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NEW CLASS OF DIGITAL MALMQUIST-TYPE ORTHOGONAL FILTERS BASED ON THE GENERALIZED INNER PRODUCT; APPLICATION TO THE MODELING DPCM SYSTEM

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Abstract. *A new class of cascade digital orthogonal filters of the Malmquist type based on bilinear transformation for mapping poles to zeroes and vice versa is presented in this paper. In a way, it is a generalization of the majority of the classical orthogonal filters and some newly designed filters as well. These filters are orthogonal with respect to the generalized inner product which is actually a generalization of the classical inner product. Outputs of these filters are obtained by using polynomials orthogonal with respect to the new inner product. The main quality of these filters is that they are parametric adaptive. The filter with six sections is practically realized in the Laboratory for Modeling, Simulation and Control Systems. Performances of the designed filter are proved on modeling and identification of the system for differential pulse code modulation. Real response and response from the proposed filter are compared with regard to the chosen criteria function. Also, a comparative analysis of the proposed filter with some existing filters is performed.*

Key Words: *Digital Orthogonal Filters, Malmquist Functions, Müntz Polynomials, Bilinear Transformation, Inner Product, Differential Pulse Code Modulation*

1. INTRODUCTION

The majority of the already existing classes of orthogonal rational functions are obtained by using one of the two simple transformations. The first one is a linear transformation $s \rightarrow as+b$ which was used for obtaining classical orthogonal functions as Legendre, Müntz-Legendre, Jacobi, and some other classes of orthogonal functions and appropriate filters as almost orthogonal [1–3], quasi-orthogonal [4, 5], and some generalized classes of

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orthogonal functions and filters [6]. The second transformation is a reciprocal transformation of poles to zeroes $s \to 1/s$ (Malmquist, Laguerre functions) [7, 8].

In [9] and [10] the relation between generalized Malmquist functions and new Müntz polynomials is given. In [11] this relation is generalized using bilinear transformation for mapping poles to zeroes and *vice versa*. A new class of Müntz polynomials, orthogonal with respect to the inner product obtained by this transformation of poles to zeroes, is derived. The filters based on the generalized Malmquist orthogonal functions are designed in [12]. These filters are orthogonal with respect to the new inner product which is derived from symmetric reciprocal transformation $s \to b/(cs)$ which is a generalization of the reciprocal transformation given in [7, 8]. Two types of these filters are designed in [12], namely, analogue and digital ones. Further generalization of these orthogonal functions is based on the above mentioned symmetric bilinear transformation $s \to (as+b)/(cs-a)$ [11]. This transformation includes the above mentioned linear and reciprocal transformations. In [13], analogue orthogonal filters based on these functions are designed.

Since the first pulse-code modulation transmission of digitally quantized speech, in World War II, digital signal processing (DSP) began to proliferate to all areas of human life. Classical digital systems are known to possess poor parameters under finite-precision arithmetic, like frequency response sensitivity to changes in the structural parameters, noise, inner oscillations, and limit cycles [14]. These effects have led to development of wave filters [15] and orthogonal filters. Rapid development of the digital orthogonal filters started in early 1980's [16, 17]. The most common approaches to the orthogonal filter synthesis are transfer function decomposition [16, 18] and the state space approach [19, 20]. Various realizations of the digital orthogonal filters developed in the last several decades. For example, in [18], the development of the pipelined structure of digital orthogonal filters started. It continued in the 1990's till present days [14]. When the two most common approaches are compared, state-space realization has advantages in the case of MIMO filters; it provides a better insight into their structure. However, in this paper the authors use transfer function realization of the proposed filters that is suitable for our purpose of modeling real systems. It was noted that IIR digital filter can be applied to modern DSP applications (e.g. mobile communication), to multichannel prediction, etc. Because of all this, the main goal of this paper is design and later practical realization of a new class of digital orthogonal filters.

In this paper a bilinear transformation for mapping poles to zeroes and *vice versa* in the case of digital system is performed: $z \to (az+b)/(cz-a)$. In this way, the filters, orthogonal with respect to a more generalized inner product, are obtained both in z-domain and discrete-time domain. The class of discrete orthogonal polynomials derived from [9, 10] will be used for determining outputs of the proposed filters. The proof of this orthogonality is given in the Appendix. In this way, the designed orthogonal digital filter is a generalization of the majority of classical orthogonal filters (Legendre, Müntz-Legendre, Laguerre, Jacobi, Malmquist) as well as the most recently designed filter based on the reciprocal transformation [12]. The generalized Malmquist filters designed in this paper are parametric adaptive.

The proposed digital orthogonal filter is practically realized in the Laboratory for Modeling, Simulation and Control Systems [21] and it will be applied to modeling of the well-known digital system for signal transmission, Differential Pulse Code Modulation (DPCM) system [22–24]. Performances of the new filter are verified by comparing them with some other classes of digital orthogonal filters.

The rest of the paper is organized as follows. In Section 2, new digital orthogonal filters based on bilinear transformation are developed; first theoretically, later by simulation; finally, they are practically realized. The proposed filter is used to modeling a linear part of the DPCM system in Section 3. Comparison between the practically realized filter and some other filters is performed in Section 4. In Section 5, the authors give conclusions and discuss a possibility for further development in this area. Finally, at the end, the Appendix includes the proof of orthogonality of these filters in discrete-time domain, and also relations for the inner products and norms.

2. Digital Malmquist-Type Orthogonal Filters: Mathematical Background, Design and Practical Realization

The generalized Malmquist filters based on bilinear transformation in discrete-time domain can be derived from corresponding generalized Malmquist filters in continuous-time domain [13] with the same procedure given in [12]. Namely, operator *s* (operator of differentiation) is substituted by operator *z* (operator of prediction) to design corresponding digital filters. The transfer function of new digital filters has the following form:

$$W_n(z) = \frac{z}{z-\alpha_0}\prod_{k=1}^{n}\frac{z-\alpha_{k-1}^*}{z-\alpha_k},\quad \alpha_k^* = \frac{a\alpha_k + b}{c\alpha_k - a},\quad \alpha_k \in R. \tag{1}$$

Cascade scheme based on (1) is given in Fig. 1, where K is the number of samples, and T is a sample period. The authors assume the sample period is one second because it is not important for further analysis, i.e. $K \equiv KT$ (discrete time).

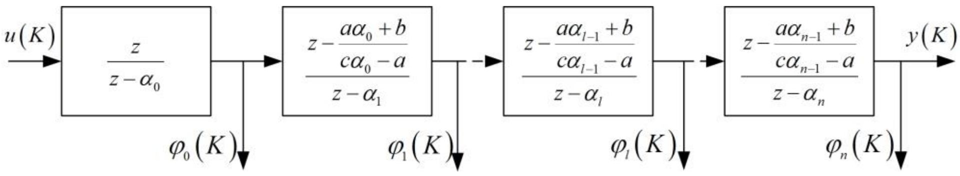

Fig. 1 Block diagram of a digital orthogonal filter based on bilinear transformation

A sequence of functions on the outputs of cascades of the proposed digital filter (1) obtained mathematically corresponds to responses by simulation and from practically realized orthogonal digital filter. These outputs for the specific case a=0, b=1, c=1 (reciprocal transformation) are already given (α_0=1/2, α_1=1/3, α_2=1/4, α_3=1/5) in [12] illustratively.

These filters are orthogonal in complex z-plane:

$$(W_n(z)W_m(z)) = \frac{1}{2\pi i}\oint_{\Gamma} W_n(z)W_m^*(z)ds = N_n{}^2\delta_{n,m}\,, \tag{2}$$

where $W_n(z) = \dfrac{z}{z-\alpha_0}\displaystyle\prod_{k=1}^{n}\dfrac{z-\dfrac{a\alpha_{k-1}+b}{c\alpha_{k-1}-a}}{z-\alpha_k}$, $\delta_{n,m}$ represents Kronecker symbol, and contour Γ surrounds all the poles of $W_n(s)$.

The proof of orthogonality is similar as in the case of continuous-time systems [11, 13]. If in transfer function $W_n(z)$ bilinear transformation is performed, the authors obtain $W_n^*(z)$ whose poles are equal to zeroes of $W_n(z)$ and zeroes of $W_n^*(z)$ are equal to poles of $W_n(z)$. Thereby, all poles of $W_n^*(z)$ are outside contour Γ, and zeroes of $W_n^*(z)$ are inside contour Γ.

If $m \neq n$ due to symmetry of the bilinear transformation, all the poles of the integrand (2) that lie inside contour Γ are annulled with appropriate zeros of $W_m^*(s)$, so the contour integral (2) is equal to zero. In the case of $m=n$, there exists one first-order pole inside contour Γ. After applying the Cauchy theorem, the following expression is obtained: $(W_n(s), W_m(s))=N_n^2 \neq 0$. Finally, all the expressions stated above imply (2).

Practical realization of the generalized Malmquist digital orthogonal filter is shown in Fig. 2.

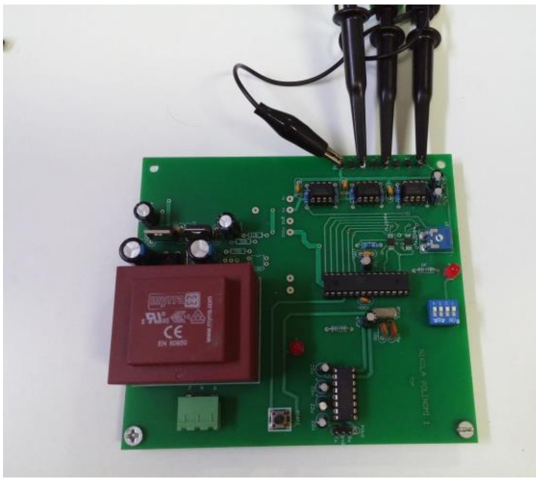

Fig. 2 Practical realization of the generalized digital orthogonal filter based on bilinear transformation – printed circuit board

Outputs from the filter in z-domain $\Phi_l(z)=U(z)W_l(z)$, l=0, 1, 2, ..., n are orthogonal [12, 13, 25]:

$$(\Phi_n(z)\Phi_m(z)) = N_n{}^2\delta_{n,m}\,. \tag{3}$$

Outputs from this digital filter in time domain are obtained using inverse z-transformation:

$$\varphi_l(K) = Z^{-1}\{\Phi_l(z)\} = \frac{1}{2\pi i}\oint_{\Gamma}\Phi_l(z)z^{K-1}dz\,, \tag{4}$$

where contour Γ surrounds all the poles of $W_l(z)$. This contour can be obtained by mapping s- to z-domain: $\Gamma=\{c|z|^2\text{-2aRe}z\text{-}b=0\}$[25].

A new inner product for the filter outputs is:

$$J_{n,m} = (\varphi_n(K),\varphi_m(K))_{\otimes} = (\varphi_n^*(K),\varphi_m^*(K)) = \sum_{K=1}^{\infty}\varphi_n^*(K)\varphi_m^*(K) = N_n{}^2\delta_{n,m}\,. \tag{5}$$

The relation for $J_{n,m}$ and N_n^2 is given in the Appendix. Otherwise, this inner product is generated in the practically realized filter thanks to its structure.

These filters are adjustable. It means that their parameters can be changed: numerical values of poles and parameters of bilinear transformation. For example, for c=0 well-known classical orthogonal filters based on linear transformation are obtained, and for a=0 orthogonal filters based on the reciprocal transformation (Malmquist type). Finally, in the case of $a \neq 0$, $b \neq 0$, $c \neq 0$ the most generalized orthogonal digital filter can be obtained.

This digital cascade orthogonal filter will be practically applied to modeling a prediction filter in a well-known digital system in telecommunications, the DPCM system [22]. Because of the cascade structure of the realized filter, i.e. a possibility to append more sections, the authors suppose that the filter will be suitable for modeling DPCM systems which can theoretically have an arbitrary high order predictor. A cascade structure is already verified in the case of appropriate class of analogue generalized Malmquist filters [4].

3. Application to Modeling DPCM System

A new digital cascade orthogonal filter based on bilinear transformation will be applied to modeling and signal identification of a linear part of the DPCM system [22]. The DPCM is a well-known and commonly used technique for signal transmission in telecommunications. This system has a wide usage in different areas, starting from speech and image coding to the latest medical research [23]. An estimate, i.e. a prediction of the present value of the input signal is based on the knowledge of its earlier values [24]. That is why one of the most important parts of every DPCM and ADPCM (Adaptive Differential Pulse Code Modulation) is a predictor (a linear part of the system).

For modeling of the prediction filter the authors use an adjustable model based on the proposed generalized digital filter (Fig. 1). A block diagram of the digital orthogonal adjustable model based on bilinear transformation is shown in Fig. 3. In this case the authors use a filter with six sections and real poles $\alpha_k^* = (a\alpha_k + b)/(c\alpha_k - a)$, k=0, 1, ..., n.

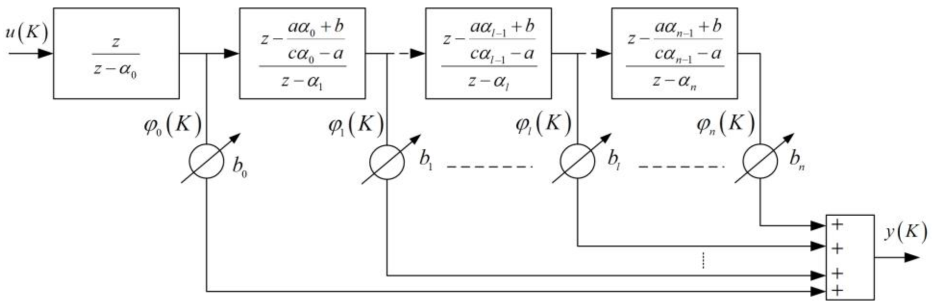

Fig. 3 Block diagram of an adjustable model with the proposed orthogonal digital filter based on bilinear transformation

It can be noticed from Fig. 3 that the orthogonal model output is:

$$y_M(K) = \sum_{k=0}^{n} b_k \varphi_k(K)\,, \tag{6}$$

where K is the number of samples, n=5 in our case.

The desired model of the linear part of DPCM system (exactly, the prediction filter in the encoder) is obtained by adjusting the following parameters: α_k (k=0,1,…,5), summation coefficients b_k (k=0,1,…,5), and parameters of bilinear transformation a, b, and c. In the case of modeling a particular unknown system, the parameters of the model should be adjusted in such a way that the model (Fig. 3) corresponds as closely as possible to the unknown system. The process of modeling is performed in the well-known manner by introducing the same input to the system itself and to its adjustable model based on the new cascade orthogonal digital filter [12]. This input signal is shown in Fig. 4.

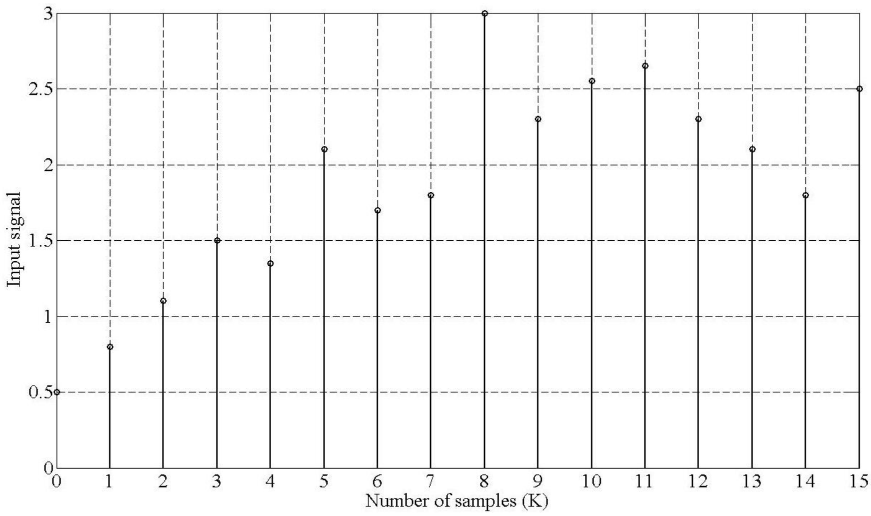

Fig. 4 The input of DPCM linear part and the adjustable model

The next step is measuring the outputs from system $y_s(t)$ and filter $y_M(t)$ and calculating the mean squared error (criteria function):

$$J = \frac{1}{N}\sum_{K=0}^{N}(y_S(K) - y_M(K))^2 \ . \tag{7}$$

Optimal values of unknown parameters which lead to minimization of mean squared error can be obtained by using genetic algorithm with J is fitness function.

The specific genetic algorithm used in experiments has the following parameters: initial population of 1000 individuals, a number of generations of 300, a stochastic uniform selection, a reproduction with 12 elite individuals, and Gaussian mutation with shrinking. The used structure of chromosome was with eight parameters coded by real numbers: a, b, c, b_0, b_1, b_2, b_3, b_4, and b_5 (Eq. 1). Poles a_0, a_1, a_2, a_3, a_4, a_5 are also adjustable, and in these experiments their values are fixed. A series of experiments can be performed with other values of poles until the obtained model fill requirements in advance. The main goal of the experiment was to obtain the best model of the unknown system in regard to the criteria function, i.e. mean squared error.

The original signal (output from the DPCM linear part) and the signal from the adjustable model based on the orthogonal digital filter of the generalized Malmquist type are given in Fig. 5.

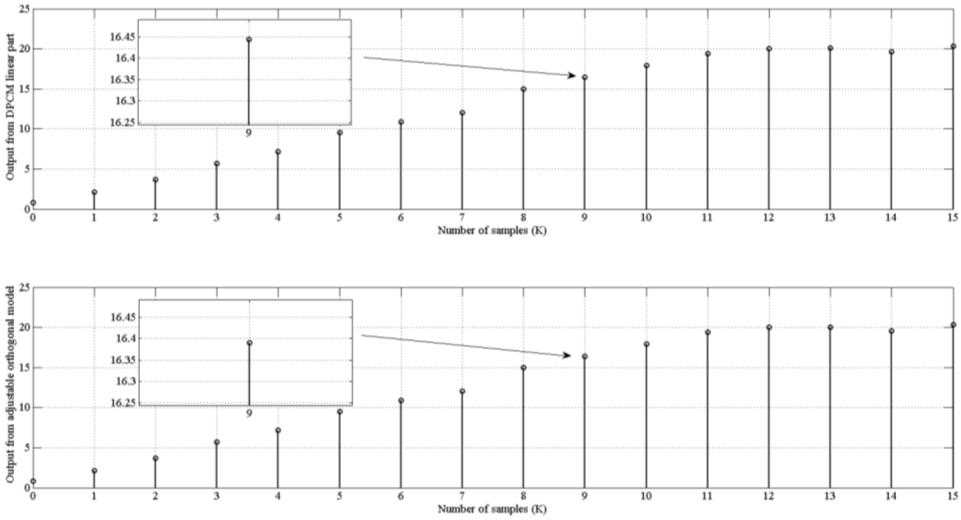

Fig. 5 Outputs from the DPCM linear part and the adjustable model

The authors performed experiments with six sections. In the case of the digital orthogonal filter based on reciprocal transformation the filter with six sections is verified as better related to the criteria function than one with four or five sections [12].

Obtained optimal values for parameters of adjustable orthogonal model are presented in Table 1. Mean squared error is: J_{min}=5.7216·10^{-3}.

Table 1 Values for parameters of the adjustable orthogonal model

Parameters	Numerical values
b_0	0.56713
b_1	0.59111
b_2	0.32604
b_3	0.61674
b_4	-0.21408
b_5	0.25053
α_0	0.91286
α_1	0.87552
α_2	0.77119
α_3	0.82101
α_4	-0.15647
α_5	0.79444
A	0.28327
B	0.09449
C	0.73071

In Fig. 5 one representative sample is zoomed for illustrative purposes because of very small differences between sample values. Measured outputs in discrete-time periods (sample periods) of the prediction filter and the adjustable model based on the new digital orthogonal filter are given in Table 2.

Table 2 Obtained outputs from the DPCM prediction filter and the adjustable model based on the digital orthogonal filter

K	Output from the DPCM prediction filter	Output from the adjustable orthogonal model
0	0.82500	0.82500
1	2.09125	2.09215
2	3.68806	3.68806
3	5.71962	5.70912
4	7.18302	7.18302
5	9.53761	9.51500
6	10.89502	10.86011
7	12.05639	12.03209
8	14.99215	14.97022
9	16.44441	16.39001
10	17.93314	17.91510
11	19.39597	19.36826
12	19.98442	19.97384
13	20.04930	20.03347
14	19.59736	19.57832
15	20.33942	20.31451

From Fig. 5 and Table 2 a high level of matching can be noticed between signals from the DPCM linear part and the proposed orthogonal digital filter.

Finally, the model of the prediction filter in the DPCM encoder is formed as:

$$W_M(z)=\sum_{k=0}^{5} b_k \prod_{i=0}^{k} \frac{z-\alpha_{i-1}^{*}}{z-\alpha_i},\ \ \alpha_{-1}^{*}=0, \tag{8}$$

where $\alpha_k^*=(a\alpha_k+b)/(c\alpha_k-a)$, and appropriate values of parameters are given in Table 1. The proposed filter model (8) using numerical values in Table 1 can be written in the following form:

$$W_M(z)=0.619\left(\frac{0.527z^5+0.661z^4+0.893z^3+1.131z^2+1.432z+1}{0.077z^6+0.125z^5+0.592z^4+0.958z^3+1.732z^2+1.214z+1}\right). \tag{9}$$

The relation (9) for the transfer function of the system is more suitable for control system theory analysis, while the relation (8) is more suitable for system modeling.

4. Comparative Analysis between the Proposed Filter and Some Other Classes of Digital Orthogonal Filters

In order to verify the quality of the model based on the new filter with six sections, a comparison with the models based on the some already existing digital orthogonal filters (generalized Legendre and generalized Malmquist filters) is performed. The criteria function is mean squared error again and the number of sections is six. The transfer functions of all the filters used in experiments are shown in Table 3 (α_{i-1} is assumed to have constant value equal to zero).

Table 3 Transfer functions of digital orthogonal filters used in experiments

Orthogonal filter type	Transfer function
Legendre (Müntz-Legendre)	$W(z)=\sum_{k=0}^{5} b_k \prod_{i=0}^{k} \frac{(z-\alpha_{i-1}+\lambda)}{(z-\alpha_i)}$
Generalized Malmquist	$W(z)=\sum_{k=0}^{5} b_k \prod_{i=0}^{k} \frac{z-\frac{b}{\alpha_{i-1}}}{z-\alpha_i}$
Filter based on bilinear transformation	$W(z)=\sum_{k=0}^{5} b_k \prod_{i=0}^{k} \frac{z-\frac{a\alpha_{i-1}+b}{c\alpha_{i-1}-a}}{z-\alpha_i}$

The outputs of the models based on these filters are calculated as $y_M(t)=\sum_{l=0}^{5} b_l \varphi_l(t)$.

Table 4 Values for parameters of the adjustable orthogonal models based on new and existing types of digital orthogonal filters

Criteria function value and orthogonal model	i	0	1	2	3	4	5
J=16.370·10^{-3} orthogonal model with **Legendre filter** (λ=0.67)	α_i	0.1677	0.3162	0.4777	0.6164	0.7011	0.8168
	b_i	0.7821	1.6234	0.9102	0.8648	0.5519	0.3271
J=13.022·10^{-3} orthogonal model with **generalized Malmquist filter** (b=0.92)	α_i	0.2681	0.1155	-0.1733	0.3124	0.2178	0.1665
	b_i	3.6410	-0.9131	2.1616	1.9872	0.8764	0.3558
J=5.721·10^{-3} orthogonal model with a new filter based on **bilinear transformation** (a=0.2834, b=0.094, c=0.731)	α_i	0.9129	0.8755	0.7712	0.8210	-0.1565	0.7944
	b_i	0.5671	0.5911	0.3260	0.6167	-0.2141	0.2505

The structure of chromosome that is used in experiments is with 6 standard parameters coded by real numbers: $\alpha_0, \alpha_1, \ldots, \alpha_5$, and one additional; for Legendre filter λ and for generalized Malmquist filter b. In Table 4 the obtained parameters for proposed and existing filters are given.

From Table 4 it can be seen that the mean squared error is much bigger for the generalized Malmquist [12] and the Legendre digital orthogonal filters than for the filter presented in this paper. The excellent matching between the system output and the output of the adjustable model based on the proposed filter has shown the quality and need for new filters described in this paper.

Of course, the model of the linear part of DPCM system can be derived by using other methods, but in this paper the authors used new orthogonal filter to verify its performances.

5. CONCLUSION AND FUTURE WORK

In this paper the authors gave mathematical background, simulation and practical realization of new digital orthogonal filters based on bilinear transformation of poles to zeroes and *vice versa*. It is an extension of generalizations of traditional orthogonal filters starting with filters based on reciprocal transformation. All good performances of already existing filters are included in this class of filters.

The great quality of this filter is parametric adaptivity, i.e., possibility of adjusting values of poles and parameters of bilinear transformation. Also, it is demonstrated that by setting specific values for parameters of bilinear transformation, most of the classes of realized filters can be obtained. In the case when c=0, these filters degenerate into classical orthogonal filters (Legendre, Laguerre, Jacobi), and when is a=0, they degenerate into classical Malmquist and generalized Malmquist filters.

In the future work, the authors could try to derive appropriate classes of orthogonal filters with complex poles which in some practical cases could be even better. Further generalization could also be in the usage of more general symmetric transformations than the bilinear one. In this way, the study of generalized class of orthogonal analogue and digital filters will be concluded.

APPENDIX

Polynomials obtained by using bilinear transformation of poles to zeroes are orthogonal on the contour:

$$\Gamma = \left\{ c\left|z\right|^2 - 2a\,\mathrm{Re}\,z - b = 0 \right\}. \tag{A1}$$

Contour (A1) is a circle with radius $R = \sqrt{\left(\frac{a}{c}\right)^2 + \frac{b}{c}}$ and center in $\left(\frac{a}{c}, 0\right)$.

Using transformation $z=R_z^*+a/c$, i.e., $z^*=(z-a/c)/R$, the circle is mapped into the unit circle with the center at the origin.

The model of these polynomials is shown in Fig. 1.

Outputs $\varphi_m(z)$ for the unit input are:

$$\varphi_0(z) = \frac{z}{z-\alpha_0} \quad ,..., \quad \varphi_m(z) = \frac{1}{z-\alpha_0}\prod_{i=1}^{m}\frac{z - \frac{a\alpha_i + b}{c\alpha_i - a}}{z-\alpha_i}, \; m = 1,2,...,n\,. \tag{A2}$$

By development of Eq. (A2) in partial fraction it is obtained:

$$\varphi_m(z) = \sum_{j=0}^{m}\frac{A_{m,j}}{z-\alpha_j}, \tag{A3}$$

where : $A_{m,j} = \lim_{z\to\alpha_i}(z-\alpha_j)\varphi_m(z)$, $A_{m,j} = \dfrac{\prod_{i=0}^{m-1}\left(z - \frac{a\alpha_i + b}{c\alpha_i - a}\right)}{\prod_{\substack{i=0\\ i\neq j}}^{m}(\alpha_j - \alpha_i)}$.

By using inverse z-transformation of $\varphi_m(z)$ outputs in time-domain are obtained:

$$\varphi_m(K) = \sum_{j=0}^{m} A_{m,j}\alpha_j^{-(K-1)} . \tag{A4}$$

By applying linear transformation onto linear systems, orthogonality is held.

When linear transformation $z=R_z^*+a/c$ is used, the region of orthogonality of filters based on bilinear transformation is mapped into the unit circle with the center at the origin, i.e. these polynomials are mapped into classical discrete-time orthogonal polynomials where orthogonality in the classical sense is valid:

$$\oint_{\Gamma} \varphi_n(z) \otimes \varphi_m(z) dz = \oint_{\Gamma_1} \varphi_n(z^*)\varphi_m(z^*) dz^* , \tag{A5}$$

where: $\varphi_n(z^*) = \sum_{j=0}^{n} \frac{A_{n,j}}{Rz^* + \frac{a}{c} - \alpha_j}, \ \varphi_m(z^*) = \sum_{j=0}^{m} \frac{A_{m,j}}{Rz^* + \frac{a}{c} - \alpha_j}$.

By using inverse z-transformation it is obtained:

$$\varphi_n^*(K) = \sum_{j=0}^{n} A_{n,j}\alpha_j^{-(K-1)}, \ \ \varphi_m^*(K) = \sum_{j=0}^{m} A_{m,j}\alpha_j^{-(K-1)} . \tag{A6}$$

The norm for outputs of filters obtained by bilinear transformation is:

$$N_n^{\ 2} = (\varphi_n(K), \varphi_n(K))_{\otimes} = (\varphi_n^*(K), \varphi_n^*(K)) = \sum_{K=1}^{\infty} \varphi_n^*(K)\varphi_n^*(K) . \tag{A7}$$

Therefore, a new inner product for outputs of the filter is obtained:

$$J_{n,m} = (\varphi_n(K), \varphi_m(K))_{\otimes} = (\varphi_n^*(K), \varphi_m^*(K)) = \sum_{K=1}^{\infty} \varphi_n^*(K)\varphi_m^*(K) = N_n^{\ 2}\delta_{n,m} . \tag{A8}$$

Let the authors note that relations for the new inner product and norm N_n^2 are used in mathematical analysis of these filters. In the case when the filters are practically realized (see Case Study), these relations are calculated thanks to their structure.

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