that $z = Bv$. We prove that $Bv = Tv$.

By (4.1), for $x = u$ and $y = v$ we obtain

$$F\left(\begin{array}{c} p(Su, Tv), p(Au, Bv), p(Su, Au), \\ p(Tv, Bv), p(Su, Bv) + p(Au, Tv) \end{array}\right) \leq 0,$$

$$F(p(z, Tv), 0, 0, p(z, Tv), 0 + p(z, Tv)) \leq 0.$$

By (F_{2a}) we have $p(z,Tv)=0$, which implies $z=Tv=Bv$. Hence, $z=Au=Su=Bv=Tv$ with $p(z,z)=0$.

Moreover, if S is pointwise A - absorbing, there exists $R_1>0$ such that

$$p(Au,ASu)\leq R_1 p(Au,Su)=R_1 p(z,z)=0.$$

Hence, $z=Au=ASu=Az$ and z is a fixed point of A.

By (4.1) we have

$$F\left(\begin{array}{c} p(Sz,Tv),p(Az,Bv),p(Sz,Az), \\ p(Tv,Bv),p(Sz,Bv)+p(Az,Tv) \end{array}\right)\leq 0,$$

$$F(p(Sz,z),0,p(Sz,z),0,p(Sz,z)+p(Sz,z))\leq 0,$$

which implies by (F_{2b}) that $p(z,Sz)=0$. Hence, $z=Sz$ and z is a common fixed point of A and S.

If T is pointwise B - absorbing, then there exists $R_2>0$ such that

$$p(Bv,BTv)\leq R_2 p(Bv,Tv)=R_2 p(z,z)=0.$$

Hence, $z=Bv=BTv=Bz$ and z is a fixed point of B.

By (4.1) we have

$$F\left(\begin{array}{c} p(Su,Tz),p(Au,Bz),p(Su,Au), \\ p(Tz,Bz),p(Su,Bz)+p(Au,Tz) \end{array}\right)\leq 0,$$

$$F(p(z,Tz),0,0,p(z,Tz),0+p(z,Tz))\leq 0,$$

which implies by (F_{2a}) that $p(z,Tz)=0$. Hence, $z=Tz$ and z is a common fixed point of B and T.

Therefore, z is a common fixed point of S,T,A and B with $p(z,z)=0$.

Suppose that A,B,S and T have two common fixed points z_i, $i=1,2$ with $p(z_i,z_i)=0$.

By (4.1) we obtain

$$F\left(\begin{array}{c} p(Sz_1,Tz_2),p(Az_1,Bz_2),p(Sz_1,Az_1), \\ p(Tz_2,Bz_2),p(Sz_1,Bz_2)+p(Az_1,Tz_2) \end{array}\right)\leq 0,$$

$$F(p(z_1,z_2),p(z_1,z_2),0,0,2p(z_1,z_2))\leq 0,$$

a contradiction of (F_3) if $p(z_1,z_2)>0$. Hence, $p(z_1,z_2)=0$ which implies $z_1=z_2$. $\square$

Example 4.1. Let $X = [0,1]$ be and $p(x,y) = \frac{x+y}{2}$, which implies $d_w(x,y) = \frac{1}{2}|x-y|$. Hence, (X,p) is a complete weak partial metric space. Let the mappings $Sx = 0, Ax = \frac{x}{x+2}, Bx = x, Tx = \frac{x}{3}$. Since $A(X) = \left[0, \frac{1}{3}\right], B(X) = [0,1], S(X) = \{0\}, T(X) = \left[0, \frac{1}{3}\right]$, then $T(X) \subset A(X)$, $S(X) \subset B(X)$ and $A(X), B(X)$ and $T(X)$ are closed subsets of X.

$$\begin{aligned} p(Ax, ASx) &= p\left(\frac{x}{x+2}, 0\right) = \frac{x}{2(x+2)}, \\ p(Ax, Sx) &= p\left(\frac{x}{x+2}, 0\right) = \frac{x}{2(x+2)}. \end{aligned}$$

Hence, $p(Ax, ASx) \leq R_1 p(Sx, Ax)$ with $R_1 \geq 1$ and S is pointwise A - absorbing.

Similarly,

$$p(Bx, BTx) = p\left(x, \frac{x}{3}\right) = \frac{\frac{x}{3}+x}{2} = \frac{2x}{3}, \; p(Bx, Tx) = p\left(x, \frac{x}{3}\right) = \frac{2x}{3}.$$

Hence, $p(Bx, BTx) \leq R_2 p(Bx, Tx)$ with $R_2 \geq 1$ and T is pointwise B - absorbing.

On the other hand,

$$p(Sx, Ty) = \frac{Sx+Ty}{2} = \frac{0+\frac{y}{3}}{2} = \frac{y}{6}, \; p(Ty, By) = \frac{\frac{y}{3}+y}{2} = \frac{2y}{3}.$$

Hence,

$$p(Sx, Ty) \leq kp(Ty, By),$$

where $k \in \left[\frac{1}{4}, 1\right]$. Therefore,

$$p(Sx, Ty) \leq k \max\{p(Ax, By), p(Sx, Ax), p(Ty, By), p(Sx, By) + p(Ax, Ty)\}$$

with $k \in \left[\frac{1}{4}, 1\right]$.

By Theorem 4.1 and Example 3.1, A, B, S and T have a unique common fixed point $z = 0$ and $p(z,z) = 0$.

If $A = B = Id$, by Theorem 4.1 and Remark 2.4 (2), we obtain

Theorem 4.2. *Let (X,p) be a weak partial metric space and S and T be self mappings on X such that for all $x, y \in X$*

$$F\begin{pmatrix} p(Sx,Ty), p(x,y), p(x,Sx), \\ p(y,Ty), p(x,By) + p(Ty,x) \end{pmatrix} \leq 0. \tag{4.7}$$

for some $F \in \mathcal{F}$.

If $S(X)$ or $T(X)$ is a closed subset of X, then S and T have a unique common fixed point.

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