

**THERMAL BUCKLING ANALYSIS OF FUNCTIONALLY
GRADED CIRCULAR PLATE RESTING ON THE PASTERNAK
ELASTIC FOUNDATION VIA THE DIFFERENTIAL
TRANSFORM METHOD**

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Abstract. *In this paper, we propose a thermal buckling analysis of a functionally graded (FG) circular plate exhibiting polar orthotropic characteristics and resting on the Pasternak elastic foundation. The plate is assumed to be exposed to two kinds of thermal loads, namely, uniform temperature rise and linear temperature rise through thickness. The FG properties are assumed to vary continuously in the direction of thickness according to the simple power law model in terms of the volume fraction of two constituents. The governing equilibrium equations in buckling are based on the Von-Karman nonlinearity. To obtain the critical buckling temperature, we exploit a semi-numerical technique called differential transform method (DTM). This method provides fast accurate results and has a short computational calculation compared with the Taylor expansion method. Furthermore, some numerical examples are provided to consider the influence of various parameters such as volume fraction index, thickness-to-radius ratio, elastic foundation stiffness, modulus ratio of orthotropic materials and influence of boundary conditions. In order to predict the critical buckling temperature, it is observed that the critical temperature can be easily adjusted by appropriate variation of elastic foundation parameters and gradient index of FG material. Finally, the numerical results are compared with those available in the literature to confirm the accuracy and reliability of the DTM to determine the critical buckling temperature.*

Key Words: *Thermal Buckling, Orthotropic Plate, Functionally Graded Materials (FGM), Pasternak Elastic Foundation, Differential Transform Method (DTM)*

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1. INTRODUCTION

One of the most important undesired phenomena in a mechanical structure as observed in plates is thermal buckling. The response of the plate to buckling depends on its mechanical properties. Functionally graded materials (FGM) are made of ceramic and metal constituents, including high mechanical strength, good machinery ability, and high thermal resistance. FGMs are a good choice to be employed as the material constituents in a plate exposed to high thermal gradients. They are less likely to delaminate at high temperatures due to the continuity of their physical and mechanical properties. In addition, the orthotropic properties of FGMs have received great attention because they increase the tolerance of plates to various types of loading. Numerous studies have been conducted on thermal plate buckling. In what follows, we mention the studies related to the subject of this paper. Dewey and Costello [1] proposed an analytical and experimental method to investigate the thermal buckling of flat plates where the modulus of elasticity changes due to the thermal gradient. Najafizadeh and Eslami [2] discussed the thermo-mechanical response of plates based on the first-order shear deformation theory. They derived nonlinear and linear governing equations from the energy method and by using the calculus of variations. In another work, Najafizadeh and Eslami [3] discussed the thermoelastic buckling of the circular orthotropic composite plates under various kinds of thermal loading. They obtained the governing equations based on the Love-Kirchhoff hypothesis and the Sanders' nonlinear strain-displacement relation. Li *et al.* [4] examined the nonlinear vibration and thermal buckling of orthotropic annular plates with a centric rigid mass. They employed the Hamilton's principle to derive the governing equations based on the Von-Karman nonlinearity. Najafizadeh and Heydari [5] assessed the thermal buckling of circular plates in functionally graded materials under uniform radial compression subject to various types of thermal loads. They established the governing equations in buckling, using variational method and solved them *via* Bessel functions. Prakash and Ganapathy [6] applied the finite element method to analyze the vibrations and thermal buckling of circular FGM plates. Zhao *et al.* [7] studied the thermal and mechanical buckling behavior of plates with arbitrary geometry, including plates containing square and circular holes in the center. Zenkour and Sobhy [8] investigated the thermal buckling of FG sandwich plates, using sinusoidal shear deformation plate theory. The thermal loads are assumed to have uniform, linear, and non-linear distribution through thickness. Jalali *et al.* [9] assessed the thermal buckling of circular FGM plates with varying thickness, using the pseudo-spectral method (PSM). Evaluating the reaction of plates resting on elastic foundations and subject to different types of loads has a great scientific importance, particularly in modern engineering structures. Kiani and Eslami [10] studied the exact solution of thermal buckling in annular FGM plates resting on the Pasternak elastic foundation. They examined the effects of geometrical parameters, power-law index, and coefficients of the elastic foundation on the critical buckling temperature. Jabbari *et al.* [11] studied the buckling of a solid porous circular plate subjected to thermal loading. They derived the governing equations based on the Sanders' nonlinear strain-displacement relation and determined the pre-buckling temperatures and the critical buckling temperatures. Yaghoobi and Fereidoon [12] proposed the thermal and mechanical buckling of functionally graded (FG) plates resting on elastic foundation, using *n*th-order shear deformation theory. They obtained the governing equations *via* exploiting the minimum total potential energy method. Mansouri and Shariat [13] predicted the thermal buckling response of heterogeneous orthotropic plates based on

the high-order theories. They employed a new version of the DQM (Differential Quadrature Method) to solve the governing differential equations. Mirzaei and Kiani [14] investigated the thermally induced bifurcation buckling of rectangular composite plates reinforced with single-walled carbon nanotubes. They revealed that in most cases, the FG-X pattern of CNTs is the most influential case since it results in higher critical buckling temperatures. Yu *et al.* [15] evaluated the new numerical results of thermal buckling of functionally graded plates (FGPs) with internal defects (for example, crack or cutout), using an effective numerical method. They employed the new formulation of the first-order shear deformation plate theory associated with extended isogeometric analysis (XIGA) and level sets. Moreover, they investigated the influences of various aspect ratios, including gradient index, crack length, plate thickness, cutout size, and boundary conditions on the critical buckling temperature rise (CBTR). Tung [16] studied the nonlinear bending and post-buckling behavior of functionally graded sandwich plates resting on elastic foundations and subject to uniform external pressure, thermal loading, and uniaxial compression in the thermal environment. Sun *et al.* [17] numerically investigated the thermomechanical buckling and post-buckling of a functionally graded material (FGM) Timoshenko beam resting on a two-parameter non-linear elastic foundation and subject to only a temperature rise, using the shooting method.

In the present research, we accomplish the thermal buckling of an orthotropic FG circular plate. To that end, the Differential Transformation Method (DTM) is applied to solve the governing equation of thermal buckling. The literature review indicates that the present work is the first attempt to exploit DTM to evaluate the critical buckling temperature. A number of studies have been carried out by this method [18-24]. The results are presented in four categories based on linear and uniform thermal load and simply supported and clamped edge conditions in the proceeding sections. The effects of parameters such as volume fraction index, stiffness of the Pasternak elastic foundation, thickness-to-radius ratio, and modulus ratio of orthotropic material to critical temperature are also investigated.

2. MATHEMATICAL FORMULATION OF CONSTITUTIVE EQUATIONS

Consider a functionally graded circular solid supported on the Pasternak elastic foundation (Fig. 1). A polar coordinate system (r, θ, z) is employed to label the material points of the plate in radial, circumferential, and thickness directions.

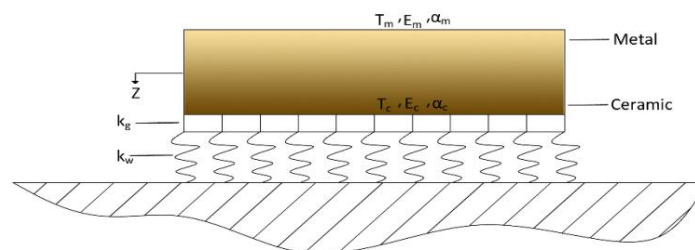


Fig. 1 Schematic of problem

Based on the Sanders' kinematic relations and the Von-Karman nonlinear assumption, the strain-displacement relationships are written in polar coordinates system as follows:

$$\varepsilon_{rr} = \bar{\varepsilon}_{rr} + zk_{rr}, \quad \varepsilon_{\theta\theta} = \bar{\varepsilon}_{\theta\theta} + zk_{\theta\theta}, \quad \gamma_{r\theta} = \bar{\gamma}_{r\theta} + 2zk_{r\theta} \quad (1)$$

where $\bar{\varepsilon}_{rr}$, $\bar{\varepsilon}_{\theta\theta}$, and $\bar{\gamma}_{r\theta}$ are mid-surface strains and k_{rr} , $k_{\theta\theta}$, and $k_{r\theta}$ are the curvatures defined as follows:

$$\begin{aligned} \bar{\varepsilon}_{rr} &= \frac{\partial u}{\partial r} + \frac{1}{2} \left(\frac{\partial w}{\partial r} \right)^2, \quad k_{rr} = -\frac{\partial^2 w}{\partial r^2} \\ \bar{\varepsilon}_{\theta\theta} &= \frac{1}{r} \frac{\partial v}{\partial \theta} + \frac{u}{r} + \frac{1}{2r^2} \left(\frac{\partial w}{\partial \theta} \right)^2, \quad k_{\theta\theta} = -\frac{1}{r} \frac{\partial w}{\partial r} + \frac{1}{r^2} \frac{\partial^2 w}{\partial \theta^2} \end{aligned} \quad (2)$$

$$\bar{\gamma}_{r\theta} = \frac{1}{r} \frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial r} - \frac{v}{r} + \frac{1}{r} \left(\frac{\partial w}{\partial r} \right) \left(\frac{\partial w}{\partial \theta} \right), \quad k_{r\theta} = \frac{1}{r^2} \frac{\partial w}{\partial \theta} - \frac{1}{r} \frac{\partial^2 w}{\partial r \partial \theta}$$

where u , v , and w represent the middle-plane displacements in the polar coordinates. Stress-strain relationships in polar coordinates are expressed as follows:

$$\begin{aligned} \sigma_{rr} &= a_{11}(\varepsilon_{rr} - (\varepsilon_T)_{rr}) + a_{12}(\varepsilon_{\theta\theta} - (\varepsilon_T)_{\theta\theta}) \\ \sigma_{\theta\theta} &= a_{21}(\varepsilon_{rr} - (\varepsilon_T)_{rr}) + a_{22}(\varepsilon_{\theta\theta} - (\varepsilon_T)_{\theta\theta}) \\ \tau_{r\theta} &= a_{66}\gamma_{r\theta} \end{aligned} \quad (3)$$

in which,

$$\begin{aligned} a_{11} &= \frac{E_r(z)}{1 - \mathcal{G}_{r\theta}\mathcal{G}_{\theta r}}, \quad a_{21} = \frac{\mathcal{G}_r E_\theta(z)}{1 - \mathcal{G}_{r\theta}\mathcal{G}_{\theta r}}, \quad a_{12} = \frac{\mathcal{G}_\theta E_r(z)}{1 - \mathcal{G}_{r\theta}\mathcal{G}_{\theta r}}, \quad a_{22} = \frac{E_\theta(z)}{1 - \mathcal{G}_{r\theta}\mathcal{G}_{\theta r}}, \\ a_{66} &= G_{r\theta}, \quad \frac{\mathcal{G}_{r\theta}}{E_r(z)} = \frac{\mathcal{G}_{\theta r}}{E_\theta(z)}, \quad \beta = \frac{\alpha_\theta}{\alpha_r}, \\ (\varepsilon_T)_{rr} &= \alpha_r \Delta T, \quad (\varepsilon_T)_{\theta\theta} = \alpha_\theta \Delta T, \quad \Delta T = T - T_0 \end{aligned} \quad (4)$$

where ε_T is the thermal strain, α_r and α_θ are the thermal expansion coefficients in radial and circumferential directions, respectively. Moreover, T and T_0 are the current and reference temperatures, respectively. The readers interested in the thermo-elastic stress-strain relations in a symmetrical case of deformation can find more details in the study conducted by Kiani et al. [10]. Upon substituting Eq. (4) into Eq. (3), the constitutive relations for orthotropic FG plate can be re-written as follows:

$$\begin{aligned} \sigma_{rr} &= a_{11}\varepsilon_{rr} + a_{12}\varepsilon_{\theta\theta} - (a_{11}\alpha_r + a_{12}\alpha_\theta)\Delta T \\ \sigma_{\theta\theta} &= a_{21}\varepsilon_{rr} + a_{22}\varepsilon_{\theta\theta} - (a_{21}\alpha_r + a_{22}\alpha_\theta)\Delta T \\ \tau_{r\theta} &= a_{66}\gamma_{r\theta} \end{aligned} \quad (5)$$

In addition, $G_{r\theta}$ is shear modulus, and E_r and E_θ are Young's modulus of the plate in radial and circumferential directions, respectively. Based on the power-law model in polar coordinates, the material properties are as follows:

$$E_r(z) = E_{rcm} \left(\frac{1}{2} + \frac{z}{h} \right)^n + E_{rm}, \quad E_{rcm} = E_{rc} - E_{rm} \quad (6)$$

$$\alpha_r(z) = \alpha_{rcm} \left(\frac{1}{2} + \frac{z}{h} \right)^n + \alpha_{rm}, \quad \alpha_{rcm} = \alpha_{rc} - \alpha_{rm} \quad (7)$$

$$\alpha_{\theta}(z) = \alpha_{\theta cm} \left(\frac{1}{2} + \frac{z}{h}\right)^n + \alpha_{\theta m}, \quad \alpha_{\theta cm} = \alpha_{\theta c} - \alpha_{\theta m} \quad (8)$$

$$G_{r\theta}(z) = G_{r\theta cm} \left(\frac{1}{2} + \frac{z}{h}\right)^n + G_{r\theta m}, \quad G_{r\theta cm} = G_{r\theta c} - G_{r\theta m} \quad (9)$$

where h , n and subscripts m and c represent the thickness, FG power index, the metal and ceramic properties, respectively. By assuming the polar orthotropic characteristics of plate, Young's modulus in the circumferential coordinate is defined as follows:

$$E_{\theta}(z) = \mu E_r(z) \Rightarrow E_{\theta}(z) = \mu \left(E_{cm} \left(\frac{1}{2} + \frac{z}{h}\right)^n + E_m \right) \quad (10)$$

where μ is the orthotropic modulus ratio.

3. GOVERNING EQUATIONS IN THERMAL BUCKLING

Stress resultants and stress couples are obtained as follows:

$$(N_{rr}, N_{\theta\theta}, N_{r\theta}) = \int_{-\frac{h}{2}}^{\frac{h}{2}} (\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{r\theta}) dz \quad (11)$$

$$(M_{rr}, M_{\theta\theta}, M_{r\theta}) = \int_{-\frac{h}{2}}^{\frac{h}{2}} z(\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{r\theta}) dz \quad (12)$$

Substituting Eq. (5) into Eqs. (11) and (12) yields the stress resultants and stress couples as follows:

$$N_{rr} = A_1 \bar{\epsilon}_{rr} + B_1 k_{rr} + A_2 \bar{\epsilon}_{\theta\theta} + B_2 k_{\theta\theta} - C_1 \Delta T \quad (13)$$

$$N_{\theta\theta} = A_2 \bar{\epsilon}_{rr} + B_2 k_{rr} + A_3 \bar{\epsilon}_{\theta\theta} + B_3 k_{\theta\theta} - C_2 \Delta T \quad (14)$$

$$M_{rr} = B_1 \bar{\epsilon}_{rr} + D_1 k_{rr} + B_2 \bar{\epsilon}_{\theta\theta} + D_2 k_{\theta\theta} - C_3 \Delta T \quad (15)$$

$$M_{\theta\theta} = B_2 \bar{\epsilon}_{rr} + D_2 k_{rr} + B_3 \bar{\epsilon}_{\theta\theta} + D_3 k_{\theta\theta} - C_4 \Delta T \quad (16)$$

where,

$$\begin{bmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ D_1 & D_2 & D_3 \end{bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{bmatrix} 1 \\ z \\ z^2 \end{bmatrix} [a_{11} \quad a_{12} \quad a_{22}] dz$$

$$(C_1, C_3) = \int_{-\frac{h}{2}}^{\frac{h}{2}} (1, z)(a_{11}\alpha_r + a_{12}\alpha_{\theta}) dz, \quad (C_2, C_4) = \int_{-\frac{h}{2}}^{\frac{h}{2}} (1, z)(a_{21}\alpha_r + a_{22}\alpha_{\theta}) dz$$

When the plate is subjected to the mechanical loading, the total energy is given by:

$$V = U + \Omega \quad (17)$$

where Ω and U are the potential energy of the external loading and the strain energy, respectively. Ω is the summation of the potential energy of mechanical loading and two parameters elastic foundation reaction. The elastic foundation is exerted on the lower surface of the plate, as shown in Fig. 1. In this research, the mechanical loading is absent. U is the summation of thermal strain energy, membrane strain energy, bending strain energy, coupled bending-membrane strain energy, and elastic foundation strain energy:

$$U = \int_0^{2\pi R} \int_0^{h/2} \int_{-h/2}^0 \frac{1}{2} (\sigma_{rr} \varepsilon_{rr} + \sigma_{\theta\theta} \varepsilon_{\theta\theta} + \sigma_{r\theta} \varepsilon_{r\theta} - \sigma_{rr} \alpha_r \Delta T - \sigma_{\theta\theta} \alpha_\theta \Delta T) r dz dr d\theta \quad (18)$$

By integrating through thickness, U can be expressed as follows:

$$U = \int_0^{2\pi R} \int_0^2 \frac{1}{2} (\bar{\varepsilon}_{rr} N_r + k_r M_r + \bar{\varepsilon}_{\theta\theta} N_\theta + k_\theta M_\theta + \bar{\gamma}_{r\theta} N_{r\theta} + 2k_{r\theta} M_{r\theta} + k_w w^2 + k_g \left(\frac{\partial w}{\partial r} \right)^2 + \frac{1}{r^2} k_g \left(\frac{\partial w}{\partial \theta} \right)^2) r dr d\theta \quad (19)$$

where k_w and k_g denote the stiffness of Pasternak elastic foundation (tension and shear foundation parameters, respectively). By substituting Eqs. (13-16) into Eq. (19), setting the resultant expression into the expression of the total potential energy function, Eq. (17), and employing Euler equations [5], the governing equilibrium equations in the buckling of the plate resting on the Pasternak foundation based on the von-Karman nonlinearity are obtained as follows [10]:

$$\begin{aligned} N_{r,r} + \frac{1}{r} N_{r\theta,\theta} + \frac{N_r - N_\theta}{r} &= 0 \\ N_{r\theta,r} + \frac{1}{r} N_{\theta,\theta} + \frac{2}{r} N_{r\theta} &= 0 \\ M_{r,rr} + \frac{2}{r} M_{r,r} + \frac{1}{r^2} M_{\theta,\theta\theta} - \frac{1}{r} M_{\theta,r} + \frac{2}{r} M_{r\theta,r\theta} + \frac{2}{r^2} M_{r\theta,\theta} - N_r w_{,rr} \\ - N_\theta \left(\frac{1}{r^2} w_{,\theta\theta} + \frac{1}{r} w_{,r} \right) - k_w w + k_g \left(\frac{1}{r} w_{,r} + w_{,rr} + \frac{1}{r^2} w_{,\theta\theta} \right) &= 0 \end{aligned} \quad (20)$$

The governing equations of equilibrium can be expressed in terms of displacement components by assuming the symmetric state of buckling which gives the variation with respect to the circumferential direction set to zero, substituting Eqs. (13-16) into Eqs. (20), and utilizing Eq. (2):

$$\begin{aligned} \frac{1}{(1-k\nu_r^2)} \left\{ E_1 \left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{\mu u}{r^2} + \frac{\partial^2 w}{\partial r^2} \frac{\partial w}{\partial r} + \frac{(1-k\nu_r)}{2r} \left(\frac{\partial w}{\partial r} \right)^2 \right] \right. \\ \left. + E_2 \left[-\frac{\partial^3 w}{\partial r^3} - \frac{1}{r} \frac{\partial^2 w}{\partial r^2} + \frac{\mu}{r^2} \frac{\partial w}{\partial r} \right] \right\} + \frac{N_\theta^T - N_r^T}{r} = 0 \end{aligned} \quad (21)$$

$$\begin{aligned} & \frac{1}{(1-k\nu_r^2)} \left\{ E_2 \left[\frac{\partial^3 u}{\partial r^3} + \frac{2}{r} \frac{\partial^2 u}{\partial r^2} - \frac{\mu}{r^2} \frac{\partial u}{\partial r} + \frac{\mu u}{r^3} + \frac{\partial^3 w}{\partial r^3} \frac{\partial w}{\partial r} + \left(\frac{\partial^2 w}{\partial r^2} \right)^2 + \frac{(2-k\nu_r)}{r} \frac{\partial^2 w}{\partial r^2} \frac{\partial w}{\partial r} \right] \right. \\ & \left. + E_3 \left[-\frac{\partial^4 w}{\partial r^4} - \frac{2}{r} \frac{\partial^3 w}{\partial r^3} + \frac{\mu}{r^2} \frac{\partial^2 w}{\partial r^2} - \frac{\mu}{r^3} \frac{\partial w}{\partial r} \right] \right\} + N_r^T w_{,rr} + \frac{1}{r} N_\theta^T w_{,r} \\ & -k_w w + k_g \left(\frac{1}{r} w_{,r} + w_{,rr} \right) = 0 \end{aligned} \quad (22)$$

where,

$$\begin{aligned} E_1 &= h \left[E_m + \frac{E_{cm}}{n+1} \right], E_2 = h^2 E_{cm} \left[\frac{1}{n+2} - \frac{1}{2n+2} \right], \\ E_3 &= h^3 \left[\frac{E_m}{12} + E_{cm} \left[\frac{1}{n+3} + \frac{1}{4n+4} - \frac{1}{n+2} \right] \right] \end{aligned}$$

3.1. Types of thermal loading

In this study, two cases of temperature rise are considered: uniform and linear temperature differences. When the plate is not thick enough, the assumption of linear distribution is reasonable [10]. But when the temperature on the top and bottom surfaces of the plate is constant and no source of heat is available in it, the temperature variation can be defined as a linear function of thickness coordinate; for instance, the temperature distribution in aircraft window, walls of the building, or furnace. By assuming that the reference temperature of plate is T_0 and the displacement in the radial direction is prevented due to restraint on the edge of the plate, N_r^T and N_θ^T as the representatives of stress resulting from the thermal gradients can be determined in two categories due to the temperature rise across thickness as follows:

– Uniform thermal loading:

$$N_r^T = C_1 \Delta T = h \frac{(1+\beta\mu\nu_r)}{(1-\mu\nu_r^2)} \left[E_m \alpha_m + \frac{E_{cm} \alpha_m + E_m \alpha_{cm}}{n+1} + \frac{E_{cm} \alpha_{cm}}{2n+1} \right] \Delta T \quad (23)$$

$$N_\theta^T = C_2 \Delta T = h \frac{(\beta\mu + \mu\nu_r)}{(1-\mu\nu_r^2)} \left[E_m \alpha_m + \frac{E_{cm} \alpha_m + E_m \alpha_{cm}}{n+1} + \frac{E_{cm} \alpha_{cm}}{2n+1} \right] \Delta T \quad (24)$$

– U Linear thermal loading:

$$T(z) = T_m + (T_c - T_m) \left(\frac{1}{2} + \frac{z}{h} \right) \quad (25)$$

$$\begin{aligned} N_r^T &= h \frac{(1+\beta\mu\nu_r)}{(1-\mu\nu_r^2)} (T_m - T_0) \left[E_m \alpha_m + \frac{E_{cm} \alpha_m + E_m \alpha_{cm}}{n+1} + \frac{E_{cm} \alpha_{cm}}{2n+1} \right] \\ &+ h \frac{(1+\beta\mu\nu_r)}{(1-\mu\nu_r^2)} \left[\frac{E_m \alpha_m}{2} + \frac{E_{cm} \alpha_m + E_m \alpha_{cm}}{n+2} + \frac{E_{cm} \alpha_{cm}}{2n+2} \right] \Delta T \end{aligned} \quad (26)$$

$$N_{\theta}^T = h \frac{(\beta\mu + \mu\nu_r)}{(1 - \mu\nu_r^2)} (T_m - T_0) \left[E_m \alpha_m + \frac{E_{cm} \alpha_m + E_m \alpha_{cm}}{n+1} + \frac{E_{cm} \alpha_{cm}}{2n+1} \right] \\ + \frac{h(\beta\mu + \mu\nu_r)}{(1 - \mu\nu_r^2)} \left[\frac{E_m \alpha_m}{2} + \frac{E_{cm} \alpha_m + E_m \alpha_{cm}}{n+2} + \frac{E_{cm} \alpha_{cm}}{2n+2} \right] \Delta T \quad (27)$$

where $\Delta T = T(z) - T_0$. By eliminating the radial displacement components of u and after performing some mathematical manipulations, the equations of the equilibrium can be summarized in one equation in terms of out-plane displacement component of w as follows:

$$D_E \left[\frac{\partial^4 w}{\partial r^4} + \frac{2}{r} \frac{\partial^3 w}{\partial r^3} - \frac{\mu}{r^2} \frac{\partial^2 w}{\partial r^2} + \frac{\mu}{r^3} \frac{\partial w}{\partial r} \right] + (N_r^T + k_g) \frac{\partial^2 w}{\partial r^2} + \frac{1}{r} (N_{\theta}^T + k_g) \frac{\partial w}{\partial r} - k_w w = 0 \quad (28)$$

where, $D_E = (E_2^2 - E_1 E_3) / E_1 (1 - \mu\nu_r^2)$. Based on the adjacent equilibrium criteria, the state of stable equilibrium may be designated by w_0 ; in addition, it is $w_0 + w_1$ in the neighborhood of stability state when w_1 can be represented to any small increment of displacement. Similar to out-plane displacement, the stress resultants are divided into two terms representing the stable equilibrium and the neighboring state [10]. Upon substituting $w_0 + w_1$ and stress resultants in two terms into the governing equation (28) and performing some mathematical manipulations, the stability equation is obtained as follows:

$$D_E \left[\frac{\partial^4 w_1}{\partial r^4} + \frac{2}{r} \frac{\partial^3 w_1}{\partial r^3} - \frac{\mu}{r^2} \frac{\partial^2 w_1}{\partial r^2} + \frac{\mu}{r^3} \frac{\partial w_1}{\partial r} \right] + (N_{r0}^T + k_g) \frac{\partial^2 w_1}{\partial r^2} \\ + \frac{1}{r} (N_{\theta 0}^T + k_g) \frac{\partial w_1}{\partial r} - k_w w_1 = 0 \quad (29)$$

where N_{r0}^T and $N_{\theta 0}^T$ are the pre-buckling thermal loading, and the following relation can be established: $N_{r0}^T = N_{\theta 0}^T = -N^T$. The following dimensionless quantities are defined to deal with the problem under consideration in the dimensionless forms:

$$\Phi = \frac{w_1}{h}, \quad \xi = \frac{r}{R}, \quad n_r^T = \frac{N_r^T R^2}{D_E}, \quad n_{\theta}^T = \frac{N_{\theta}^T R^2}{D_E}, \quad K_g = \frac{k_g R^2}{D_E}, \quad K_w = \frac{k_w R^4}{D_E} \\ \xi^3 \frac{d^4 \Phi}{d\xi^4} + 2\xi^2 \frac{d^3 \Phi}{d\xi^3} - \mu \xi \frac{d^2 \Phi}{d\xi^2} + \mu \frac{d\Phi}{d\xi} + (n_r^T + K_g) \xi^3 \frac{d^2 \Phi}{d\xi^2} \\ + (n_{\theta}^T + K_g) \xi^2 \frac{d\Phi}{d\xi} - K_w \xi^3 \Phi = 0 \quad (30)$$

4. SOLVING THE GOVERNING EQUATION BY DTM

The differential transform method (DTM) is a numerical method based on the Taylor series expansion that proposes the solution in the form of polynomials [24]. This method is a fast convergent method in comparison with the Taylor series in order to solve the differential equations. The advantage of this method is its low computational manipulation and its applicability to handle linear and non-linear ordinary and partial differential equations. By

exploiting the DTM, the differential equations are reduced to the recurrence relations and convert the boundary conditions into a set of algebraic equations. The differential transform of the k -th derivative of function $f(r)$ is defined as follows:

$$F[k] = \frac{1}{k!} \left(\frac{d^k f(r)}{dr^k} \right)_{r=r_0} \tag{31}$$

where $f(r)$ and $F[k]$ are the original function and transform function, respectively. The inverse differential transform of $F[k]$ is defined as follows [5]:

$$f(r) = \sum_{k=0}^{\infty} F[k] (r - r_0)^k \tag{32}$$

In the domain of R , original function $f(r)$ is considered to be analytical, and $r=r_0$ represents any point in R . $f(r)$ is represented by power series whose center is located at r_0 . From Eqs. (31) and (32), it can be concluded that:

$$f(r) = \sum_{k=0}^{\infty} \frac{(r - r_0)^k}{k!} \left(\frac{d^k f(r)}{dr^k} \right)_{r=r_0} \tag{33}$$

Table 1 demonstrates the fundamental mathematical properties of the differential transform method (DTM):

Table 1 Fundamental theorems of DTM [22]

| Original Function | Transformed Function |
|------------------------------------|--|
| $f(r) = y(r) \pm z(r)$ | $F[k] = Y[k] \pm Z[k]$ |
| $f(r) = \lambda y(r)$ | $F[k] = \lambda Y[k]$ |
| $f(r) = y(r).z(r)$ | $F[k] = \sum_{k_1=0}^k Y[k_1] Z[k - k_1]$ |
| $f(r) = \frac{(d^m y(r))}{(dr^m)}$ | $F[k] = \frac{(m+k)!}{k!} Y[k+m]$ |
| $f(r) = r^n$ | $F[k] = \delta(k-n) = \begin{cases} 0 & \text{if } k \neq n \\ 1 & \text{if } k = n \end{cases}$ |

Hence, function $\Phi[\xi]$ is obtained as follows:

$$\Phi[\xi] = \sum_{k=0}^N \phi[k] \xi^k = \phi[0] \xi^0 + \phi[1] \xi^1 + \phi[2] \xi^2 + \dots \tag{34}$$

To solve Eq. (35), we use the differential transform relationships of the k -th derivative of the function of non-dimensional out-plane displacement Φ ; moreover, DTM theorems are listed in Table 1:

$$\begin{aligned}
\xi^3 \frac{d^4 \Phi}{d\xi^4} &= \sum_{k_1=0}^k \delta(k_1-3)(k-k_1+4)(k-k_1+3)(k-k_1+2)(k-k_1+1)\phi[k-k_1+4] \\
\xi^2 \frac{d^3 \Phi}{d\xi^3} &= \sum_{k_1=0}^k \delta(k_1-2)(k-k_1+3)(k-k_1+2)(k-k_1+1)\phi[k-k_1+3] \\
\xi \frac{d^2 \Phi}{d\xi^2} &= \sum_{k_1=0}^k \delta(k_1-1)(k-k_1+2)(k-k_1+1)\phi[k-k_1+2] \\
\frac{d\Phi}{d\xi} &= (k+1)\phi[k+1] \\
\xi^3 \frac{d^2 \Phi}{d\xi^2} &= \sum_{k_1=0}^k \delta(k_1-3)(k-k_1+2)(k-k_1+1)\phi[k-k_1+2] \\
\xi^2 \frac{d\Phi}{d\xi} &= \sum_{k_1=0}^k \delta(k_1-2)(k-k_1+1)\phi[k-k_1+1] \\
\xi^3 \Phi &= \sum_{k_1=0}^k \delta(k_1-3)\phi[k-k_1]
\end{aligned} \tag{35}$$

Substituting the aforementioned relationships into the governing equation in thermal buckling Eq. (28) yields:

$$\begin{aligned}
&\sum_{k_1=0}^k \delta(k_1-3)(k-k_1+4)(k-k_1+3)(k-k_1+2)(k-k_1+1)\phi[k-k_1+4] \\
&+ 2 \sum_{k_1=0}^k \delta(k_1-2)(k-k_1+3)(k-k_1+2)(k-k_1+1)\phi[k-k_1+3] \\
&- \mu \sum_{k_1=0}^k \delta(k_1-1)(k-k_1+2)(k-k_1+1)\phi[k-k_1+2] + \mu(k+1)\phi[k+1] \\
&+ A_1 \sum_{k_1=0}^k \delta(k_1-3)(k-k_1+2)(k-k_1+1)\phi[k-k_1+2] \\
&+ A_2 \sum_{k_1=0}^k \delta(k_1-2)(k-k_1+1)\phi[k-k_1+1] - K_w \sum_{k_1=0}^k \delta(k_1-3)\phi[k-k_1] = 0
\end{aligned} \tag{36}$$

By utilizing the appropriate theorems of the DT method (see Table 1 and the simplified form of Eq. 36), we have the following recurrence relation:

$$\phi[k+1] = \frac{(A_1(k-2) + A_2)(k-1)\phi[k-1] - K_w \phi[k-3]}{(-k^2 + \mu)(k^2 - 1)}, \quad k \geq 3 \tag{37}$$

where $A_1 = n_r^T + K_g$ and $A_2 = n_\theta^T + K_g$.

The following boundary conditions are imposed as clamped and simply supported boundary conditions on the edges of the plate. However, by applying differential transformation method to boundary conditions, we can obtain:

- Non-dimensional clamped edge:

$$\phi|_{\xi=1} = 0, \quad \left. \frac{d\phi}{d\xi} \right|_{\xi=1} = 0 \quad (38)$$

- Differential transform of clamped edge condition with DTM:

$$\sum_{k=0}^N \phi[k] = 0, \quad \sum_{k=0}^N k\phi[k] = 0 \quad (39)$$

- Non-dimensional simply supported boundary condition:

$$\phi|_{\xi=1} = 0, \quad -D_E \left[\frac{d^2\phi}{d\xi^2} + \frac{\nu}{r} \frac{d\phi}{d\xi} \right] \Big|_{\xi=1} = 0 \quad (40)$$

- Differential transform of simply supported edge condition with DTM:

$$\sum_{k=0}^N \phi[k] = 0, \quad \sum_{k=0}^N k(k-1+\nu)\phi[k] = 0 \quad (41)$$

In this study, the symmetrical thermal buckling behavior of a plate is considered, and the regularity condition is imposed besides the boundary conditions. The non-dimensional form of regularity condition and its differential transform are as follows:

$$\left. \frac{d\phi}{d\xi} \right|_{\xi=0} = 0, \quad \phi[1] = 0 \quad (42)$$

Upon substituting $k=3,5,7,\dots$ into the recurrence Eq. (37), we get:

$$\begin{aligned} \varphi[3] &= -\frac{A_2\varphi[1]}{3\mu+12} = 0 \\ \varphi[5] &= \frac{(6A_1+3A_2)\varphi[3]-K_w\varphi[1]}{15\mu-240} = 0, \quad \varphi[7] = \frac{(20A_1+5A_2)\varphi[5]-K_w\varphi[3]}{35\mu-1260} = 0, \dots \end{aligned} \quad (43)$$

It can be concluded that for odd values of k in Eq. (34), $\varphi[k]$ equals zero [23-24]. Therefore, by using recurrence Eq. (37), we can find that $\varphi[k]$ can be determined in terms of $\varphi[0]$ and $\varphi[2]$. By using recurrence relation (37) for $k=2, 4, 6,\dots$, we can obtain the following equations:

$$\begin{aligned} \varphi[4] &= \frac{2(A_1+A_2)\varphi[2]-K_w\varphi[0]}{8\mu-72} \\ \varphi[6] &= \frac{(12A_1+4A_2)\varphi[4]-K_w\varphi[2]}{24\mu-600}, \quad \varphi[8] = \frac{(30A_1+6A_2)\varphi[6]-K_w\varphi[4]}{48\mu-2352}, \dots \end{aligned} \quad (44)$$

Hence, all the $\varphi[k]$ with even values of k in Eq. (34) can be expressed in terms of $\varphi[2]$, $\varphi[0]$.

In order to determine the critical buckling temperature, recurrence relation (42) and imposed boundary conditions of Eq. (39) are simultaneously employed for clamped edges. Therefore, a set of two homogenous equations are established in terms of $\phi[0]$ and $\phi[2]$, as follows:

$$\begin{cases} H_{11}\phi[0] + H_{12}\phi[2] = 0 \\ H_{21}\phi[0] + H_{22}\phi[2] = 0 \end{cases} \quad (45)$$

where H_{11} , H_{12} , H_{21} , and H_{22} are the coefficients of polynomials of N -th order.

$$\begin{aligned} H_{11} &= 1 - \frac{K_w}{8\mu - 72} - \frac{(3A_1 + A_2)K_w}{(6\mu - 150)(8\mu - 72)} + \dots \\ H_{12} &= 1 + \frac{A_1 + A_2}{4\mu - 36} - \frac{K_w}{24\mu - 600} + \frac{(3A_1 + A_2)(A_1 + A_2)}{(6\mu - 150)(4\mu - 36)} + \dots \\ H_{21} &= \frac{-K_w}{2\mu - 18} - \frac{6(3A_1 + A_2)K_w}{(6\mu - 150)(2\mu - 36)} + \dots \\ H_{22} &= 2 + \frac{A_1 + A_2}{\mu - 9} - \frac{K_w}{4\mu - 100} + \frac{(12A_1 + 4A_2)(A_1 + A_2)}{(4\mu - 100)(4\mu - 36)} + \dots \end{aligned} \quad (46)$$

For a non-trivial solution of Eq. (45), the determinant of coefficients must vanish, leading to the eigenvalue problem. Hence, we have:

$$\begin{vmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{vmatrix} = 0 \quad (47)$$

In a similar manner, we can obtain the solution for the simply supported plate by utilizing the recurrence relation (37) and the imposed boundary conditions of Eq. (41).

5. NUMERICAL RESULTS

In this section, some numerical results are presented for the thermal buckling of orthotropic FG circular plates of two categories of uniform and linear temperature rise under clamped and simply supported boundary conditions. Due to the high volume of the calculations required, the system of algebraic equations presented in the preceding section are implemented in a computer code in MATLAB software, and the numerical results are presented in a tabulated form. For the numerical results, an FG plate composed of aluminum (as metal) and alumina (as ceramic) is considered. Young's modulus of aluminum (Al) and alumina are $E_m=70 \text{ Gpa}$ and $E_c=380 \text{ Gpa}$, respectively. The thermal expansion coefficients are $\alpha_m=23 \times 10^{-6} \text{ K}^{-1}$ and $\alpha_c=7.4 \times 10^{-6} \text{ K}^{-1}$ for metal and ceramic constituents, respectively. Poisson's ratio remains constant at $\nu=0.3$. It is assumed that the material properties are assumed to be temperature-independent. To examine the convergence rate of DT method, we obtain the results for a clamped isotropic homogeneous circular plate with the thickness-to-radius ratio of $h/R=0.01$. We observe that the number of terms ($k=20$) is sufficient to get precise values of the critical temperature. This trend remains constant in other numerical results as presented in the following section.

5.1. Uniform loading - clamped boundary condition

Table 2 shows the variation of the critical buckling temperature with respect to FG power-law index, thickness-to-radius ratio, and orthotropic ratio. As it is observed, the critical temperature increases as the value of μ rises. According to Table 2, for a specific h/R ratio, the buckling critical temperature is reduced by increasing the value of n ; however, increasing the h/R ratio increases the values of the critical temperature.

Table 2 Variation of the critical temperature ($^{\circ}K$) of the clamped plate versus FG power-law index, orthotropic ratio, and thickness-to-radius ratio in case of uniform thermal loading

| μ | h/R | n | | | | | |
|-------|-------|----------|---------|---------|---------|---------|---------|
| | | 0 | 0.5 | 1 | 2 | 5 | 10 |
| 0.50 | 0.010 | 7.070 | 4.081 | 3.163 | 2.914 | 2.151 | 2.002 |
| | 0.150 | 15.911 | 9.104 | 7.390 | 6.552 | 6.060 | 5.950 |
| | 0.020 | 28.280 | 16.081 | 12.851 | 11.652 | 11.141 | 10.860 |
| | 0.030 | 63.641 | 36.153 | 28.934 | 26.214 | 26.042 | 25.851 |
| | 0.040 | 113.132 | 64.214 | 52.560 | 46.603 | 46.142 | 45.823 |
| 0.70 | 0.010 | 13.780 | 7.813 | 6.422 | 5.684 | 5.164 | 5.010 |
| | 0.150 | 31.020 | 17.570 | 14.414 | 12.780 | 11.181 | 10.551 |
| | 0.020 | 55.131 | 31.232 | 25.611 | 22.714 | 21.421 | 21.090 |
| | 0.030 | 124.061 | 70.280 | 55.412 | 51.091 | 52.732 | 50.261 |
| | 0.040 | 220.552 | 124.944 | 98.503 | 90.831 | 93.721 | 92.361 |
| 0.9 | 0.010 | 21.381 | 12.112 | 9.5404 | 8.812 | 9.150 | 9.040 |
| | 0.150 | 48.110 | 27.260 | 21.480 | 19.823 | 20.444 | 20.121 |
| | 0.020 | 85.541 | 48.460 | 38.881 | 35.234 | 36.341 | 33.370 |
| | 0.030 | 192.462 | 109.033 | 89.412 | 79.270 | 81.841 | 80.124 |
| | 0.040 | 342.152 | 193.842 | 152.872 | 140.920 | 145.382 | 142.534 |
| 1 | 0.010 | 24.950 | 14.142 | 11.594 | 10.280 | 10.001 | 8.901 |
| | 0.150 | 56.150 | 31.813 | 25.083 | 23.132 | 21.863 | 20.520 |
| | 0.020 | 99.824 | 56.552 | 46.370 | 41.113 | 38.411 | 38.604 |
| | 0.030 | 224.581 | 127.240 | 104.343 | 92.504 | 89.432 | 88.104 |
| | 0.040 | 399.270 | 226.214 | 185.480 | 164.451 | 161.652 | 160.412 |
| 1.5 | 0.010 | 51.633 | 29.262 | 23.480 | 21.260 | 21.941 | 22.561 |
| | 0.150 | 114.280 | 64.744 | 53.091 | 47.120 | 48.562 | 49.933 |
| | 0.020 | 203.141 | 115.081 | 94.421 | 83.662 | 80.311 | 78.760 |
| | 0.030 | 456.602 | 258.660 | 213.192 | 188.150 | 194.121 | 199.480 |
| | 0.040 | 807.412 | 457.411 | 360.590 | 332.542 | 343.150 | 352.761 |
| 2 | 0.010 | 87.150 | 49.371 | 40.634 | 35.890 | 33.151 | 31.134 |
| | 0.150 | 200.051 | 113.361 | 89.381 | 82.393 | 78.050 | 77.412 |
| | 0.020 | 349.374 | 197.942 | 162.401 | 143.881 | 141.450 | 140.661 |
| | 0.030 | 794.632 | 450.160 | 355.991 | 327.270 | 322.644 | 320.180 |
| | 0.040 | 1408.221 | 797.630 | 651.631 | 579.880 | 558.262 | 555.180 |

Table 3 exhibits the critical buckling temperature with respect to elastic foundation coefficient, orthotropic ratio, and FG power-law index. As elastic foundation coefficients increase by increasing the orthotropic ratio, the critical temperature increases. On the other hand, when the FG power-law index increases, the critical temperature decreases. This shows that the pure ceramic plate, compared to the metal-ceramic plate, is more stable at the elevated working temperature.

Table 3 Variation of the critical temperature ($^{\circ}K$) of the clamped plate versus FG power-law index, orthotropic ratio, and elastic foundation on the critical temperature in case of uniform thermal loading, $h/R=0.020$

| μ | n | k_w, k_g | | | | | |
|-------|------|------------|---------|---------|---------|----------|----------|
| | | (0,0) | (100,0) | (200,0) | (500,0) | (100,10) | (200,20) |
| 0.7 | 0 | 55.131 | 63.121 | 67.341 | 81.470 | 77.470 | 96.911 |
| | 0.50 | 31.232 | 35.690 | 38.150 | 46.151 | 43.880 | 54.880 |
| | 1 | 25.611 | 29.341 | 31.280 | 37.851 | 35.903 | 44.841 |
| | 2 | 22.714 | 26.030 | 27.741 | 33.581 | 31.960 | 39.801 |
| | 5 | 21.422 | 23.822 | 24.633 | 31.610 | 30.841 | 37.250 |
| 1 | 0 | 99.824 | 114.293 | 121.780 | 147.531 | 140.050 | 175.360 |
| | 0.5 | 56.552 | 64.520 | 69.270 | 83.812 | 79.512 | 99.5281 |
| | 1 | 46.370 | 53.141 | 56.343 | 68.301 | 65.381 | 81.334 |
| | 2 | 41.113 | 47.282 | 50.480 | 61.012 | 57.722 | 73.982 |
| | 5 | 38.411 | 44.690 | 47.742 | 59.553 | 53.801 | 71.552 |
| 1.5 | 0 | 203.142 | 232.570 | 248.150 | 300.222 | 285.450 | 357.153 |
| | 0.5 | 115.081 | 131.764 | 140.580 | 170.244 | 161.711 | 202.300 |
| | 1 | 94.421 | 108.111 | 115.342 | 139.544 | 132.680 | 165.823 |
| | 2 | 83.662 | 95.781 | 102.233 | 123.644 | 117.583 | 147.221 |
| | 5 | 80.313 | 92.812 | 100.433 | 120.810 | 115.283 | 145.342 |

5.2. Linear loading - clamped boundary conditions

Table (4) represents the effect of increasing the thickness-to-radius ratio on increasing the critical temperature in case of linear thermal loading condition. This table clearly shows that the trend of variation of the critical temperature is confirmed in Table 2 as well. The comparison of the two tables indicates that the rate of increase is higher for linear thermal loading as compared to the uniform one. Nevertheless, for small h/R and non-zero quantities of n , this trend is reversed.

Table 4 Variation of the critical buckling temperature ($^{\circ}K$) for the clamped plate in case of linear thermal loading, $(k_w, k_g)=0$

| μ | h/r | n | | | | |
|-------|-------|----------|----------|---------|---------|---------|
| | | 0 | 0.5 | 1 | 2 | 5 |
| 0.50 | 0.010 | 8.480 | 2.652 | 0.843 | 0.351 | 0.305 |
| | 0.150 | 26.531 | 12.780 | 8.673 | 6.640 | 6.062 |
| | 0.020 | 50.740 | 26.521 | 19.272 | 15.472 | 14.763 |
| | 0.030 | 67.592 | 37.052 | 27.860 | 22.873 | 20.160 |
| | 0.040 | 220.670 | 122.801 | 93.292 | 77.060 | 73.893 |
| 0.70 | 0.010 | 15.173 | 4.750 | 1.511 | 0.523 | 0.451 |
| | 0.150 | 43.640 | 21.021 | 14.272 | 10.920 | 10.284 |
| | 0.020 | 93.142 | 48.682 | 35.372 | 28.403 | 27.932 |
| | 0.030 | 234.551 | 128.581 | 96.700 | 79.370 | 77.370 |
| | 0.040 | 432.540 | 240.714 | 182.851 | 151.231 | 150.681 |
| 0.9 | 0.010 | 22.772 | 7.123 | 2.274 | 0.650 | 0.563 |
| | 0.150 | 76.230 | 36.710 | 24.922 | 19.110 | 18.713 |
| | 0.020 | 151.082 | 78.962 | 57.371 | 46.122 | 44.920 |
| | 0.030 | 364.921 | 200.153 | 150.444 | 123.480 | 122.212 |
| | 0.040 | 664.300 | 369.6821 | 280.832 | 231.901 | 231.493 |
| 1 | 0.010 | 29.912 | 8.273 | 2.980 | 0.731 | 0.660 |
| | 0.150 | 92.291 | 43.632 | 30.171 | 23.101 | 21.860 |
| | 0.020 | 179.632 | 93.131 | 68.214 | 54.772 | 53.790 |
| | 0.030 | 429.171 | 234.563 | 176.932 | 145.231 | 143.053 |
| | 0.040 | 778.532 | 432.552 | 329.124 | 271.880 | 264.810 |
| 1.5 | 0.010 | 62.510 | 19.562 | 15.032 | 10.080 | 9.840 |
| | 0.150 | 197.501 | 95.113 | 71.954 | 57.064 | 55.860 |
| | 0.020 | 355.670 | 185.882 | 151.512 | 118.450 | 115.042 |
| | 0.030 | 931.304 | 510.520 | 390.711 | 322.131 | 315.793 |
| | 0.040 | 1572.631 | 875.182 | 671.040 | 555.614 | 551.260 |
| 2 | 0.010 | 101.694 | 31.813 | 30.290 | 22.394 | 21.350 |
| | 0.150 | 325.971 | 156.981 | 124.653 | 100.291 | 98.140 |
| | 0.020 | 617.394 | 322.672 | 251.290 | 205.614 | 205.380 |
| | 0.030 | 1511.632 | 828.650 | 639.903 | 528.782 | 524.433 |
| | 0.040 | 2266.591 | 1261.371 | 971.882 | 805.652 | 803.570 |

Table (5) shows the effect of the presence of elastic foundation on the critical buckling temperature for linear loading condition. As it is observed, the critical temperature decreases when the elastic foundation coefficient gets more values.

In order to validate the present solutions, we compare them with the results obtained from the study conducted by Ghiasian et al. [25]. Table (6) shows the critical temperature for the clamped plate without elastic foundation. The values are $E_m=201 \text{ Gpa}$, $\alpha_m=12.33 \times 10^{-6} \text{ K}^{-1}$, $E_c=350 \text{ Gpa}$, and $\alpha_c=5.87 \times 10^{-6} \text{ K}^{-1}$, and the Poisson's ratio is $\nu=0.3$.

Table 5 Effect of elastic foundation on the critical buckling temperature for linear temperature rise, $h/R=0.020$

| μ | n | (k_w, k_g) | | | | | |
|-------|------|--------------|---------|---------|---------|----------|----------|
| | | (0,0) | (100,0) | (200,0) | (500,0) | (100,10) | (200,20) |
| 0.7 | 0 | 93.140 | 107.621 | 114.160 | 136.523 | 142.211 | 163.120 |
| | 0.50 | 48.681 | 56.560 | 60.163 | 71.362 | 74.610 | 54.442 |
| | 1 | 35.371 | 41.212 | 43.363 | 51.851 | 54.661 | 61.681 |
| | 2 | 28.401 | 33.280 | 35.222 | 41.634 | 43.362 | 49.742 |
| | 5 | 27.931 | 33.111 | 35.022 | 41.411 | 43.331 | 48.040 |
| 1 | 0 | 179.632 | 207.291 | 219.152 | 264.062 | 274.664 | 312.511 |
| | 0.5 | 93.131 | 105.611 | 111.761 | 134.290 | 141.562 | 158.322 |
| | 1 | 68.210 | 79.390 | 83.900 | 100.812 | 104.361 | 118.692 |
| | 2 | 54.772 | 64.361 | 66.274 | 80.2380 | 85.331 | 117.322 |
| | 5 | 54.390 | 65.830 | 65.182 | 79.012 | 83.030 | 98.190 |
| 1.5 | 0 | 355.670 | 416.954 | 439.520 | 521.322 | 545.123 | 622.433 |
| | 0.5 | 185.880 | 217.913 | 229.722 | 272.253 | 284.620 | 325.342 |
| | 1 | 151.510 | 177.622 | 187.244 | 222.363 | 234.163 | 265.253 |
| | 2 | 118.450 | 138.863 | 146.370 | 173.632 | 180.853 | 207.283 |
| | 5 | 117.841 | 140.721 | 145.342 | 172.884 | 180.052 | 206.220 |

Table 6 Comparison of the results of the present study with those of Ref. [25]

| | n | $\frac{h}{R}$ | | | | | | | |
|-----------|-----|---------------|-------|--------|-------|---------|-------|---------|-------|
| | | 0.010 | p.e.* | 0.020 | p.e.* | 0.030 | p.e.* | 0.040 | p.e.* |
| Present | 0 | 12.731 | 1.11 | 49.910 | 0.893 | 112.291 | 0.884 | 199.644 | 0.857 |
| Ref. [25] | | 12.591 | | 50.360 | | 113.293 | | 201.369 | |
| Present | 0.5 | 9.532 | 2.88 | 37.001 | 0.146 | 83.142 | 0.264 | 141.149 | 0.015 |
| Ref. [25] | | 9.265 | | 37.055 | | 83.362 | | 141.170 | |
| Present | 1 | 8.536 | 2.41 | 33.120 | 0.654 | 74.921 | 0.105 | 133.358 | 0.038 |
| Ref. [25] | | 8.335 | | 33.338 | | 75.000 | | 133.307 | |
| Present | 2 | 7.761 | 0.792 | 30.554 | 0.792 | 68.752 | 0.768 | 122.314 | 0.676 |
| Ref. [25] | | 7.700 | | 30.798 | | 69.284 | | 123.147 | |
| Present | 5 | 7.370 | 2.718 | 28.486 | 0.732 | 64.326 | 0.356 | 114.452 | 0.253 |
| Ref. [25] | | 7.175 | | 28.696 | | 64.556 | | 114.742 | |

*The percentage of error

5.3. Effect of boundary conditions on the critical buckling temperature

In circular FG plates, due to the stretching-bending coupling exposed to uniform thermal loading, the asymmetric material distribution induces the pre-buckling thermal moments. Therefore, the bifurcation buckling may not occur, and the buckling critical temperature is not available [26]. However, the plates with clamped edges can tolerate the bending moments and remain in an un-deformed configuration. In this study, we consider that the variation of the critical buckling temperature is tabulated for a homogeneous orthotropic circular plate under simply supported boundary condition, as shown in Table 7.

Table 7 Variation of the critical buckling temperature for a homogeneous orthotropic circular plate under simply supported boundary condition and linear and uniform temperature rises

| $\frac{h}{R}$ | μ | | | | | |
|---------------|---------|--------|---------|--------|---------|--------|
| | 0.50 | | 0.70 | | 0.90 | |
| | Uniform | Linear | Uniform | Linear | Uniform | Linear |
| 0.010 | 2.06 | 4.12 | 3.72 | 7.45 | 5.37 | 10.74 |
| 0.150 | 4.63 | 9.26 | 8.37 | 16.74 | 12.08 | 24.17 |
| 0.020 | 8.23 | 16.46 | 14.88 | 29.76 | 21.48 | 42.96 |
| 0.030 | 18.52 | 37.04 | 33.49 | 66.97 | 48.34 | 96.67 |
| 0.040 | 32.92 | 65.84 | 59.53 | 119.04 | 85.93 | 171.86 |
| $\frac{h}{R}$ | 1.0 | | 1.50 | | 2.00 | |
| | Uniform | Linear | Uniform | Linear | Uniform | Linear |
| 0.010 | 6.03 | 12.07 | 10.31 | 20.76 | 15.80 | 34.28 |
| 0.150 | 13.58 | 27.15 | 23.29 | 46.24 | 39.64 | 79.55 |
| 0.020 | 24.13 | 48.27 | 41.46 | 73.41 | 70.46 | 126.31 |
| 0.030 | 54.30 | 108.60 | 93.10 | 187.88 | 159.56 | 318.12 |
| 0.040 | 96.54 | 193.08 | 165.95 | 333.14 | 253.41 | 569.59 |

6. CONCLUSIONS

In this paper, we have analyzed the thermal buckling of a circular FGM plate resting on the Pasternak elastic foundation and subjected to uniform and linear thermal loading by employing the differential transform method (DTM) to obtain the solutions. Some remarkable conclusions obtained in this study are as follows:

- By utilizing the differential transform method (DTM), the governing differential equation in the thermal buckling can be transformed to algebraic equations in the sub-domains. DTM is capable of deriving the analytical solution to determine the critical buckling temperature for FG orthotropic plate resting on two-parameter (Pasternak) foundations.
- Comparing the results with those existing in the literature indicates that DTM is a fast convergent, precise, and cost-efficient tool to analyze the thermal buckling behavior of functionally graded plates.

- By increasing volume fraction n , the flexural rigidity is reduced as the plate becomes more metal-rich. Consequently, the critical buckling temperature is reduced. This leads to the conclusion that for a ceramic-rich plate, the value of the critical temperature is the maximum.
- The present study has investigated the effect of the presence of elastic foundation, as a controlling parameter, on the critical temperature for two boundary conditions, namely clamped and simply supported edges. The numerical results indicate that increasing the parameters of elastic foundation increases the critical temperature. Hence, the critical temperature in buckling can be adjusted effectively.
- According to the results, when the temperature rises linearly, the critical temperature gets higher values in comparison with the uniform temperature rise.
- As the ratio of Young's modulus in the circumferential direction to that in the radial one increases, the plate demonstrates a more resistant behavior. Therefore, the critical buckling temperature increases.

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