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NEUTRINOS AND THE STRUCTURE OF SPACE-TIME

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Abstract. The phenomenon of neutrino oscillations is studied usually as a mixing between the flavor neutrinos and the neutrinos having a definite mass. The mixing angles and the mass eigenvalues are treated independently in order to accommodate the experimental data. We suggest that neutrino oscillations are connected to the structure of spacetime. We expand on a recently proposed model, where two “mirror” branes coexist. One brane hosts left-handed particles (our brane), while the other brane hosts right-handed particles. Majorana-type couplings mixes neutrinos in an individual brane, while Dirac-type couplings mixes neutrinos across the brares. We first focus our attention in a single brane. The mass matrix, determined by the Majorana mass, leads to mass eigenstates and further to mixing angles identical to the mixing angles proposed by the tri-bimaximal mixing. When we include the Dirac-type coupling, connecting the two branes, we obtain a definite prediction for the transition to a sterile neutrino (right-handed neutrino). With m_L (m_R) the Majorana mass for the left (right) brane, we are able to explain the solar and the atmospheric neutrino data with $m_L = 2m_R$ and $m_R = 10^{-2}$ eV.

1. Introduction

Neutrinos are very numerous in the universe, trailing just behind the most dominant particles, the photons. Since they interact only through the weak interactions, detecting them is a most difficult task. The energy spectrum of neutrinos ranges from the few MeV of solar neutrinos to the high PeV energies of astrophysical neutrinos. The neutrinos traverse unhindered huge distance and they always point back to their sources. This last fact is the ground for the emergence of a new field, “neutrino astronomy”.

Neutrino oscillations is a purely quantum mechanical phenomenon, where a neutrino of a given flavor is transformed to a neutrino of a different flavor. The study of neutrino oscillations reveals essential parameters, like the neutrino masses and the neutrino mixing angles. Understanding the values of these parameters would

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imply that we have a handle on the underlying neutrino physics. The neutrinos act also like messengers, conveying important information about the original source and the medium traversed.

In the present work we indicate that neutrinos may provide information about the structure of space-time. The starting point is to view spinors as the building blocks of space-time, expanding on ideas first proposed by Cartan and Penrose. The ensuing geometry , two "mirror" branes, is best studied by neutrinos. Let us remind the thrust of the argument.

It has been suggested that the relational logic of C. S. Peirce (or equivalently category theory) may serve as the foundation for quantum mechanics [1]. The algebra of logical relations leads to discrete geometrical patterns [1], thus reviving

Wheeler's pioneering idea of "pregeometry". Wheeler suggested that geometry is preceded by pregeometry, which is based on a calculus of logical propositions [2, 3]. A relation, considered as a logical proposition receiving a "yes" or a "no" answer, resembles to a spinor, which under measurement is revealed as "spin up" or "spin down" [1, 4]. We may view then the spinor as the building block of our logical construction and a spinor network would be equivalent to a pregeometry.

The profound connection between spinors and geometry was established a hundred years ago by Cartan, who introduced spinors as linear representation of the rotation group [5]. Penrose used spinor as the building block of discrete space-time and as a powerful tool to study physics issues [6]. A single spinor gives rise to the Riemann-Bloch sphere, which is topologically equivalent to the null cone of Minkowski spacetime [6]. It is quite natural then to wonder what kind of geometry we obtain when we entangle two spinors. We carried out this work in reference [7].

There are two ways to couple two spinors. The first way relies on Majorana's recipe to create a spinor [8]. Given a left-handed spinor $|\psi_L\rangle$, we may construct a right-handed spinor $|\chi_R\rangle$ by

$$|\chi_R\rangle = \sigma_2 \, |\psi_L\rangle^* \tag{1.1}$$

Starting with two independent left-handed Weyl spinors, we may induce a coupling between them by establishing a four-component Majorana spinor

$$|\Psi_M\rangle = \begin{pmatrix} |\chi_L\rangle \\ \sigma_2 \, |\psi_L\rangle^* \end{pmatrix} \tag{1.2}$$

Defining $X_i = \langle \Psi_M \, |\gamma_i| \, \Psi_M\rangle$ $(i = 0, 1, 2, 3)$ we find that X_i is not a null vector [7]

$$X_1^2 + X_2^2 + X_3^2 - X_0^2 = M_M^2 \tag{1.3}$$

with

$$\begin{aligned} X_4 &= i \, \langle \Psi_M | \, \Psi_M\rangle \\ X_5 &= \langle \Psi_M \, |\gamma_5| \, \Psi_M\rangle \end{aligned}$$

$$M_M^2 = -\left(X_4^2 + X_5^2\right) \tag{1.4}$$

Thus among two left-handed Weyl spinors (or two right-handed Weyl spinors), the Majorana's coupling induces a mass term. Notice that usually a Majorana spinor represents a particle which is itself an antiparticle and it is reduced to a two-component spinor. In our case we managed to connect two distinct left-handed Weyl spinors to a four component spinor.

The Dirac coupling involves a left-handed Weyl spinor and a right-handed Weyl spinor. Writing

$$|\Psi_D\rangle = \begin{pmatrix} |\chi_L\rangle \\ |\psi_R\rangle \end{pmatrix} \tag{1.5}$$

we obtain

$$X_1^2 + X_2^2 + X_3^2 - X_0^2 = -M_D^2 \tag{1.6}$$

with

$$M_D^2 = \left(X_4^2 + X_5^2\right) \tag{1.7}$$

Let us define $T = X_0$, $t = M_D$. The Dirac entanglement, equ.(1.6), takes the form of a space-like hyperboloid

$$T^2 - \sum_{i=1}^{3} X_i^2 = t^2 \tag{1.8}$$

A comparison with the null cone geometry, indicates that quantum entanglement, specified and quantified by t, generates an extra dimension. The distance along this extra dimension indicates how far we are from the null cone. Furthermore our space-time acquires a double-sheet structure, reminding the ekpyrotic model where two branes coexist [9, 10, 11]. There is though a distinct difference. In our model, by construction, one brane hosts left-handed particles (our brane), while the other brane hosts right-handed particles. We should indicate that we use the term "brane" in a broad sense. Usually brane models and the ensuing phenomenology are studied within string theory and general relativity [12, 13, 14]. In the present case the "mirror" brane structure is obtained by making appeal to the Cartan-Penrose connection of spinors to geometry. Notice that "mirror" brane models have been also proposed elsewhere [15].

The conventional way to restore left-right symmetry is to introduce an extra $SU(2)_R$ gauge group in the energy desert above the scale of the standard $SU(2)_L$ interactions. The right-handed gauge bosons are more massive compared to the left-handed gauge bosons, leading to parity violation at low energies [16, 17]. Within our approach the left-right symmetry is achieved with the extra dimension hosting two "mirror" branes, a left-handed brane and a right-handed brane. The most prominent candidate for mediation between the two branes is the neutrino particle. The left-handed neutrino, an essential ingredient of the standard model, resides in our brane, while its counterpart, the right-handed neutrino, resides in the other brane. Within our approach neutrino oscillations acquire a novel character. Majorana-type coupling mixes the left-handed flavor neutrinos residing in our brane, as well as the right-handed neutrinos residing in the other brane. Dirac-type coupling connects the left-handed neutrinos of our brane to the right-handed neutrinos of the other brane. From our point of view, right-handed neutrinos appear as sterile neutrinos, and the transition flavor neutrino - sterile neutrino - flavor neutrino amounts to a swapping between the two branes. Let us study first the mixing among the left-handed neutrinos, or focus our attention into our brane.

2. Single brane

In our model Majorana mass couplings connect distinct left-handed Weyl spinors (there is no self-coupling). We assume a "democratic principle" attributing the same value to all Majorana mass couplings. Then the mass matrix for the left-handed neutrinos will take the form

$$M = \begin{pmatrix} 0 & m & m \\ m & 0 & m \\ m & m & 0 \end{pmatrix} \tag{2.1}$$

The eigenvalues of M, involving a double root, are

$$\lambda_1 = \lambda_3 = -m \qquad \lambda_2 = 2m \tag{2.2}$$

The corresponding eigenvectors are

$$\begin{aligned} N_1^T &= \frac{1}{\sqrt{6}} \begin{pmatrix} 2, & -1, & -1 \end{pmatrix} \\ N_2^T &= \frac{1}{\sqrt{3}} \begin{pmatrix} 1, & 1, & 1 \end{pmatrix} \\ N_3^T &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0, & 1, & -1 \end{pmatrix} \end{aligned} \tag{2.3}$$

Expressing the flavor left-handed neutrinos in terms of the mass eigenstates we write

$$|\nu_{f_i}\rangle = \sum_j c_{ij} \, |N_j\rangle \tag{2.4}$$

with $\nu_{f_1}, \nu_{f_2}, \nu_{f_3}$ denoting respectively $\nu_{e_L}, \nu_{\mu_L}, \nu_{\tau_L}$.

Defining $(U)_{ij} = c_{ij}$ we find that the mixing matrix U is

$$U = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \tag{2.5}$$

This type of mixing defines the celebrated tri-bimaximal mixing (TB mixing) [18-21]. We notice that the TB mixing has been proposed in order to accomodate the experimental data, while in our case emerges as the outcome of a Majorana-type coupling among the left-handed neutrinos.

Let us consider an initial flavor ν_{e_L} beam. The transitions to other flavors are given by

$$P\left(\nu_e \to \nu_\mu\right) = P\left(\nu_e \to \nu_\tau\right) = \frac{4}{9} sin^2 \left(\frac{3}{4}\frac{m^2}{\hbar E}t\right) \tag{2.6}$$

Also

$$P\left(\nu_e \to \nu_e\right) = \frac{1}{9}\left[9 - 8 sin^2 \left(\frac{3}{4}\frac{m^2}{\hbar E}t\right)\right] \tag{2.7}$$

The oscillations depend upon a single mass scale and clearly cannot reproduce the available data. The introduction of the right-handed brane allows us to have access to two more scales, the Majorana mass coupling in the right-handed brane and the Dirac mass coupling among the branes. We move then to the case of the two "mirror" branes.

3. Mirror branes

On general grounds we expect the Majorana mass coupling in the right-handed brane to be of the same order of magnitude with the corresponding parameter in the left-handed brane. For general purposes we denote them by m_L, m_R, with the obvious correspondence. Each single left-handed neutrino, residing in our brane, is connected to all the right-handed neutrinos, residing in the other brane, by the same universal Dirac mass coupling μ. Then the mass matrix involving the 6 neutrino states (3 left-handed plus 3 right-handed) will have the form

$$\mathcal{M} = \begin{pmatrix} M_L & M_+ \\ M_+ & M_R \end{pmatrix}. \tag{3.1}$$

$M_L\,(M_R)$ is a mass matrix identical to M, equ. (2.1), with m replaced by $m_L\,(m_R)$. M_+ involves the mass terms connecting the two branes and is given by

$$M_+ = \begin{pmatrix} \mu & \mu & \mu \\ \mu & \mu & \mu \\ \mu & \mu & \mu \end{pmatrix} \tag{3.2}$$

The eigenvalues, involving two double roots, are

$$\begin{aligned} \lambda_1 &= \lambda_3 = -m_L \\ \lambda_4 &= \lambda_6 = -m_R \\ \lambda_2 &= (m_L + m_R) + \left[(m_L - m_R)^2 + 9\mu^2\right]^{\frac{1}{2}} \\ \lambda_5 &= (m_L + m_R) - \left[(m_L - m_R)^2 + 9\mu^2\right]^{\frac{1}{2}} \end{aligned} \tag{3.3}$$

Let us define

$$d = \left[(m_L - m_R)^2 + 9\mu^2\right]^{\frac{1}{2}} \tag{3.4}$$

$$\delta_\pm = d \pm (m_L - m_R)$$

$$\cos\phi = \left(\frac{\delta_+}{2d}\right)^{\frac{1}{2}} \qquad \sin\phi = \left(\frac{\delta_-}{2d}\right)^{\frac{1}{2}} \tag{3.5}$$

The corresponding eigenvectors are

$$\begin{aligned}
N_1^T &= \frac{1}{\sqrt{6}}\left(\ 2,\quad -1,\quad -1,\quad 0,\quad 0,\quad 0\ \right)\\
N_2^T &= \frac{1}{\sqrt{3}}\left(\ \cos\phi,\quad \cos\phi,\quad \cos\phi,\quad \sin\phi,\quad \sin\phi,\quad \sin\phi\ \right)\\
N_3^T &= \frac{1}{\sqrt{2}}\left(\ 0,\quad 1,\quad -1,\quad 0,\quad 0,\quad 0\ \right)\\
N_4^T &= \frac{1}{\sqrt{6}}\left(\ 0,\quad 0,\quad 0,\quad 1,\quad -2,\quad 1\ \right)\\
N_5^T &= \frac{1}{\sqrt{3}}\left(\ \sin\phi,\quad \sin\phi,\quad \sin\phi,\quad -\cos\phi,\quad -\cos\phi,\quad -\cos\phi\ \right)\\
N_6^T &= \frac{1}{\sqrt{2}}\left(\ 0,\quad 0,\quad 0,\quad -1,\quad 0,\quad 1\ \right)
\end{aligned} \tag{3.6}$$

The similarities and the differences with the case of a single brane, equ. (2.3), are apparent. $N_1, N_3\,(N_4, N_6)$ involve mixing within the individual left (right) brane. N_2 and N_5 connect the two branes. For $m_L = m_R$, $\phi = \frac{\pi}{4}$ and the two branes are equally present in the mixing phenomenon. For small Dirac coupling compared to the Majorana couplings we obtain $\phi \simeq 0$.

The mixing matrix connecting the flavor eigenstates (left-handed and right-handed) to the six eigenvectors takes the form

$$U = \begin{pmatrix}
\sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}}\cos\phi & 0 & 0 & \frac{1}{\sqrt{3}}\sin\phi & 0\\
-\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}}\cos\phi & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{3}}\sin\phi & 0\\
-\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}}\cos\phi & -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{3}}\sin\phi & 0\\
0 & \frac{1}{\sqrt{3}}\sin\phi & 0 & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}}\cos\phi & -\frac{1}{\sqrt{2}}\\
0 & \frac{1}{\sqrt{3}}\sin\phi & 0 & -\sqrt{\frac{2}{3}} & -\frac{1}{\sqrt{3}}\cos\phi & 0\\
0 & \frac{1}{\sqrt{3}}\sin\phi & 0 & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}}\cos\phi & \frac{1}{\sqrt{2}}
\end{pmatrix} \tag{3.7}$$

Again for $\phi = 0$ the upper left part of the matrix gives the previous result, equ. (2.5), for the single brane.

Imagine that at $t = 0$ we start with a pure ν_{e_L} beam. The probability to find later another flavor is given by

$$P\left(\nu_{e_L} \to \nu_{\mu_L}\right) = P\left(\nu_{e_L} \to \nu_{\tau_L}\right)$$

$$= \frac{1}{9}\left\{1+\cos^4\phi+\sin^4\phi+2\left[\cos^2\phi\sin^2\phi\cos\frac{(\omega_+-\omega_-)\,t}{2\hbar E}-\cos^2\phi\cos\frac{\omega_+ t}{2\hbar E}-\sin^2\phi\cos\frac{\omega_- t}{2\hbar E}\right]\right\}$$

where

$$\begin{aligned}\omega_+ &= m_L^2+2m_R^2+9\mu^2+2d\,(m_R+m_L)\\ \omega_- &= m_L^2+2m_R^2+9\mu^2-2d\,(m_R+m_L)\\ \omega_+-\omega_- &= 4d\,(m_R+m_L)\end{aligned} \tag{3.8}$$

The transition to a generic sterile neutrino (an incoherent sum of all right-handed neutrinos) is given by

$$P\,(\nu_e\to\nu_s)=\frac{1}{9}\sin 2\phi\sin^2\left(\frac{1}{4\hbar E}\,(\omega_+-\omega_-)\,t\right) \tag{3.9}$$

Notice that for the transition of the ν_{μ_L} we find

$$P\,(\nu_{\mu_L}\to\nu_{e_L})=P\,(\nu_{\mu_L}\to\nu_{\tau_L})=P\,(\nu_{e_L}\to\nu_{\mu_L}) \tag{3.10}$$

We may recall the neutrino oscillation data [22]. Solar and atmospheric neutrino oscillations define two distinct mass scales

$$\Delta m_s^2\simeq 5*10^{-5}eV^2 \qquad \Delta m_a^2\simeq 2*10^{-3}eV^2 \tag{3.11}$$

A neutrino oscillation experiment defines a specific value for the parameter $\frac{t}{E}$ (the distance traveled by the neutrino over its energy). Large values of $\frac{t}{E}$ allow to explore small values of Δm^2, or correspondingly small ω. Solar neutrinos correspond to low energy neutrinos covering huge distance, therefore their oscillation is determined by ω_-. Atmospheric neutrinos involve higher energies and smaller distances and their oscillation is controlled by ω_+. Accordingly we assign

$$\begin{aligned}\omega_- &\simeq \Delta m_s^2\\ \omega_+ &\simeq \Delta m_a^2\end{aligned} \tag{3.12}$$

There is a conflicting evidence for the existence of a sterile neutrino [23]. At any rate the amplitude for a transition to a sterile neutrino is expected to be small and correspondingly $\sin\phi$, see equ. (3.9), and the Dirac coupling μ are small. Adopting the hierarchy $(m_L-m_R)>\mu$ we find that the values

$$m_L\simeq 2m_R \qquad m_R\simeq 10^{-2}eV \tag{3.13}$$

reproduce the observed scales, equ. (3.12). The precise smallness of μ will fix the magnitude of $\sin\phi$ and therefore the probability to a sterile neutrino oscillation. Notice however that within our scheme the mass scale for the transition to a sterile neutrino is at a sub-eV scale ($3*10^{-2}$ eV), rather far from the value suggested by the LSND experiment.

4. Conclusion

The conventional approach to the phenomenon of neutrino oscillations is to consider it as a manifestation of a mixing between the flavor eigenstates and the mass eigenstates. The mixing angles and the masses of the mass eigenstates are treated independently and are determined largely by the experimental data. There is also an effort to accommodate the available data by making appeal to discrete groups [24, 25, 26]. We offer an alternative approach, by proposing that neutrino oscillations are connected to the structure of space-time. Space-time hosts two branes, one brane where the left-handed particles reside (our brane) and another brane where the right-handed particles reside. The long sought left-right symmetry is achieved through the geometry of space-time. Majorana-type couplings connect the neutrinos living in an individual brane, while Dirac-type couplings connect neutrinos across the branes. We managed to treat at the same time both the masses involved and the mixing angles, by making appeal to first principles. Is this success fortuitous? We consider that even if the theoretical construction is not fully accepted, its phenomenological implication of the existence of two mirror branes is highly interesting. But clearly further indications are needed.

Finally we would like to remind the experimental evidence for a small non-vanishing value for the matrix element c_{13} [27-30]. It seems that this small value indicates a hidden substructure and work along this line is in progress.

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