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ON SOME MODELS OF THE EXOTIC HADRON STATES *

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Abstract. There are several kinds of exotic hadron states. In this paper, we are concerned with the flavor exotic meson states: the tetraquarks and meson molecules. Two distinct models of tetraquarks are studied: diquark-antidiquark bound states and uncorrelated quarks. Although diquark is not a color singlet, there is a strong theoretical evidence that such states do exist within hadrons. Both tetraquark models as well as methods used to study such states are reviewed. The meson-meson molecules are also mentioned. It is remarked that hadron spectroscopy *per se* may not always be sufficient to discern between physically distinct models.

1. Introduction

Conventional mesons are made of one quark and one antiquark and conventional (anti)baryons of three (anti)quarks. From its inception in 1964, it is known that the Quark Model can accommodate unconventional hadrons. The first detailed analysis of exotic hadrons was done by R. L. Jaffe in 1977 [1]. The exotic hadrons have been studied theoretically ever since, but there had been no experimental evidence of unconventional states until ten years ago.

The first hadron states that could plausibly be interpreted as exotic ones were discovered in 2003, but the supposed evidence for pentaquarks turned out to be spurious [2]. However, some meson states – especially certain scalar mesons like $f(980)$ and $a_0(980)$ and heavy mesons like $X(3872)$ – are difficult to interpret as conventional mesons [3]. This is even more true for the newly discovered $Z_c(3900)$ state. Interpretation of some mesons that contain a c-quark – primarily $D_s^+(2317)$ and $D_s^+(2632)$ – as tetraquarks is not excluded either [4].

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All the unconventional hadrons are often termed *exotic hadrons*. There are, however, several physically distinct types of states that can be considered exotic: multiquarks, glueballs, hybrids (states containing both quark and glueballs) and meson molecules. Multiquark is any meson that consists of more than two quarks/antiquarks or any baryon that consists of more than three quarks/antiquarks. Meson molecules are bound states of conventional hadrons and were postulated before the quark model itself.

There is also an alternative classification proposed by R. L. Jaffe [5], based on theoretical considerations. Jaffe divides hadron states into two classes: ordinary and extraordinary. Theoretical behavior of the ordinary and extraordinary hadrons is opposite in the $N_c \to +\infty$ limit: ordinary states decouple from the scattering channels and become infinitely narrow while extraordinary ones become infinitely broad and disappear into the continuum. Multiquark states and meson molecules belong to the extraordinary states. Perhaps unfortunately, Jaffe's classification has not been broadly accepted, although there is a broadly accepted term "flavor-exotic states" [3] that can be considered synonymous to some extent (provided we include the cryptoexotic states, e.g. those considered in Sec. 2).

This review is concerned with the extraordinary mesons: the tetraquarks and, to much smaller extent, the meson molecules. The term "exotic hadrons" in a restricted sense is sometimes reserved to the extraordinary hadrons and this is the meaning of the term in the title of this paper.

Basically, there are three distinct models of the extraordinary mesons:

1) meson molecule (also called meson-meson molecule, sometimes 'deuson' state etc.), which is a bound state of two conventional mesons, i.e. $\langle \bar{q}q \rangle$, $\langle \bar{q}q \rangle$ state;

3) tetraquark as a bound state of a diquark and an antidiquark, $\langle qq \rangle$, $\langle \bar{q}\bar{q} \rangle$ states.

2) tetraquark that consists of uncorrelated quarks, $\langle qq\bar{q}\bar{q} \rangle$ states;

The models are schematically shown on fig. 1.1. Of course, there are also many possible mixings of these states with the conventional meson states.

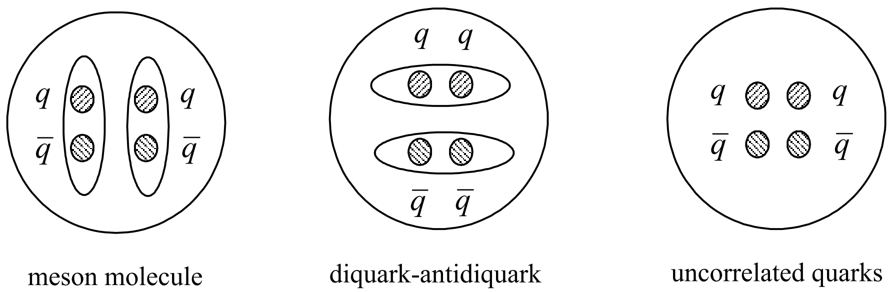

Fig. 1.1: Schematic view of three models of exotic mesons

Several theoretical methods have been used within each model. The phenomenological problem is how to connect the QCD-based calculations with the analysis of the experimental data. There is a growing evidence that some of the experimentally observed states cannot be explained as purely conventional mesons. Hence it is of some interest to compare the results obtained by various theoretical methods to the available experimental data.

One of the most important theoretical concepts in the study of tetraquarks is the diquark/antidiquark. Even though such state is not a color singlet, there is some theoretical evidence that such states can exist within hadrons. Diquarks have been analyzed using various approaches derived from, or inspired by, the QCD. Besides being relevant for the study of tetraquark states, they can be relevant for other particle physics problems, such as quark-gluon plasma.

Along with their relevance for Standard model phenomenology, the heavy exotic states might allow search for certain signatures of new physics.

In this survey we consider the physical models and theoretical methods used to study exotic meson states. In particular, we give some details about the diquark-antidiquark system and on schematic interactions used in hadron spectroscopy.

2. Tetraquark as a Diquark-Antidiquark System

Diquark is a system that consists of two quarks and antidiquark – a system of two antiquarks. Such systems were postulated almost from the beginning of the Quark Model. Shortly thereafter, models of baryons as bound states of quarks and diquarks appeared [6]. Back in 1967, Lichtenberg and Tassi mention hypothetical exotic mesons as diquark-antidiquark systems, but such states were not studied in detail before 1980's since the exotic states in general were not given much attention.

The significance of the concept of a diquark has long surpassed the structure of conventional baryons; it is used in some models of the dense quark matter, analysis of the hypothetical dibaryons (hexaquarks) etc. Diquark can be considered as a particle of negligible size, but also of a size comparable to the hadron size [6].

Quantum Chromodynamics (QCD), as a non-abelian gauge theory, is quite different from the Quantum Electrodynamics. For example, bound state of two quarks is possible unlike bound state of two electrons. In the space of color as the QCD charge the symmetry group is $\mathrm{SU}(3)_c$. In color space, the diquarks belong to the multiplets of the direct product of two triplets: $3 \otimes 3 = \overline{3} \oplus 6$, the antitriplet states being antisymmetric and the sextet states symmetric. The antitriplet is much more tightly bound than the sextet. Clearly, these states *are not color singlets so they can exist only within hadrons* (or hadronic matter in general).

For antisymmetric color wave functions, the spin-flavor wave functions need to be symmetric, which can be achieved if both components are either symmetric or antisymmetric. The second case gives more tightly bound states [3, 7]. For light flavors (u, d, s) the flavor symmetry group is $\mathrm{SU}(3)_f$ so that $3 \otimes 3 = \overline{3} \oplus 6$ for diquarks, fig. 2.1, and $\overline{3} \otimes \overline{3} = 3 \oplus \overline{6}$ for antidiquarks, fig. 2.2. Flavor triplets correspond to spin singlets; these states are more favorable energetically (scalar or 'good' diquarks). The flavor sextets, which correspond to spin triplets, are also admissible (vector or 'bad' diquarks). The mass differences are on the order of 200 MeV [7].

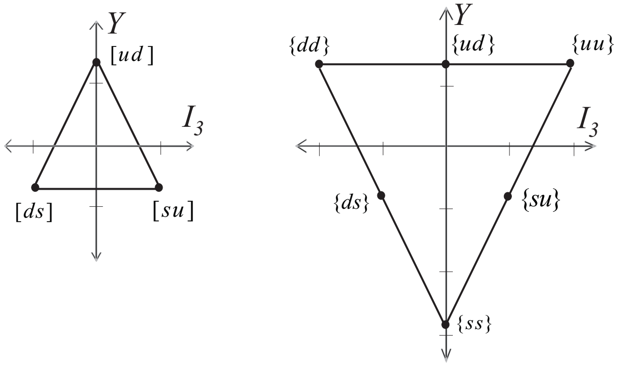

FIG. 2.1: Light flavor diquarks

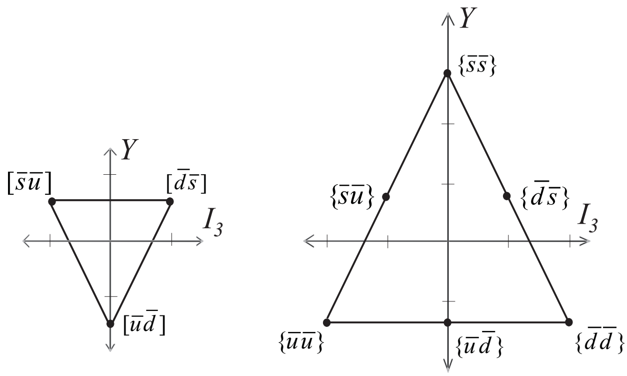

FIG. 2.2: Light flavor antidiquarks

The possible tetraquark states are obtained by expanding the direct product:

$$(\overline{3} \oplus 6) \otimes (3 \oplus \overline{6}) = \overline{3} \otimes 3 \oplus \overline{3} \otimes \overline{6} \oplus 6 \otimes 3 \oplus 6 \otimes \overline{6}, \tag{2.1}$$

where the terms on the right hand side expand as follows: the first term into an octet and a singlet, the second and third ones into decouplets and octets and the fourth one into a 27-plet, an octet and a singlet.

In the simplified Jaffe's model [7], out of all the multiplets in (2.1) only the states from the nonet $\overline{3} \otimes 3 = 8 \oplus 1$ shown on fig. 2.3 are observable while the (much more massive) other ones would probably disappear into the meson-meson continuum. The nonet states are not (openly) exotic but cryptoexotic as they carry the same quantum numbers as the conventional scalar nonet states. They can be distinguished from the conventional states because they have considerably lower masses, which is consistent with some of the experimentally observed states.

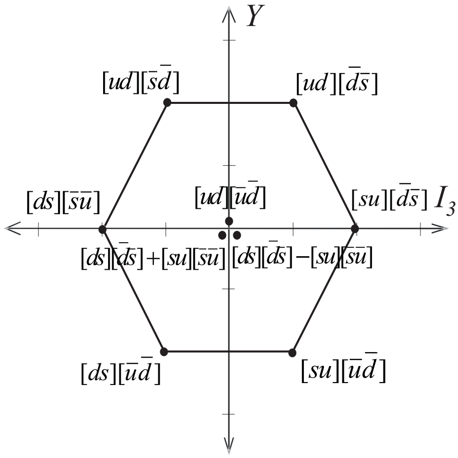

FIG. 2.3: The nonet of light flavor tetraquarks

The isosinglet states shown on fig. 2.3 are given for the $SU(3)_f$ *broken* by the s-quark mass, so they should correspond to physical states. The state $[ud][\bar{u}\bar{d}]$, often identified with the elusive σ-meson, would then be the lightest state. The other non-strange states are a_0 and f_0, supposedly identifiable with the observed $a_0(980)$ and $f_0(980)$ mesons. The strange states are the two doublets of the so-called κ-mesons. The predictions for masses are in a qualitative agreement with the experiment [5]. The interpretation of the $a_0(980)$ and $f_0(980)$ mesons – and scalar mesons in general – is still an open question; there are attempts to explain them as conventional mesons or meson molecules. One should also notice that not all the naturally occurring flavor multiplets need to be complete.

While it is not difficult to build tetraquarks with the c-quark(s) it is obvious not only that the proliferation of states becomes enormous (256 states altogether!), but also that many results derived for the light flavors will be invalidated due to the strong breaking of the $SU(4)_f$ group. On the other hand, if we restrict ourselves to one and only one heavy quark the proliferation of states is moderate.

2.1. Non-Perturbative QCD

Since Quantum Chromodynamics is non-perturbative at low energy, several alternative methods have been developed for the analysis of bound states. As nearly all the methods have been used to analyze the diquarks, they are enumerated here.

The now well-known *MIT bag model* was used by Jaffe in his seminal papers [1].

Chiral perturbation theory is based on the chirally invariant massless QCD, which is then broken by pseudoscalar mesons as Goldstone bosons. The theory gives some results for light quarks in accordance with the experiment.

Lattice QCD starts from the QCD Lagrangian and the effects are computed numerically. The method is computationally extensive. Some results are very promising and they are also used to justify approximations in other – especially purely phenomenological – approaches.

QCD sum rules are based on the subasymptotic QCD [8], which is derived from QCD Lagrangian using Operator Product Expansion (OPE) method. QCD sum rules give many results in quantitative agreement with the experiment, albeit often with fairly large uncertainties. There are many different sum rules, although some of them have been used only by very few researchers.

Constituent quark model is, of course, older than the QCD itself, but it is still used today, although with various model interactions which improve the inaccurate predictions of the naive quark model.

Heavy Quark Effective Theory (HQET), developed in the 1990's, uses the large mass difference between light and heavy quarks.

There are several other methods, e.g. those influenced by more recent theoretical concepts: quark interaction mediated by strings (inspired by the lattice QCD), the theory based on analogy between massless QCD and conformal field theory etc.

3. Uncorrelated Quarks

Tetraquarks made of uncorrelated quarks – where every pair of quarks/antiquarks is treated equally – can be analyzed within the constituent quark model with hyperfine interactions. For example, we can use the following so-called Fermi-Breit Hamiltonian for the interaction of the quarks/antiquarks [9, 10, 11]:

$$H_{\mathrm{FB\ int}} = C \sum_{i<j} \frac{\sigma_i \cdot \sigma_j}{m_i m_j} \lambda_i^c \lambda_j^c, \tag{3.1}$$

where m_i are constituent quark masses (usually, one takes $m_d = m_u$), C a constant, σ_i are Pauli spin matrices and λ_i^c Gell-Mann matrices that belong to $\mathrm{SU}(3)_c$ group.

It is remarkable that a formally very similar Hamiltonian, the so-called Glozman-Riska Hamiltonian, can be constructed with the Gell-Mann matrices belonging to $\mathrm{SU}(3)_f$ group [9, 10]:

$$H_{\mathrm{GR\ int}} = -C_g \sum_{i<j} (-1)^{\alpha_{ij}} \frac{\sigma_i \cdot \sigma_j}{m_i m_j} \lambda_i^f \lambda_j^f. \tag{3.2}$$

The only formal difference with respect to the Hamiltonian (3.1) is that signs differ for quark-antiquark interaction as opposed to quark-quark or antiquark-antiquark interaction [9, 10]:

$$(-1)^{\alpha_{ij}} \equiv \begin{cases} -1 & \text{for } q\overline{q} \\ +1 & \text{for } qq \text{ or } \overline{qq}, \end{cases}$$

quark-antiquark interaction being energetically more favorable. These two interactions are physically completely different; the Hamiltonian (3.1) is a *s*pin-color while the similar Hamiltonian (3.2) is a *s*pin-flavor interaction.

Given the different nature of the spin-color and spin-flavor interactions, it was surprising that the two Hamiltonians lead to almost the same masses [10]. We have shown this to be the case for the conventional mesons as well [9].

Interactions of the type (3.1,3.2) are called schematic. There are also models that include an additional potential that depends on the distance between the quarks/antiquarks within a pair. It is possible to use the more realistic methods enumerated in the previous section, but in this case there are 6 pairs of interactions instead of the 3 in the case of the diquark-antidiquark model.

4. Meson Molecules

Forces between mesons can, especially near the kinematic threshold in meson-meson scattering, be strong enough to form a bound state called a meson molecule, or more precisely *meson-meson* molecule. Meson-meson molecules were considered even before the quark model itself; within the QCD context they have been studied since 1976. Annihilation of such molecular state is not possible so that the interaction is dominated by the exchange of quarks/antiquarks that belong to *different* mesons within the molecule. Molecular states can mix with conventional mesons [3]. Moreover, there is no clear distinction between a tetraquark and a meson molecule [5].

The meson molecules are usually denoted by the constituent mesons. For example, the four states that contain s *and* $\bar{s}$ in the nonet shown in fig. 2.3 can also be interpreted as $K\overline{K}$ molecules, K being the kaon. Given how close the $a_0(980)$ and $f_0(980)$ mesons are to the $K\overline{K}$ threshold, this meson-meson molecule is, as noted above, considered a viable alternative interpretation of these scalar mesons.

Various methods have been used to study the meson molecules as well: effective Lagrangians, potentials etc.

5. Conclusions

Theoretical studies of flavor exotic hadron states within QCD date back to 1976, although the experimental evidence did not appear until 2003. We have considered the flavor exotic (or "extraordinary") mesons. There are three distinct physical models of the flavor exotic meson states: diquark-antidiquark tetraquarks, tetraquarks consisting of uncorrelated quarks and meson-meson molecules. Out of possible 81 light flavor diquark-antidiquark states, only the cryptoexotic nonet states are likely to be observable. Many theoretical methods have been used to study all the three models of the four-quark states. Our previous analysis of tetraquark states using the constituent quark model with hyperfine interactions hints that hadron spectroscopy alone may not be sufficient to discern among physically distinct model interactions.

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