FINITE-ELEMENT MODELLING OF THE TYMPANIC MEMBRANE RETRACTION POCKET UNDER NEGATIVE PRESSURE IN THE TYMPANIC CAVITY

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Abstract. The finite-element calculation of the static stress-strain state of the middle ear was made in this paper. The malleus, incus and stapes models were constructed on the basis of tomographic data. The tympanic membrane model was obtained using the equations of elliptic hyperboloids. The tympanic membrane consists of the pars tensa and pars flaccida, which have different thicknesses and elasticity moduli. Absolute deformations of the tympanic membrane were defined at different values of negative pressure in the tympanic cavity. The critical values of elastic modulus for the pars tensa posterosuperior quadrant were found for the point at which the tympanic membrane touches the auditory ossicles. Obtained results can be used to predict the thickness of a cartilaginous graft which is overlaid on the posterosuperior quadrant of the pars tensa in order to eliminate the retraction pocket.

Key Words: Middle Ear, Tympanic Membrane, Retraction Pocket, Finite Element Analysis, Strain-state State, Negative Pressure

1. INTRODUCTION

The retraction pocket of the tympanic membrane (TM) is a clinical manifestation of atelectatic otitis media. For many reasons this pathology of the middle ear deserves special attention. In particular, it is necessary to correctly assess the nature of the pathogenesis and best formulate a treatment strategy to repair retraction pockets that have formed as a result of the cholesteatoma growth in the middle ear [1, 2]. Two factors may lead to the formation of the TM retraction: a) pressure imbalance in the middle ear cavity
The thickness of the middle ear TM itself (thinning and changing physical properties) [3]. The TM consists of two parts [4]: pars tensa (which is mainly responsible for sound transmission) and pars flaccida (for the detection of pressure differentials). The elasticity of the pars tensa is associated with its fibrous layer, which is absent in the pars flaccida. The posterosuperior quadrant of the pars tensa has a weak network of only circular collagen fibers, which is why this part of the TM has a thickness of about 60 µm, while the thickness of the rest of pars tensa is approximately 90 µm [5, 6]. Therefore, the pars flaccida and the posterosuperior quadrant of the pars tensa are the most anatomically vulnerable areas of the TM for the formation of retraction pockets. The process of retraction is accompanied with irreversible changes in the TM structure and consequently, with a significant reduction of its elastic characteristics and thickness. For this reason, the weakened parts of pars tensa come into contact with the underlying auditory ossicles (the long crus of incus, the incudo-stapedial articulation). This inevitably leads to their erosion [7] and epidermal ingrowth into the middle ear cavity with cholesteatoma formation. Retraction pockets are therefore dangerous because they are asymptomatic for a long time but ultimately lead to a permanent irreversible hearing loss.

An «early» surgical treatment of the retraction pocket is a preventive measure against middle ear cholesteatoma development. Surgery typically aims at mounting a cartilaginous graft in the area of the posterosuperior quadrant of the pars tensa in order to increase the rigidity of this part of the TM and preserve its auditory function [8]. Therefore, it is useful to calculate the magnitude of the TM deformation at different levels of negative pressure in the tympanic cavity (TC) in order to determine a suitable thickness for the cartilaginous graft.

This paper is dedicated to finite-element modeling of the stress-strain state (SSS) of the middle ear TM at different values of negative pressure as well as those of elasticity moduli of the pars tensa. Two cases are taken into consideration: when the elastic characteristics of pars tensa are within normal values; and when the elasticity modulus values are low.

2. THE GEOMETRIC AND THE FINITE-ELEMENT MIDDLE EAR MODELS

A solid 3D model of the TM was obtained by gluing different parts of the pars tensa and pars flaccida. The thickness of the pars flaccida was 30 µm. The posterosuperior quadrant of the pars tensa consisted of two layers, and the final part consisted of three layers. The thickness of the posterosuperior quadrant of the pars tensa was 60 µm, and the final part was 90 µm. The dimensions of the TM are indicated in Fig. 1 (all dimensions are in mm).

Solid 3D models of the malleus, incus and stapes were generated in SolidWorks 2010 (SolidWorks Corporation, USA) using a computed tomography of the middle ear. The characteristic dimensions of some middle ear models are shown in Fig. 2.

The finite-element middle ear model is shown in Fig. 3. Contact between the TM, handle of the malleus and middle ear ossicles is described by means of the ‘Bonded–type’ (owing to the absence of slip and penetration). All the gaps between contacting surfaces are closed in this type of contact, i.e. the contacting surfaces are bonded. The finite-
element segmentation was carried out in a semi-automatic mode. Maximal dimensions of the finite-element rib for the middle ear ossicles and TM were 0.3 mm and 0.1 mm, respectively. The number of nodes and elements used for the middle ear model components is shown in Table 1.

The transverse dimension of the finite element rib for each layer of the TM was 30 µm – both for the pars tensa and pars flaccida. The TM is rigidly fixed in an external circuit in the tympanic ring (TR) [8]. The variance of anchorage corresponding to ossification of the TM and TR contact area was considered. Negative pressure in the TC, leading to TM retraction and retraction pocket formation was modeled by applying a static, evenly distributed load on the outer side of the TM.

![Fig. 1 Geometrical dimensions of the TM: a) external view, b) plane section of membrane x = 0 (0A1A2A3 is the posterosuperior quadrant of the pars tensa, A2A4A5 is the pars flaccida)](image)

![Fig. 2 Geometric model of the middle ear and its characteristic dimensions: a) internal view; b) side view; c) external view](image)
Table 1 Number of nodes and elements in the middle ear model

<table>
<thead>
<tr>
<th>Middle ear model component</th>
<th>Number of nodes</th>
<th>Number of elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>TM</td>
<td>274 341</td>
<td>135 852</td>
</tr>
<tr>
<td>Pars flaccida</td>
<td>3 984</td>
<td>1 849</td>
</tr>
<tr>
<td>Malleus</td>
<td>12 097</td>
<td>7 631</td>
</tr>
<tr>
<td>Incus</td>
<td>16 308</td>
<td>10 424</td>
</tr>
<tr>
<td>Stapes</td>
<td>4 600</td>
<td>2 449</td>
</tr>
</tbody>
</table>

Fig. 3 Discrete model of the middle ear: a) finite element models of the TM, malleus, incus and stapes; b) finite-element segmentation of TM layers in the area of pars tensa.

The elastic modulus of the malleus, incus and stapes is 13.7 GPa, Poisson’s ratio was 0.3 (this corresponds to compact bone tissue [9]). The elastic moduli of the pars tensa (excluding the posterosuperior quadrant 0A1A2A3) and pars flaccida are 33 MPa and 11 MPa, respectively. The Poisson’s ratio for these two areas is 0.4 [10, 11]. The elastic modulus of the posterosuperior quadrant 0A1A2A3 has different values when modeling middle ear SSS for the normal and pathologically affected states of the pars tensa.

3. TYMPANIC MEMBRANE DEFORMATIONS IN THE NORMAL STATE

The finite-element calculation of the middle ear SSS was carried out using ANSYS Workbench 14 (ANSYS Inc., USA) for different values of pressure on the TM. Fig. 4 shows simulated deformations of the TM and middle ear ossicles for the standard pressure, \( p_S = 20 \) Pa \( (p_{SPL} = 120 \) dB), which is caused by the static pressure difference between the ear canal and the TC. Critical sound pressure in decibels (dB) is defined based on the Weber–Fechner law [12], as \( p_{SPL} = 20 \log (p_S/p_{RS}) \), \( p_{RS} = 2 \times 10^{-5} \) Pa. The elastic modulus of the posterosuperior quadrant 0A1A2A3 is 33 MPa.

Fig. 4 shows that maximal absolute deformations (approximately 3.060 µm) are observed with negative pressure of 20 Pa in the central part of the pars flaccida (see Fig. 4, a, b). A large deformation also occurs in the pars tensa with maximal values of approximately 1.7 µm (see Fig. 4, b). Other parts of the TM also undergo deformation besides the pars flaccida and the pars tensa (the range of absolute deformations is from 1.0 µm to 1.7 µm).
Fig. 5 shows the middle ear SSS under negative pressure of 34.0 kPa (184.6 dB). This corresponds to the pressure that develops when the incus touches the TM (i.e. when the minimal distance between them is 0.1 nm).

Fig. 4 Absolute deformations of middle ear under negative pressure of 20 Pa:
   a) internal view; b) external view; c) side view

Fig. 5 Absolute deformations of middle ear under negative pressure of 34.0 kPa:
   a) internal view; b) external view; c) side view

Fig. 5 shows that the maximal value of *pars tensa* displacements is approximately 5.20 mm. The direction of maximal deformation is that in which the TM makes contact with the side part of incus. This corresponds to a clinical manifestation of the retraction pocket in the posterosuperior quadrant of the TM (pre-cholesteatoma) [13, 14]. The obtained values of displacements correspond to the documented results [12], according to which the distance between the posterior point on the TM and head of the stapes is approximately 3.4521 mm.

Maximal deformation of the *pars flaccida* is also relatively high at approximately 3.468 mm. Values for the *pars flaccida* and *pars tensa* maximal deformation and distribution (see Fig. 5, c) show that significant sagging of these parts may be observed in overpressure. The maximum displacement of other TM parts does not exceed 2.31 mm.

4. TM Deformations in Pathological Changes of Pars Tensa Elastic Properties

Negative pressures are defined as the point where the incus touches the TM, as in the case when recurrent inflammatory diseases in the middle ear cause a decrease in the elastic modulus of the *pars tensa* [15]. The condition of the incus and TM contact...
remained unchanged. Tables 2 and 3 show values for the maximal displacements of the TM in the posterosuperior quadrant with an elastic modulus between 9 MPa and 11 MPa.

Absolute middle ear displacements under pressures between 6.7 kPa and 8.3 kPa are shown in Figs. 6 and 7. Posterosuperior quadrant elasticity moduli are 9 MPa and 11 MPa, (the elastic moduli of other parts of the pars tensa and pars flaccida are 33 MPa and 11 MPa, respectively).

### Table 2

Maximum deformations of the TM, when the elastic modulus of the posterosuperior quadrant of the pars tensa is 9 MPa, for different values of negative pressure (the pressure level in the last column corresponds to contact between the incus and TM).

<table>
<thead>
<tr>
<th>Pressure $p_S$, kPa</th>
<th>0.5</th>
<th>1.0</th>
<th>2.0</th>
<th>4.0</th>
<th>6.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure $p_{SPL}$, dB</td>
<td>148.0</td>
<td>154.0</td>
<td>160.0</td>
<td>166.0</td>
<td>170.5</td>
</tr>
<tr>
<td>Maximum deformation of the TM, mm</td>
<td>0.2443</td>
<td>0.4886</td>
<td>0.9771</td>
<td>1.9542</td>
<td>3.2733</td>
</tr>
</tbody>
</table>

### Table 3

Maximum deformations of the TM, when the elastic modulus of the posterosuperior quadrant of the pars tensa is 11 MPa, for different values of negative pressure (the pressure level in the last column corresponds to contact between the incus and TM).

<table>
<thead>
<tr>
<th>Pressure $p_S$, kPa</th>
<th>0.5</th>
<th>1.0</th>
<th>2.0</th>
<th>4.0</th>
<th>6.7</th>
<th>8.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure $p_{SPL}$, dB</td>
<td>148.0</td>
<td>154.0</td>
<td>160.0</td>
<td>166.0</td>
<td>170.5</td>
<td>172.4</td>
</tr>
<tr>
<td>Maximum deformation of the TM, mm</td>
<td>0.2023</td>
<td>0.4046</td>
<td>0.8093</td>
<td>1.6186</td>
<td>2.7111</td>
<td>3.3585</td>
</tr>
</tbody>
</table>

Figs. 6 and 7 show that the qualitative distribution of the absolute deformations in the posterosuperior quadrant is virtually identical for models with different elastic moduli of the pars tensa. The quantitative difference lies in absolute deformations of TM.

Practical interest lies in the assessment of the elastic modulus of the posterosuperior quadrant of the pars tensa at which contact occurs between the TM and incus at pressure of 20 Pa. Finite-element calculations show that contact between the middle ear ossicles and the posterosuperior part of the TM is observed below an elastic modulus of 32.5 MPa. The
distribution of absolute deformations in the middle ear under negative pressure of 20 MPa, and with a posterosuperior quadrant elastic modulus of 32.5 MPa, is shown in Fig. 8.

**Fig. 7** Absolute deformations of the middle ear under negative pressure of 8.3 kPa: a) internal view; b) external view; c) side view

Fig. 8 shows the deformation of only the posterosuperior quadrant of the pars tensa for negative pressure of 20 Pa and a posterosuperior elastic modulus of 32.5 MPa. There are no absolute deformations of other middle ear parts.

**Fig. 8** Absolute deformations of the middle ear (with an elastic modulus of 32.5 MPa for the posterosuperior quadrant of TM): a) internal view; b) external view; c) side view

5. Mesh Convergence

The grid convergence was studied for the TM finite element models with the maximum thickness of the finite element rib equal to 0.2 mm, 0.1 mm and 0.05 mm. The increase in the membrane maximal deformation in the second segmentation variant was 5% when compared to the first one. The increase in the maximal deformation in the third variant of segmentation was 0.3% compared to the second one. The second variant of the TM sampling was accepted on this basis in the middle ear finite-element calculation. The decrease in the element rib thickness for the middle ear ossicles models (less than 0.3 mm) did not lead to the TM maximal deformation change.
5. Conclusion

The following results were obtained during the finite-element simulation of a middle ear model with negative pressure in the TC:

- Maximal deformation of the TM is observed in the posterosuperior quadrant of the *pars tensa* for all values of negative pressure.
- The largest deformation observed for the standard negative pressure ($p_s = 20$ Pa) was $3.060 \, \mu$m.
- The negative pressure corresponding to contact between the TM and incus was $34$ kPa; the maximal deformation of the TM in this case was $5.2$ mm (normal elastic properties).
- The negative pressure corresponding to contact between the TM and incus, with an elastic modulus between $9$ MPa and $11$ MPa for the posterosuperior quadrant of *pars tensa*, was $6.7$ kPa and $8.3$ kPa, respectively (the elastic moduli for the final part of *pars tensa* and *pars flaccida* were $33$ MPa and $11$ MPa, respectively).
- The elastic modulus of the posterosuperior quadrant of the *pars tensa* corresponding to contact between the TM and middle ear ossicles under the standard pressure of $20$ Pa was $32.5$ MPa.
- The distribution of absolute deformations of the TM with different and identical elasticity moduli for the *pars flaccida* and *pars tensa* under various negative pressures differ from each other. This shows the need to describe the elastic characteristics of the *pars flaccida* and *pars tensa* with different elasticity moduli when calculating the stress-strain state in the middle ear.

The qualitative analysis of the above results allows us to make the following conclusions which could assist both researchers and clinicians:

- It is the reduction of the TM elastic properties in the posterosuperior quadrant of the *pars tensa* that may result in the retraction formation.
- To increase the rigidity of a weakened segment of the eardrum it is recommended to apply an early surgical approach involving the installation of a cartilage transplant.
- One needs to perform further finite-element simulations in order to study the effect of the cartilage transplant and estimate an optimal thickness of a cartilaginous graft.

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