FACTA UNIVERSITATIS
Series: **Physics, Chemistry and Technology** Vol. 14, N° 3, 2016, Special Issue, pp. 307 – 318
DOI: 10.2298/FUPCT1603307S

ON THE ELLIPTIC TIME IN THE ADELIC GRAVITY†

UDC 511.225:531.26:512.6/.7:514.74

George Shabat*

Russian State University for the Humanities, Moscow, Russia

Abstract. *The paper is devoted to the algebraic and arithmetic structures related to the two-body problem and discuss the possible generalizations. The role of the points of finite order on the elliptic curves is emphasized.*

Key words: *elliptic time, adelic gravity, two-body problem*

1. Introduction

The standard physical models, as a rule, were created as transcendental ones and were initially elaborated in transcendental terms. However, during the twentieth century (among lots of complicated processes) we have observed a kind of *algebraisation* of physics. The domains of mathematics providing tools used in the central physical theories have changed several times: the physicists' attention first moved from differential equations and functional analysis to differential geometry and topology, then to the group theory and, finally, to algebraic geometry – see, e.g., [1] for the general discussion.

The ground field is one of the central issues in algebraic geometry. Initially the field of complex numbers, being the closest to the transcendental world, was the most natural partner of physical theories[1]; among the most known early results of this

Received May 27th, 2016; accepted August 12nd, 2016.
†**Acknowledgement:** Supported in part by the Simons Foundation
***E-mail:** george.shabat@gmail.com
[1] The other kind of relations between physics and transcendental algebraic geometry was typical for the nineteenth century: theta functions emerged as the fundamental solutions of the heat equation, and Riemann surfaces made of conducting foil provided the initial intuition for the study of Abelian integrals ...

period one can mention the (motivated by Yang-Mills equations) description of the instantons over the 4-sphere in terms of vector bundles over the complex projective 3-space in [2]. This paper was followed by a stream of others, relating physics with various parts of algebraic geometry: the seminal [16] and [6] started the marriage of string theory with the geometry of moduli spaces of curves, the monograph [4] had summarized the early stage of the interaction of conformal Field theory with toric varieties, etc. In all these papers the algebro-geometric methods coexist naturally with the transcendental ones, so everything was considered over $\mathbb{C}$.

However, some of the popular physical models turned out to be the complexifications of the ones, defined over $\overline{\mathbb{Q}}$. Among many examples one can mention

• replacing of the integration in the string theory over the moduli spaces $\mathcal{M}_g(\mathbb{C})$ by the *summation* over $\mathcal{M}_g(\overline{\mathbb{Q}})$, [15];
• relating the black holes physics with the arithmetic of elliptic curves with complex multiplication, [12];
• Studying the instanton numbers for Calabi-Yau manifolds in terms of Frobenius map on p-adic cohomology, [7].

A general discussion on the arithmetisation of physics can be found in [11].

The present paper is devoted to the algebraic and arithmetic structures related to the physics that are a couple of centuries earlier than anything mentioned so far. We are going to find arithmetic in the two-body problem and discuss the possible generalizations. The author is indebted to P. Dunin-Barkovsky, A. Yu. Morozov, S. Nedic and A. Sleptsov for the useful discussions and criticism.

2. An algebraic theory of Kepler-Newton dynamics

1.0. Setup. All the relations between physics and the complex algebraic geometry that we are ultimately going to consider in this paper, are based on the construction of the form

$$\boxed{(\text{physical quantity in a point } P) = \int_{P_0}^{P} \omega} \tag{1}$$

where the integrand $\omega \in \Omega^1(\mathbf{V})$ is an appropriate rational 1-form on some complex algebraic manifold $\mathbf{V}$, a fixed point $P_0 \in \mathbf{V}$ on it and a variable point $P \in \mathbf{V}$.

Thus the only transcendent component of our models will be *integration*, while the basic components will be algebro-geometric, and there will be a certain flexibility in choosing the ground field.

2.1. Two-body problem: solving over $\mathbb{R}$

We present a self-contained exposition of the well-known theory, and this exposition is addressed to mathematicians; the physical concepts will be used only on the level of terminology.

The *Kepler-Newton* equations are:

$$\begin{cases} \ddot{x} = -\gamma \dfrac{x}{(x^2+y^2)^{\frac{3}{2}}} \\ \ddot{y} = -\gamma \dfrac{y}{(x^2+y^2)^{\frac{3}{2}}}. \end{cases} \tag{2}$$

for a real $\gamma > 0$. We consider this system of ODE's in the phase space

$$\mathcal{P} := \mathrm{Spec}(\mathbb{R}[x, y, \dot{x}, \dot{y}]) \setminus \{(0,0,0,0)\}. \tag{3}$$

The *sector velocity integral*

$$\Sigma := x\dot{y} - \dot{x}y \tag{4}$$

fibers $\mathcal{P}$ in the integral quadrics (Σ-levels)

$$\mathcal{P} = \coprod_{\sigma \in \mathbb{R}} \mathbf{Q}_{\Sigma}. \tag{5}$$

A solution (x, y) of Kepler-Newton equations is called *non-catastrophic*, if $(\dot{x}, \dot{y}) \notin \mathbb{R} \cdot (x, y)$. It can be shown that local solution that exists by the main ODE theorem, can be extended to the whole real line:

$$\mathbb{R} \longrightarrow \mathcal{P} \setminus \mathbf{Q}_0. \tag{6}$$

These solutions are terribly trascendental, and we are not going to work with them explicitly.

In the polar coordinates $x = r\cos\varphi,\ y = r\sin\varphi$ the Kepler-Newton system implies

$$\ddot{r} - r\dot{\varphi}^2 = -\frac{\gamma}{r^2}, \tag{7}$$

while the sector velocity integral results in $\dot{\varphi} = \frac{\Sigma}{r^2}$, so the variables separate:

$$\boxed{\ddot{r} = \frac{\Sigma^2}{r^3} - \frac{\gamma}{r^2}} \tag{8}$$

Solutions of this ODE are the integral curves of the *rational* vector field

$$\dot{r}\frac{\partial}{\partial r} + (\frac{\Sigma}{r^3} - \frac{\gamma}{r^2})\frac{\partial}{\partial \dot{r}} \tag{9}$$

on the $(r, \dot{r})$-affine plane; such fields can be considered over *any* field.
The boxed equation (8) admits a further rational integral (called "energy"...)

$$\boxed{E := \frac{\dot{r}^2}{2} + \frac{\Sigma^2}{2r^2} - \frac{\gamma}{r}}. \tag{10}$$

Using $\dot{r} = \frac{\mathrm{d}r}{\mathrm{d}t}$, it can be rewritten as

$$\mathrm{d}t = \frac{r\mathrm{d}r}{\sqrt{2Er^2 + 2\gamma r - \Sigma^2}}. \tag{11}$$

In a certain weak sense the 2-body problem is solved: formally integrating the above differential relation, we arrive at the relation

$$t - t_0 = \int \frac{r\mathrm{d}r}{\sqrt{2Er^2 + 2\gamma r - \Sigma^2}}, \tag{12}$$

that can be locally inverted to get the desired $r(t)$, and it can (and will soon) be shown that it is well-defined globally.

However, we'd like to have an explicit expression for $r(t)$ in a closed form, but instead arrived at $t(r)$ as a nasty multi-valued expression (the integral will be studieded below). Besides, this answer makes sense only over $\mathbb{R}$, and it is not our true goal.

Informally we come to the negative

Conclusion. Trying to parametrize everything by the *classical time* t fails.

1.2. The angular "time". The angle φ behaves better than the classical t!

According to the above and using $\frac{\mathrm{d}\varphi}{\mathrm{d}t} = \frac{\Sigma}{r^2}$, we arrive at

$$\boxed{\mathrm{d}\varphi = \frac{\Sigma \cdot \mathrm{d}r}{r\sqrt{2Er^2 + 2\gamma r - \Sigma^2}}}, \tag{13}$$

that integrates to

$$\varphi - \varphi_0 = \arccos \frac{\frac{1}{r} - \frac{\gamma}{\Sigma^2}}{\sqrt{\frac{2E}{\Sigma^2} + \frac{\gamma^2}{\Sigma^4}}} \Longrightarrow r = \frac{\frac{\Sigma^2}{\gamma}}{1 + \sqrt{\frac{2\Sigma^2 E}{\gamma^2} + 1}\cos(\varphi - \varphi_0)} \tag{14}$$

Introducing $r_0 := \frac{\Sigma^2}{\gamma}$ and $\varepsilon := \sqrt{1 + \frac{2E\Sigma^2}{\gamma^2}}$, we get the 2-parametric family of *ellipses*

$$\boxed{r = \frac{r_0}{1 + \varepsilon \cos(\phi - \phi_0)}}. \tag{15}$$

For the **Earth**

$$r_0 \approx 150\ 000\ 000 \text{ km}, \ \ \varepsilon \approx .017 \tag{16}$$

In terms of the initial phase space $\mathcal{P}$ the answers in terms of φ can be written down in quite an explicit form

$$x = \frac{r_0 \cos \varphi}{1 + \varepsilon \cos(\varphi - \varphi_0)}, \tag{17}$$

$$y = \frac{r_0 \sin \varphi}{1 + \varepsilon \cos(\varphi - \varphi_0)}, \tag{18}$$

$$\dot{x} = \sqrt{\frac{\gamma}{r_0}}(-\sin \varphi - \varepsilon \sin \varphi_0), \tag{19}$$

$$\dot{y} = \sqrt{\frac{\gamma}{r_0}}(\cos \varphi + \varepsilon \cos \varphi_0). \tag{20}$$

It would be a *perfect* answer[2] – if φ were a TIME.

Thus rather simple and purely mathematical considerations have led us to the deep philosophical question:

what is a TIME?

We are going to discuss it from purely mathematical positions, but taking into account the above results, concerning the two-body problem.

3. Times

We start with a standard definition.

2.0. What deserves to be called a time? From now on let $\mathcal{P}$ be a *configuration space*. Basically it means that $\mathcal{P}$ is a set (usually with some structure) whose elements are called *events*. In the previous section we have considered $\mathcal{P} \simeq \big(\mathbb{R}^2 \setminus \{(0,0)\}\big) \times \big(\mathbb{R}^2 \setminus \{(0,0)\}\big)$.

Let $\mathbb{T}$ be an arbitrary *group*. In the present paper we are going to consider only the

[2] among other things, we've got for free the *periodicity* of motions in the two-body problem

commutative ones.

In a given physical model $\mathbb{T}$ will be called "the" *time* group.

A mathematician's answer to the question in the title of this subsection is simply that the *action*

$$\boxed{\mathbb{T}:\mathcal{P}}. \tag{21}$$

is assumed.

2.1. The case of rational eccentricity. From now on we assume that

$$\varepsilon \in \mathbb{Q}. \tag{22}$$

This assumption is physically harmless since all the physical measurements are approximate and, say, one can learn from the official sources that *the eccentricity of the Earth's orbit is currently about*

$$0.0167 \in \frac{1}{10^4}\mathbb{Z} \subset \mathbb{Q}. \tag{23}$$

Consider only the *nondimensionalized* coordinates of the (φ-dependent) position of the *second body*:

$$\frac{x}{r_0} = \frac{\cos\varphi}{1+\varepsilon\cos(\varphi-\varphi_0)}, \tag{24}$$

$$\frac{y}{r_0} = \frac{\sin\varphi}{1+\varepsilon\cos(\varphi-\varphi_0)}. \tag{25}$$

Assume additionally that $0 \le \varepsilon < 1$. Then, obviously,

$$\varphi, \varphi_0 \in \mathbb{R} \Longrightarrow \frac{x}{r_0}, \frac{y}{r_0} \in \mathbb{R}; \tag{26}$$

$$\varphi, \varphi_0 \in \mathbb{Q}\pi \Longrightarrow \frac{x}{r_0}, \frac{y}{r_0} \in \mathbb{Q}^{\mathsf{ab}} \bigcap \mathbb{R}; \tag{27}$$

$$N \in \mathbb{N},\ \varphi, \varphi_0 \in \mathbb{Z}\frac{2\pi}{N} \Longrightarrow \frac{x}{r_0}, \frac{y}{r_0} \in \mathbb{Q}(\sqrt[N]{1}) \bigcap \mathbb{R}. \tag{28}$$

Taking into account the approximate nature of the physical measurements, we see that *the last two cases are compatible with ALL the celestial observations!* Of course, in the very last one it is assumed that N is large enough.

2.2. How bad is the real[3] time? Trying to do the same with the *physical* time $t \in \mathbb{R}$, we have

[3]this word can be understood either in the common or in the techical sense

$$t - t_0 = \int \frac{\mathrm{d}r}{\sqrt{\gamma}\sqrt{\frac{\varepsilon^2}{r_0} - (\frac{\sqrt{r_0}}{r} - \frac{1}{\sqrt{r_0}})^2}}. \tag{29}$$

This integral can be calculated in elementary functions:

$$t - t_0 = \frac{(k+1)^2}{4k}[\frac{(k-1)z}{kz^2+1} + \sqrt{k}(k+1)\arctan(\sqrt{k}z)], \tag{30}$$

where $k = \frac{1+\varepsilon}{1-\varepsilon}$ and $z = \tan\frac{\varphi}{2}$.

But there are **no chances** to express r or φ in terms of t!

2.3. Times over various fields. The above formulas for the motion of the planet along the elliptic orbits make sense in the affine planes over almost arbitrary field $\mathbb{k}$, with minor restrictions on the char($\mathbb{k}$).

The following pairs can provide "solutions" of the 2-body problem:

Time group $\mathbb{T}$	**Field**
$\mathbb{R}$	$\mathbb{R}$
$\frac{\mathbb{R}}{2\pi\mathbb{Z}}$	$\mathbb{R}$
$\frac{\mathbb{Z}}{N\mathbb{Z}}$	$\sqrt[N]{1} \subset \mathbb{k}, \mathrm{char}(\mathbb{k}) \nmid N$
$\mathbb{Z}_p$	$\mathbb{k} = \overline{\mathbb{k}}, \mathrm{char}(\mathbb{k}) \neq p$
$\widehat{\mathbb{Z}} \cong \prod_p \mathbb{Z}_p$	$\mathbb{k} = \overline{\mathbb{k}}, \mathrm{char}(\mathbb{k}) = 0$

As we see, there are lots of possibilities to create algebro-geometric models of the Kepler-Newton gravity. The natural problem is to choose those of them that are the most compatible with the actual physical observations.

Among the candidates that are the closest to the classical models, we find the case of finite cyclic times group (say, for the beginning, with $N = 365$) and $\mathbb{k} = \mathbb{Q}^{\mathsf{ab}}$; the orbits will lie in the affine plane of the real subfield $\mathbb{Q}(\cos\frac{\pi}{4}, \cos\frac{\pi}{5}, \cos\frac{\pi}{6}, \dots) \subset \mathbb{Q}^{\mathsf{ab}}$. It would be interesting to study the behavior of heights along the orbits.

The models over $\overline{\mathbb{F}_p}$ can turn out to be quite realistic (in some ways) for the large p's. As for the p-adic and adelic times, they are interesting mathematically, and the corresponding physics deserves thinking about.

4. Elliptic fantasies

To every eccentricity from the ground field we associate an elliptic curve defined over this field. In the archimedean case all the physically meaningful quantities come out as abelian integrals on this curve.

3.0. Divine curves. To every $\varepsilon \in \Bbbk \setminus \{1\}$ we associate the main parameter of the orbit

$$k := \frac{1+\varepsilon}{1-\varepsilon} \tag{31}$$

and the elliptic curve (imaginary *Legendre* quartic) defined by the equation

$$\boxed{w^2 = (1+z^2)(1+k^2z^2)} \tag{32}$$

In the archimedean case

$$z := \tan\frac{\varphi}{2}. \tag{33}$$

We call such a curve *divine* for the following reasons: it

- depends on the planet and is defined by its orbit;
- is dimensionless;
- is nowhere (not embedded in the physical space);
- governs "all" the observable variables related to the planet.

3.1. The remarkable differentials. We introduce them both in the original notations and in the coordinates of the divine curve.

$$\mathrm{d}\varphi = \frac{2\mathrm{d}z}{1+z^2}; \tag{34}$$

$$\frac{\Sigma}{r_0^2}\mathrm{d}t = \frac{(k+1)^2}{2}\frac{1+z^2}{(1+kz^2)^2}\mathrm{d}z; \tag{35}$$

$$\frac{\mathrm{d}r}{r_0} = -(k^2-1)\frac{z}{(1-kz^2)^2}\mathrm{d}z; \tag{36}$$

The classical distance along the orbit is defined by a symbolic expression $(\mathrm{d}s)^2 := (\dot{x}^2+\dot{x}^2)(\mathrm{d}t)^2$. This quadratic differential is the square of an abelian one

$$\frac{\mathrm{d}s}{r_0} = (k+1)\frac{w\mathrm{d}z}{(1+kz^2)^2}; \tag{37}$$

A puzzling observation. *All the poles of all the above differentials lie in the points of 4th order of the divine curve.*

"Physically" it means that the torsion group of the divine curve contains the (usually imaginary) stationary points of meaningful quantities.

3.2. On the concept of elliptic time. For the time being it is ill-motivated. However, we have observed three phenomena:

• imposing algebraic structures on the Time clarifies the matters;
• the concept of Time is flexible;
• the physical quantities are naturally defined in terms of divine elliptic curves.

The hope for unifying these phenomena is related to the conjecture that the whole gravity can be governed by the group structure of the divine curves – or, rather, by the structure of a group scheme on the universal family of these curves.

5. p-adic and adelic times?

We have discussed above the restriction of the phase space from the manifolds over $\mathbb{R}$ to the ones over $\mathbb{R}\bigcap\overline{\mathbb{Q}}$. While we restrict ourselves with archimedean structures, we get no new metric effects, even if we consider discrete periodic dynamics.

However, embedding fields of definitions of orbits into p-adic fields and still considering finite time groups – or, may be, profinite, say, $\mathbb{T} = \mathbb{Z}_\ell$ – we can hope to meet something interesting. In an analogy with [13] one can hope that the chaotic behavior of the $\overline{\mathbb{Q}}$-models will occur in p-directions only for finite number of p's, so some *adelic* measure of chaos will appear. Considering all the p's together, we arrive at the *adelic time* $\mathbb{T} = \widehat{\mathbb{Z}}$.

The above mentioned 4-order points (2-isogenies, Landin transforms,...) suggests special consideration of 2-*adic time.* A simple 2-adic model of period-doubling onset of chaos was considered in [5].

6. Jumping to general relativity

Now turn for a while to the adult math, consider a much more advanced gravitational theory and discuss elliptic curves therein. We are going to present briefly the results of Tod and Hitchin, concerning the anti-self-dual metrics, including the Einstein ones.

Let three generators of the space of invariant 1-forms on the 3-sphere $\langle\sigma_1, \sigma_2, \sigma_3\rangle$ satisfy

$$\begin{cases} \mathrm{d}\sigma_1 = \sigma_2 \wedge \sigma_3 \\ \mathrm{d}\sigma_2 = \sigma_3 \wedge \sigma_1 \\ \mathrm{d}\sigma_3 = \sigma_1 \wedge \sigma_2 \end{cases} \tag{38}$$

Tod in [14] and Hitchin in [8] have found on $(0,1) \times \mathbf{S}^3$ a family of SU_2-invariant metrics

$$(\mathrm{d}s)^2 = \frac{(\mathrm{d}t)^{\otimes 2}}{t(1-t)} + \frac{\sigma_1^{\otimes 2}}{\Omega_1^2} + \frac{(1-t)\sigma_2^{\otimes 2}}{\Omega_2^2} + \frac{t\sigma_3^{\otimes 2}}{\Omega_3^2}, \tag{39}$$

where

$$\begin{cases} \Omega_1^2 = \frac{(x-t)^2 x(x-1)}{t(1-t)}[z - \frac{1}{2(x-1)}](z - \frac{1}{2x}) \\ \Omega_2^2 = \frac{x^2(x-1)(x-t)}{t}[z - \frac{1}{2(x-t)}][z - \frac{1}{2(x-1)}] \\ \Omega_3^2 = \frac{(x-1)^2 x(x-t)}{1-t}[(z - \frac{1}{2x}][z - \frac{1}{2(x-t)}], \end{cases} \tag{40}$$

the function $x(t)$ solves Painlevé-VI (with parameters $\frac{1}{8}, -\frac{1}{8}, \frac{1}{8}, \frac{3}{8}$)

$$\ddot{x} = \frac{1}{2}(\frac{1}{x} + \frac{1}{x-1} + \frac{1}{x-t})\dot{x}^2 - (\frac{1}{t} + \frac{1}{t-1} + \frac{1}{x-t})\dot{x} + \\ + \frac{x(x-1)(x-t)}{t^2(t-1)^2}[\frac{1}{8} - \frac{t}{8x^2} + \frac{t-1}{8(x-1)^2} + \frac{3x(x-1)}{8(x-t)^2}] \tag{41}$$

and $z(t)$ is defined from the relation

$$\dot{x} = \frac{x(x-1)(x-t}{t(t-1)}[2z - \frac{1}{2x} - \frac{1}{2(x-1)} + \frac{1}{2(x-t)}]. \tag{42}$$

The elements of the metric turn out to be expressible in terms of theta-functions!

In fact, in the above-quoted papers two ways of establishing relations between Einstein and Painlevé equations are presented.

Tod's way is formal. He proves that the metric

$$(\mathrm{d}s)^2 = f(t)(\mathrm{d}t)^{\otimes 2} + a_1(t)\sigma_1^{\otimes 2} + a_2(t)\sigma_2^{\otimes 2} + a_3(t)\sigma_3^{\otimes 2} \tag{43}$$

is anti-self-dual if and only if it can be rescaled to

$$(\mathrm{d}s)^2 = \frac{(\mathrm{d}t)^{\otimes 2}}{t(1-t)} + \frac{\sigma_1^{\otimes 2}}{\Omega_1^2} + \frac{(1-t)\sigma_2^{\otimes 2}}{\Omega_2^2} + \frac{t\sigma_3^{\otimes 2}}{\Omega_3^2}, \tag{44}$$

where the scalar functions $\Omega_{1,2,3}$ satisfy the system of ODE

$$\begin{cases} \dot{\Omega}_1 = -\frac{\Omega_2\Omega_3}{t(1-t)} \\ \dot{\Omega}_2 = -\frac{\Omega_3\Omega_1}{t} \\ \dot{\Omega}_3 = -\frac{\Omega_1\Omega_2}{1-t} \end{cases} . \tag{45}$$

This system reduces to Painlevé-VI with parameters $(\frac{1}{8}, \frac{1}{8}, c, d)$, where $c + d = \frac{1}{2}$; if the original metric is Einstein, then $c = \frac{1}{8}$.

Though the Painlevé-VI equation is famous for producing *essentially new transcendents*, the parameters $(\frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{3}{8})$ are known to be related to *algebraic* solutions. It reminds the claim from the introduction: physical models find the hidden islands of algebraic structure in the oceans of the transcendent objects...

Hitchin's way is geometric, based on twistors, families of rational curves on complex 3-manifolds, flat connections with log-singularities, isomonodromic deformations and so on. The analytic part of his approach is based on the century-old transformation of Schlessinger equations to Painlevé-VI, and once again we meet the physically interesting *arithmetic* structures among the algebraic ones.

Any algebraic solution $x(t)$ of Painleve-VI by definition has an *affine model* – a plane curve, defined by a polynomial equation $F(t, x) = 0$. It can be easily seen that the "time" t on this curve can have among the critical values only 0 and 1; hence, it is a *Belyi function* on the curve (see, e.g., [10]). It follows then from the so-called *easy Belyi theorem,* that the curve is defined over $\overline{\mathbb{Q}}$.

Hitchin in [9] has constructed an infinite number of algebraic solutions of Painleve-VI using the Poncelet closure theorem; the dynamic, governed by the finite-order points of elliptic curves, appears once more!

The p-adic aspects of Painleve-VI equations are studied in [3].

7. Conclusion

In the main part of this paper we have discussed the possible arithmetical theory of the simplest gravitational model, the two-body problem; in the last section the general relativity was mentioned. In both, somewhat extreme, cases the arithmetic of torsion of elliptic curves appeared. The author hopes to clarify the above hints and to find the generalizations and specializations that would connect the extreme cases by some intermediate models.

REFERENCES

1. M. Atiyah, R. Dijkgraaf, N. Hitchin: *Geometry and physics*. Phil. Trans. R. Soc. A **368** (2010) 913–926.
2. M. Atiyah, V. Drinfeld, N. Hitchin, Y. Manin: *Construction of instantons*. Physics Letters A **65** (3) (1978), 185–187.
3. A. Buium, Y. I. Manin: *Arithmetic Differential Equations of Painleve VI Type*. arXiv:1307.3841v2 [math.NT].
4. D. A. Cox, S. Katz: *Mirror Symmetry and Algebraic Geometry*. American Mathematical Soc., 1999.
5. V. Dremov, G. Shabat and P. Vytnova: *On dynamical systems with 2-adic time*. Proceedings of The Steklov Institute of Mathematics **265** (2009) 101–109.
6. M. Kontsevich: *Intersection Theory on the Moduli Space of Curves and the Matrix Airy FunctionCommun*. Math. Phys. **147** (1992) 1-23.
7. M. Kontsevich, A. Schwarz, V. Vologodsky: *Integrality of instanton numbers and p-adic B-model*. arXiv:hep-th/0603106.
8. N. J. Hitchin: *Twistor spaces, Einstein metrics and isomonodromic deformations*. J. Differential Geom. **42** (1995), no. 1, 30–112.
9. N. J. Hitchin: *Poncelet polygons and the Painlevé equations*. Geometry and analysis (Bombay, 1992), Tata Inst. Fund. Res., Bombay, 1995.
10. S. K. Lando and A. K. Zvonkin: *Graphs on Surfaces and Their Applications (with Appendix by Don B. Zagier)*. Springer-Verlag, 2004.
11. Y. Manin: *Reflections on arithmetical physics*. In Conformal invariance and string theory, pp. 293–303, Academic Press, Boston, MA, 1989.
12. G. Moore: *Arithmetic and Attractors*. arXiv:hep-th/9807087.
13. G. Shabat: *p-adic entropies of logistic maps*. Proc. Steklov Math. Inst. **245** (2004) 257-263.
14. K. P. Tod: *Self-dual Einstein metrics from the Painleve VI equation*. Phys. Lett, **A190** 221-224.
15. V. A. Voevodsky **and** G. B. Shabat: *Equilateral triangulations of Riemann surfaces, and curves over algebraic number fields*. Dokl. Akad. Nauk SSSR **304** (1989) 265–268.
16. E. Witten: *Two dimensional gravity and intersection theory on moduli space*. Surveys in Diff. Geom. **1** (1991) 243–310.

O ELIPTIČKOM VREMENU U ADELIČKOJ GRAVITACIJI

Rad je posvećen algebarskim i aritmetičkim strukturama problema dva tela i mogućim generalizacijama. Istaknuta je uloga tačaka konačnog reda na eliptičkim krivima.

Ključne reči: *eliptičko vreme, adelička gravitacija, problem dva tela*