

## DYNAMIC STABILITY OF A CRACKED PIPE CONVEYING FLUID UNDER THERMAL LOADS

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**Abstract.** *In the paper is investigated the effect of temperature load and crack position on the dynamic stability of a cracked straight pipe conveying fluid. The static scheme of the investigated pipe is a beam with restricted horizontal and vertical displacements at both of its ends. The velocity of the transported fluid is constant. The Galerkin method is applied for the solution of the differential equation of the transverse vibrations of the pipe. The differential equation is reduced to a first-order differential equation system. The system of differential equations is transformed and rewritten in a matrix form. The roots of the characteristic equation of the system are obtained by solving the generalized first order eigenvalue problem. A numerical solution for a cracked pipe conveying fluid with specified geometric and physical characteristics has been carried out. The temperature load, the position of the crack and the critical velocity of the fluid are considered as parameters of the problem. The results show that the temperature load and the crack position affect the vibrational characteristics of the pipe, as well as its critical velocity.*

**Key words:** *pipe, fluid, dynamic stability, crack, critical velocity*

### 1. INTRODUCTION

Fluid conveying pipes are considered as a fundamental problem of the dynamics in the fluid-structure interaction area. They are widely used in the petroleum industry for transportation of oil and gas. Another broad use of them is in the transport of water. Pipelines also find application in power plants, aviation, aerospace etc.

Nanoscale tubes are advanced technological products that find broad application in nanobiology and nanophysics. However, the experiments at the nanoscale level are extremely difficult and very expensive. In order to overcome such problems, the continuum elastic models are widely used to study the fluid-structure interaction at nanoscale level. The classical Euler and Timoshenko beam theories are often used as an efficient tool in the study of the carbon nanotubes conveying fluid.

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The flow of the fluid in the tube causes oscillations in it. The dynamic characteristics of the pipe's oscillations depend on the velocity and the mass of the conveyed fluid. The system is stable for flow velocities that are less than a certain value, called critical flow velocity. The research of the dynamic stability of pipes conveying fluid is branched into two directions: a) dynamic stability of pipes with a rectilinear axis and b) dynamic stability of curved pipes.

Thermal loads may induce excessive vibration in the system, leading to loss of stability. Therefore, analysis of the dynamic stability due to thermal loading is essential for the safe operation of the pipeline.

The most common methods used for dynamic analysis of the pipes conveying fluid are the Transfer matrix method (TMM) and the Generalized differential quadrature method (GDQM). Both methods have a significant advantage over the Finite element method (FEM). The conventional FEM can be very time consuming when it comes to investigation of a pipeline with a high number of spans. The order of the overall property matrices for the whole multispans pipeline increases with the number of spans. This is unlike the TMM in which the order of the overall transfer matrix is independent of the number of spans and is kept unchanged.

The GDQM approximates a derivative of a function in the partial differential equation of the lateral vibration of the pipe at any discrete point as a weighted sum of the function values at all discrete value at the domain. The main advantage of the method is its high convergence with a small number of grid points.

Cracks are the most encountered damages in the structures. They reduce the stiffness of the structural element which causes a decrease in its natural frequencies and change in the mode shapes. In pipes conveying fluid, cracks lead to a decrease in the critical velocity of the fluid. The cracks could be hazardous for the system. They might lead to a loss of stability if the reduced, due to the crack, critical velocity of the transported fluid is exceeded. That is why crack detection is a topic of great interest in the scientific research. Some of the studies for crack detection deal with the change in the natural frequencies and eigenforms, other with the dynamic response to harmonic loads.

The interaction of a tube and a fluid flowing in it is the subject of much research. One of the pioneers and the most productive in the area is M. Paidoussis [1], [2]. Numerous articles nowadays analyze the linear and nonlinear dynamics of pipes conveying fluid, proving the topicality of the problem [3]-[8].

In [9] the propagation of transverse and longitudinal waves in fluid conveying pipes with variable thickness along their length is investigated. The effect of the temperature load on the fluid and on the pipe is considered. The influence of the temperature and the fluid velocity on the dynamic characteristics of the pipes' free oscillations was investigated.

Zhou, Ni, Chen, Dai, Peng and Wang in [10] investigate a pipe with flowing fluid that have small geometric imperfections. The differential equations describing the lateral vibration of a pipe with geometric imperfections are formulated and the dynamic stability of the system is investigated. The critical velocity of the flowing fluid is determined.

In [11] is considered the influence of the thermal load and the ratio of the outer diameter of the tube to its length on the oscillations of a tube with a flowing fluid.

A cracked pipe under the influence of a tensile force along the pipe axis and a thermal gradient in the radial direction was investigated in [12]. The stresses at the crack tip were determined.

Samujlo, Longwic, and Lavorgna in [13] examined cracks in polyethylene pipes under variable load and temperature effects.

Dahmane, Boutchicha, and Adjlout presented analytical and numerical modeling for the dynamic study of a pipe with flowing fluid [14]. The circular frequencies at different fluid velocities and three cases of pipe support are obtained - pinned-pinned, clamped-pinned and clamped-clamped.

Dahmane, Zahaf, Benkhettab, and Boutchicha applied the finite element method for a parametric study of the dynamic stability of a pipe-fluid system [15].

Dahmane, Zahaf and Boutchicha carried out numerical studies of the free oscillations of a pipe conducting a hot fluid [16].

The paper is structured as follows. First, the model of the pipe and the governing differential equation of its transverse vibration is presented. The Galerkin method is employed to approach the solution of the problem. It is shown how to obtain the characteristic equation of the problem. Based on its roots conclusions could be drawn about the stability of the system. Second, it is shown how to model the crack with the help of the Castigliano's theorem. Finally, the obtained results from the numerical solution are presented and several important conclusions are summarized.

2. PROBLEM FORMULATION

The present paper uses the Euler-Bernoulli beam theory to investigate the dynamic stability of a pipe of length  $l$ , conveying fluid and subjected to thermal load  $T$ . The pipe, shown in Fig.1, is hinged at both ends. The pipe is supposed to have an open edge crack, which dimensions ( $\theta_c$  and  $b$ ) are shown in (Fig.1).  $b$  is the length of the crack.  $\theta_c$  is the half central angle corresponding to the chord  $b$ . The crack severity is usually measured by the ratio  $\theta_c / \pi$ . The crack position along the length of the tube is fixed through the coordinate  $x_c$ . The crack is modeled as a rotational spring with a lumped stiffness  $k_r$  [17] (Fig.2).

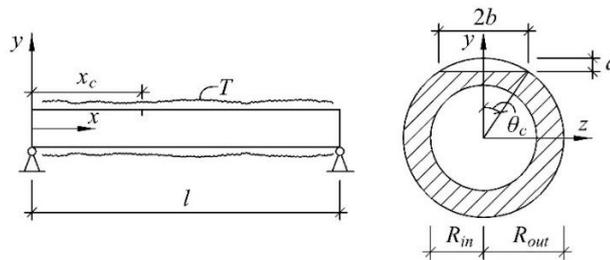


Fig. 1 Static scheme and cross-section of the investigated pipe

The pipe is divided into two segments. The first segment is the left-hand side of the crack, and the second – the right-hand side of the crack.

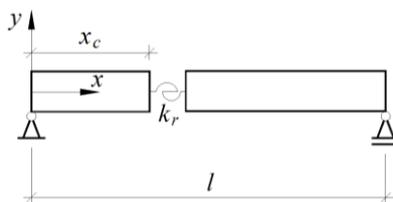


Fig. 2 Mechanical model of the crack

The transverse vibration of a straight pipe conveying inviscid fluid and under thermal load is governed by the following differential equation

$$EI \frac{\partial^4 w}{\partial x^4} + (m_f V^2 + EA\alpha T) \frac{\partial^2 w}{\partial x^2} + 2m_f V \frac{\partial^2 w}{\partial x \partial t} + (m_f + m_p) \frac{\partial^2 w}{\partial t^2} = 0 \quad (1)$$

where  $t$  is the time,  $w(x,t)$  is the lateral displacement of the pipe axis,  $x$  is the coordinate along the axis,  $EI$  is the rigidity of the pipe. The mass of the pipe per unit length is denoted by  $m_p$  and the mass of the fluid per unit length of the pipe by  $m_f$ .  $V$  is the flow velocity of the fluid in the pipe.  $A$  is the area of the cross-section of the pipe.  $\alpha$  is the coefficient of linear thermal expansion of the material of the pipe.

The spectral Galerkin method is applied to approximate the solution of the boundary value problem (1). According to this method, an approximate solution is sought in [18]:

$$w(x,t) = \sum_{i=1}^n y_i(x) z_i(t) \quad (2)$$

In this expression  $z_i(t)$  are unknown functions.  $y_i(x)$  are basic functions satisfying the boundary conditions of the tube. The eigenfunctions for the pipe with stationary fluid ( $V=0$ ) are used as basic functions in the present paper.

The boundary conditions of the cracked simply supported beam, shown in Fig.1 are:

For the left end of the beam

$$y_{1i}(0) = 0 \quad \text{and} \quad y_{1i}''(0) = 0 \quad (3)$$

For the right end of the beam

$$y_{2i}(l) = 0 \quad \text{and} \quad y_{2i}''(l) = 0 \quad (4)$$

For the cracked section of the pipe [15]

$$y_{1i}(x_c) = y_{2i}(x_c) ;$$

$$y_{1i}''(x_c) = y_{2i}''(x_c) ;$$

$$y_{1i}'''(x_c) = y_{2i}'''(x_c) ;$$

$$\left[ y_{1i}^I(x_c) - y_{2i}^I(x_c) \right] \bar{k}_r = y_{2i}''(x_c) \quad (5)$$

For a Bernoulli-Euler tubular beam filled with stationary fluid, one has

$$EI \frac{\partial^4 w}{\partial x^4} + (m_f + m_p) \frac{\partial^2 w}{\partial t^2} = 0 \quad (6)$$

The free vibration of the beam has the form

$$w(x,t) = y(x) e^{i\omega t} \quad (7)$$

where  $\omega$  is the natural frequency of the beam and  $i = \sqrt{-1}$ .

The substitution of (7) in (6) yields

$$y_i^{IV}(x) = \gamma_i^4 y_i(x) \tag{8}$$

where

$$\gamma_i = \sqrt[4]{\frac{(m_f + m_p) \omega_i^2}{EI}} \tag{9}$$

Substituting (2) in equation (1) one obtains the residual function, which does not vanish identically since  $w(x,t)$  is not exact solution of equation (1). Here, and in the sequel, dots denote derivatives with respect to  $t$  and primes denote derivatives with respect to  $x$ .

$$R(x,t) = \sum_{i=1}^n \left[ EI y_i^{IV} z_i + (m_f V^2 + EA\alpha T) y_i'' z_i + 2m_f V y_i' \dot{z}_i + (m_f + m_p) y_i \ddot{z}_i \right] \tag{10}$$

According to the standard Galerkin procedure, the residual function  $R(x,t)$  should be orthogonal to the basic functions in the area  $x \in [0;l]$ :

$$\int_0^l R(x,t) y_k(x) dx = 0 \text{ for } k = 1, \dots, n \tag{11}$$

The result of the application of (11) is a system of  $n$  differential equations about the unknown functions  $z_i(t)$ . This system for the differential equation (1) is:

$$\sum_{i=1}^n \int_0^l \left[ EI y_i^{IV} z_i + (m_f V^2 + EA\alpha T) y_i'' z_i + 2m_f V y_i' \dot{z}_i + (m_f + m_p) y_i \ddot{z}_i \right] y_k dx = 0 \tag{12}$$

For the solution of system (12) is employed the described in [18] method. The pipe is divided to sections with length of  $\Delta x$ . The integrals in (12) are expressed in the following form

$$\int_0^l y_i y_k dx = \begin{cases} 0, & k \neq i \\ \{y_i\}^T \{y_k\} \Delta x, & k = i \end{cases} \tag{13}$$

$$\int_0^l y_i' y_k dx = \{y_i'\}^T \{y_k\} \Delta x \tag{14}$$

$$\int_0^l y_i'' y_k dx = \frac{1}{EI} \{M_i\}^T \{y_k\} \Delta x \tag{15}$$

In equations (13), (14) and (15):

$\{y_i\}$  is a column vector of the lateral displacements of the nodes on the axis of the pipe, corresponding to the  $i$ -th eigenform of a pipe with stationary fluid;

$\{y_i'\}$  is a column vector of the rotations of the nodes on the axis of the pipe, corresponding to the  $i$ -th eigenform of a pipe with stationary fluid;

$\{M_i\}$  is the vector of the bending moments associated with the  $i$ -th mode shape  $\{y_i\}$ .

The substitution of (13), (14) and (15) in (12) yields

$$\sum_{i=1}^n \left\{ (m_f + m_p) \{y_i\}^T \{y_k\} \ddot{z}_i + 2m_f V \{y_i^I\}^T \{y_k\} \dot{z}_i + \left[ EI \gamma_i^4 \{y_i\}^T \{y_k\} + \frac{1}{EI} (m_f V^2 + EA \alpha T) \{M_i\}^T \{y_k\} z_i \right] \Delta x \right\} = 0 \quad (16)$$

Writing equation (16) in matrix form gives:

$$[M] \ddot{z}_i + [C] \dot{z}_i + [K] z_i = 0 \quad (17)$$

The elements of the matrices in (17) are calculated by the following formulas

$$M_{ii} = (m_f + m_p) \{y_i\}^T \{y_k\} \Delta x, \quad M_{ik} = 0 \quad (i \neq k) \quad (18)$$

$$C_{ik} = 2m_f V \{y_i^I\}^T \{y_k\} \Delta x \quad (19)$$

$$K_{ik} = \frac{1}{EI} (m_f V^2 + EA \alpha T) \{M_i\}^T \{y_k\} \Delta x + E_{ik} \quad (20)$$

$$E_{ik} = EI \lambda_i^4 \Delta x, \quad E_{ik} = 0 \quad (i \neq k) \quad (21)$$

The equation (17) could be transformed in the following form

$$\begin{vmatrix} C & M \\ M & 0 \end{vmatrix} \{ \dot{q} \} + \begin{vmatrix} K & 0 \\ 0 & -M \end{vmatrix} \{ q \} = 0 \quad (22)$$

where

$$\{ q \}^T = \{ q_1 = z_1; \dots; q_n = z_n; q_{n+1} = \dot{z}_1; \dots; q_{2n} = \dot{z}_n \} \quad (23)$$

The roots of the characteristic equation of (22) are the eigenvalues of the generalized first order eigenvalue problem

$$\left( \lambda \begin{vmatrix} C & M \\ M & 0 \end{vmatrix} + \begin{vmatrix} K & 0 \\ 0 & -M \end{vmatrix} \right) \{ u \} = 0 \quad (24)$$

In (24)  $\{ u \}$  is the eigenvector of the system.

Conclusions about the stability of the system could be drawn on the basis of obtained roots. When the real part of all the roots of the characteristic equation of (22) is negative, the system is stable.

The roots depend on all the parameters of the system. If all of them are fixed except the velocity of the conveyed fluid  $V$ , one could obtain the corresponding critical velocity.

### 3. CRACK MODELLING

It is considered that the bending vibrations of the Euler-Bernoulli beam is in the plane  $x$ - $y$  (Fig.1), which is also a plane of symmetry for the cross-section. The crack is assumed to be open. The Castigliano's theorem is used to obtain the local flexibility in the presence of the crack [20].

$$c = \frac{\partial^2 U}{\partial M^2} = \frac{1-\nu^2}{E} \int_{-b}^b \int_0^a \frac{\partial^2 (K_I^2)}{\partial M^2} dx dy \quad (25)$$

where  $E$  and  $\nu$  are respectively Young's module and Poisson's ratio.  $K_I$  is the stress intensity factor of bending.  $a$  and  $b$  are the crack dimensions as shown in (Fig.1).  $M$  is the bending moment.

$$K_I = \frac{M}{\pi R^2 t_p} \sqrt{\pi R \theta_c} F(\theta_c) \quad (26)$$

where  $R = 0,5(R_{in} + R_{out})$ ,  $t_p$  and  $\theta_c$  are respectively thickness of the pipe and the half central angle of the crack (Fig. 1).  $F(\theta_c)$  is calculated from the following formula [19]

$$F(\theta_c) = 1 + A_t \left[ 4,5967 \left( \frac{\theta_c}{\pi} \right)^{1,5} + 2,6422 \left( \frac{\theta_c}{\pi} \right)^{4,24} \right] \quad (27)$$

$$A_t = \sqrt[4]{\frac{1}{8} \frac{R}{t_p} - \frac{1}{4}} \quad \text{for } 5 \leq \frac{R}{t_p} \leq 10 \quad (28)$$

$$A_t = \sqrt[4]{\frac{2}{5} \frac{R}{t_p} - 3} \quad \text{for } 10 \leq \frac{R}{t_p} \leq 20 \quad (29)$$

The equivalent rotational spring stiffness

$$k_r = \frac{1}{c} \quad (30)$$

### 3. RESULTS AND DISCUSSION

Numerical studies have been carried out for the system in Fig. 1

The geometric and the material characteristics of the pipe are: the inner and the outer radii of the cross-section of the pipes –  $R_{in} = 0.012m$  and  $R_{out} = 0.014m$ , Young's modulus  $E = 210GPa$ , coefficient of linear thermal expansion  $\alpha = 10^{-5}C^{-1}$ , the density of the material of the pipe  $\rho = 7800 kg/m^3$ . The density of the flowing fluid is  $\rho = 1000 kg/m^3$ . The dimensions of the crack are  $a=1 mm$ ,  $b=5 mm$ .

The finite element method was used to obtain the basic functions  $y_i(x)$ . The eigenfunctions for the pipe with stationary fluid ( $V=0$ ) are used as basic functions in the present paper. The first 14 modes were used in the present calculations.

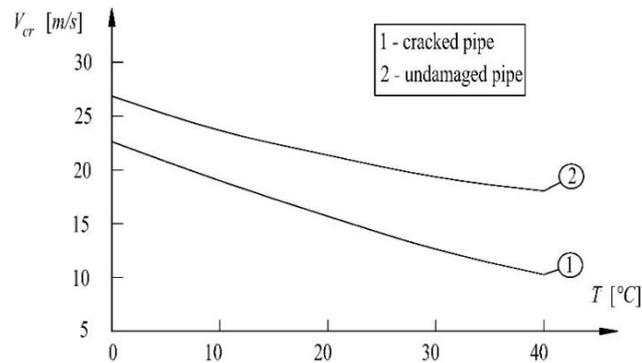
At first the position of the crack is fixed with the coordinate  $x_c/l = 0,33$  at the top edge of the beam. The aim is to investigate the influence of the temperature load on the stability of the system.

On the basis of obtained roots of characteristic equation could be drawn conclusions about the stability of the system. The system is stable if the real part of all the roots is negative. If one or more roots have positive real parts, the system is unstable. When one

or more roots of the characteristic equation have real parts equal to zero the system is at the edge of loss of stability, the corresponding fluid velocity is the critical fluid velocity. The roots depend on all the parameters of the system. If all of them are fixed, except the velocity of the conveyed fluid  $V$ , one could obtain the corresponding critical velocities.

The obtained results for a cracked pipe are compared with the results of an undamaged pipe.

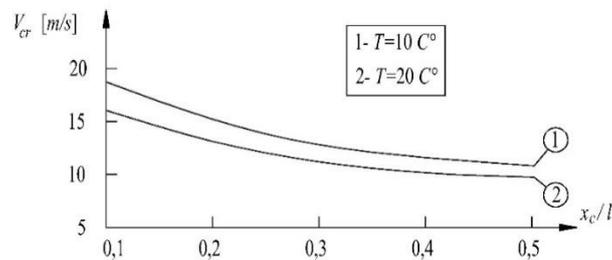
For the pipe in Fig.1 are obtained the critical velocities for different values of the temperature  $T$ . The results are shown in Fig.3.



**Fig. 3** Critical velocity versus the temperature load

The obtained results show that the temperature load has a destabilizing effect on the pipe - with increasing the temperature the critical velocity decreases. The crack also has a destabilizing effect on the system, leading to decreasing of the critical velocity.

In the second part of the survey the influence of the position of the crack on the stability of the system was investigated.



**Fig. 4** Critical velocity versus the position of the crack

When the coordinate of the crack  $x_c/l$  increases, the critical flow velocity decreases. The system is less stable when the crack is in the middle cross-section of the pipe  $x_c/l = 0,5$ .

#### 4. CONCLUSIONS

A numerical simulation procedure for predicting Thermal loads on a structure could affect its integrity if they are not taken into account in the design process. The structures are subject to daily and seasonal temperature changes due to their exposure to outdoor air temperature, solar radiation or underground temperature. In the past thermal stresses have caused failures in the structures. Understanding the effect of the thermal loads on the structures, and how to minimize them, significantly reduces the risks of failure or serious damages and prevents from high repair costs.

Cracks are most encountered damages in the structures. When a structure is cracked its stiffness is reduced, with consequent reduction in the natural frequencies and a change in the eigenforms.

In the present paper is studied the influence of the temperature on the stability of a cracked pipe conveying fluid.

The pipe is modelled as two segments connected by a rotational elastic spring at the cracked cross-section. Castigliano's theorem is employed to calculate the stiffness of the spring. The spring stiffness depends on the geometry of the cross-section of the pipe and the severity of the crack.

The results obtained in the study could be summarized as follows:

Increasing the temperature has a destabilizing effect on the system. That means that the fluid must flow through the pipe with lower velocity in order not to occur loss of stability of the system.

The position of the crack affects the stability of the system. If the crack severity remains unchanged the position of the crack along the length of the pipe affects the stability of the system. The closer the crack to the middle of the span, more unstable is the system.

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The results obtained contribute to the safety of pipes conveying fluid. In order to avoid damages, the operator of the pipe should not allow higher transportation velocities than the critical velocity of the system. As the critical velocity depends on many parameters of the system, among them the severity and position of the crack, the operator of the pipe, should strictly perform crack detection test and based on the results to correct the velocity of the fluid in order the damaged system not to lose stability.

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## **DINAMIČKA STABILNOST NAPUKNUTE CEVI KOJA PROVODI TEČNOST POD TERMIČKIM OPTEREĆENJEM**

*U radu se istražuje uticaj temperaturnog opterećenja i položaja prsline na dinamičku stabilnost napuknute prave cevi koja transportuje fluid. Statička šema ispitivane cevi je greda sa ograničenim horizontalnim i vertikalnim pomeranjima na oba kraja. Brzina transportovane tečnosti je konstantna. Za rešenje diferencijalne jednačine poprečnih vibracija cevi primenjena je metoda Galerkina. Diferencijalna jednačina se svodi na sistem diferencijalnih jednačina prvog reda. Sistem diferencijalnih jednačina se transformiše i prepisuje u matičnom obliku. Koreni karakteristične jednačine sistema dobijaju se rešavanjem generalizovanog problema sopstvenih vrednosti prvog reda. Urađeno je numeričko rešenje za napuknutu cev koja prenosi fluid sa određenim geometrijskim i fizičkim karakteristikama. Kao parametri problema se smatraju temperaturno opterećenje, položaj prsline i kritična brzina fluida. Rezultati pokazuju da temperaturno opterećenje i položaj prsline utiču na vibracione karakteristike cevi, kao i na njenu kritičnu brzinu.*

**Ključne reči:** *cev, tečnost, dinamička stabilnost, pukotina, kritična brzina*