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EFFECTS OF DIFFERENT PRESTRESS INTENSITIES ON THE DISPLACEMENT OF MEMBRANE STRUCTURES UNDER POINT LOADS

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Abstract. *Membrane structures exhibit large displacements under external loads. These are a consequence of the low stiffness of the membrane. One of the ways to increase the stiffness of the structure is to increase the prestress intensity of the membrane or of the cables. This paper presents a research done on the influence of the prestress intensities on the displacements under point loads since point loads have a significant impact on the displacements in membrane structures. In this research the prestress of the membrane and the prestress of the edge cables have been investigated separately. Ten different intensities of the membrane prestress, and five different intensities of the cable prestress, combined with four positions of the point load have been modeled. The intensity of the point load has been kept constant in all loads cases. The research is conducted on numerical models in software Sofistik. Displacements under given parameters of the membrane structure and the point loads were recorded and maximal displacements were compared. The obtained results provide a better understanding of the relation between the prestress intensities and displacements of saddle shaped membranes. The outcome of the research can be used to better predict and to reduce the displacements of membranes under point loads.*

Key words: *membrane structures, point load, prestressing, displacements, double curvature, tensile structures*

1. INTRODUCTION

Membrane structures are double curved tensile structures. They have been widely used in the world since the mid-twentieth century, while in the Republic of Serbia several membrane structures have been built in the recent years. The most common use of membranes is to cover public spaces and facilities. Since they provide low thermal resistance [1] they are less frequently used to fully enclose a space. Their main function is to protect from the sun and rain, wind and snow, and to upgrade the visual environment [2].

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Membrane structures are very specific in the structural sense. Their form is a result of a parametric process called “formfinding”. Under given parameters, a specialized software calculates the shape with respect to the minimal energy concept [3]. The form should always be double curved with negative Gaussian curvature [4]. This form enables membranes to be constantly under tension [5], which allows for a special structural material to be used [6]. The use of 1 mm thick membrane material results in extremely low self-weight, low stiffness of the structure [7] and large displacements under external loads [8]. When an external force acts on a membrane, it causes the membrane material to strain, and thus change its geometry, and the geometry of the entire structure until the equilibrium between external and internal forces is reached. These displacements can be as big as 1/10 of the span of the structure, which is much larger compared to traditional building materials. Because of their amplitude, displacements can be a problem for the membrane structures. Large deflections analysis of membrane structures has already been investigated [9-11]. Deflections of a rubber membrane under point loads were also researched [12], but this type of membrane is not used in architecture.

Fig. 1 NAVAK in Serbia, constructed by ArTech, exterior and interior view

This research explores the correlation between the prestress intensities and displacements under external point loads in membrane structures. There are two types of prestressed elements in membrane structures: one is the membrane itself, and the other one is the edge cable. These two are researched independently, and the results are shown in this paper. The point load is selected as this research is part of a wider research investigating the overall effects of point loads on membranes. The results presented in this paper lead to a better understanding, and possible reduction of displacements under external loads in membrane structures.

Fig. 2 Famous membrane structures in the world: SkySong and O2 Arena

2. METHODOLOGY

This research has been conducted on a numerical model in software Sofistik 2012. The numerical model of a membrane structure is saddle shaped, with the ground dimensions of 5×5 meters, and with two opposite corners at a height of 1.5 meters. All four point supports are fixed. The membrane material is modeled with the following properties: elastic modulus of 600 kN/m, shear modulus of 25 kN/m, compression modulus of 0 kN/m and Poisson's ratio of 0.4. The thickness of the membrane material is 1 mm. The membrane is divided into finite elements of about 0.25×0.25 meters. The direction of the warp is from one elevated corner to the other, and the weft direction is perpendicular to warp. Applied point load has the intensity of 0.5 kN, and a vertical downward direction. The position of the point load is in the middle of the structure, and at three other characteristic points. Since the structure is doubly symmetric to its diagonals, these four points represent the typical behavior of the whole membrane.

In order to research the effects of prestress intensities of the membrane and the edge cables separately, two different models were created. Model 1 has fixed edges, and Model 2 has prestressed cables on the edges. The models are shown in Figure 3. The position of the point loads is shown in Figure 4.

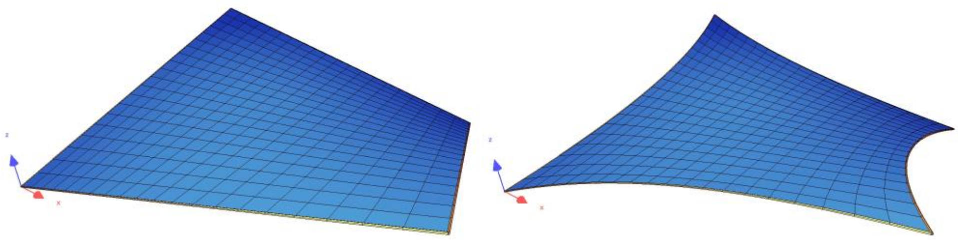

Fig. 3 Models 1 and 2

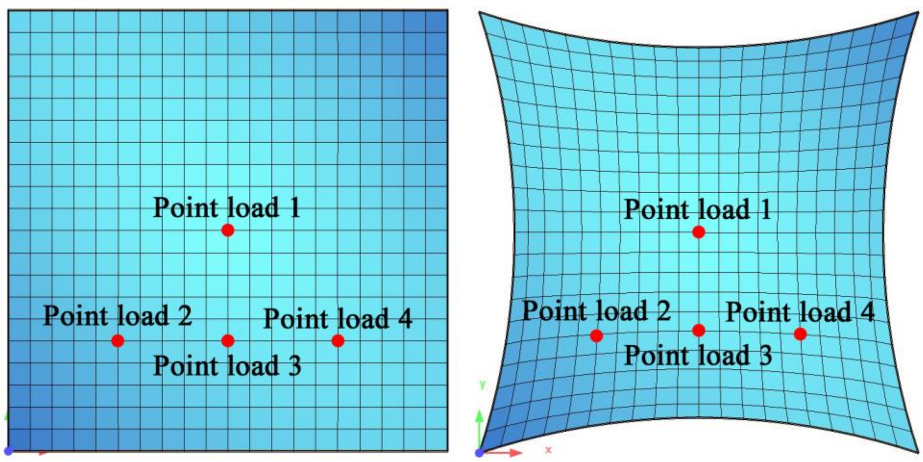

Fig. 4 Position of point loads on Models 1 and 2

The effects of different membrane prestress intensities were investigated on Model 1. The prestress of the membrane is defined through the intensities in warp and weft direction of the membrane. Recommended minimal prestress value for PVC/polyester membrane materials is 1.3%, and for PTFE/fiberglass membrane materials is 2.5% of the average tensile strip capacity in both warp and weft [13]. In this research, warp and weft prestress intensities were taken to be equal. If they are not equal, the membrane form will consequently change. Ten different intensities were taken into account, ranging from 1 kN/m to 10 kN/m, in both directions. After each change of prestress intensity, the loads were applied separately, and the displacement results were recorded. In addition to the impact on the displacements, changes in prestress also have an effect on the internal forces of the membrane during external loading [14].

The effects of different cable prestress intensities were investigated using Model 2. All four edge cables have the same values of prestress, and the diameter of cables is set to 18 mm. The intensities of the cables are taken to be 4, 8, 12, 16 and 20 kN. The membrane prestress is kept constant in all cases and it is 1 kN/m in both directions. Once again, after each change of prestress intensity, the load is applied, and the displacement results are recorded.

3. Results and Discussion

There were a total of 60 different analyzed models and load cases in this numerical experiment. The results obtained by varying the membrane and cable prestress intensities are presented and discussed separately in the next two subchapters.

3.1. Varying the prestress of the membrane

The displacements of Model 1 membrane under four different point loads are shown in Figure 5. Since the structure is doubly symmetric to its diagonals, and also centrally symmetric, these four points actually provide results for nine different positions on the structure.

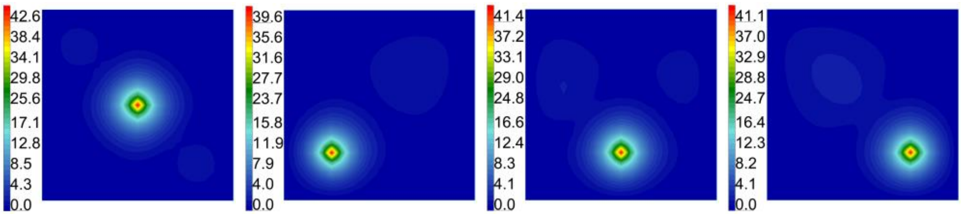

Fig. 5 Displacements of the Model 1 under different position of point loads with prestress of the membrane of 5 kN/m

Displacements of the membrane are highly concentrated around the position of the point load. Away from this position displacements rapidly decrease, so most of the membrane will suffer only minor change in geometry. The maximal displacement occurs exactly at the point where the load is applied [15]. Since maximal displacement is being checked in structural analysis of the membrane structures as the most important information about displacements, it will be elaborated in the further results. Next, the relation between the displacements and the membrane prestress is established. Figure 6 shows maximal displacements under four point loads depending on different prestress values of the membrane. The values of all four point loads are kept constant at 0.5 kN, and the point loads are applied separately.

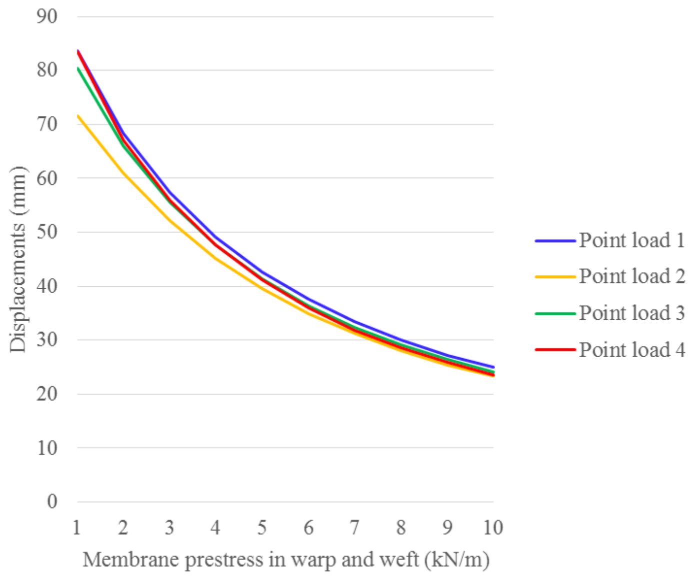

Fig. 6 Maximal displacements depending on the membrane prestress

The graph in Figure 6 shows the relation between the maximal displacements of the membrane and the intensity of the membrane prestress under different positions of the point loads. This relation is highly nonlinear. The results show that the increase of the membrane prestress will lead to reduction of the displacements under point load. This conclusion is valid regardless of the position of the point load. The position of the point load has an important impact on the position of the maximal displacements, but a less important role in defining the value of maximal displacement. It is notable that only the displacements under point load 2 have significantly lower values of maximal displacements compared to the other three. Point load 1 in the center of the membrane induces the highest displacements during all prestress intensities. At lower prestress intensities the point load 4 has somewhat higher displacements compared to the point load 3, while at higher prestress intensities the opposite occurs.

Membrane structures have very low stiffness and thus rely on internal forces and change of geometry to counteract the external loading. The results of this research show that increase of prestress intensity can be a method of reducing the displacements under point loads. If the internal forces, introduced through prestressing of the membrane, are higher, the membrane will not have to change its geometry as much in order to reach the equilibrium with external forces. However, care should be taken to avoid overloading the membrane. If the prestress forces are high, large external loads can induce critical values of the tension forces in the membrane. In addition, the membrane will elongate more under higher prestress, which can lead to wrinkling of the membrane in some cases.

3.2. Varying the prestress of the cables

The change of cable prestress has a direct effect on the form of the structure. Figure 7 shows different shapes of the membrane, depending on the cable prestress intensity. The recommendation [16] states that the cable sag divided by its span should be more than 0.1 in order to have the optimal cable forces, but this also leads to cables that are too curved from the aesthetic point of view. The value between 0.1 and 0.05 is the most commonly used, and is architecturally acceptable. Sag to span value lower than 0.05 results in large diameters of the cables due to high forces in cables. In this research five different cases are analyzed. The cable prestress forces were set to 4, 8, 12, 16 and 20 kN in the formfinding stage, thus the sag to span ratios are 0.13, 0.08, 0.06, 0.04 and 0.03 respectively. The second and third case have the optimal cable curvature from structural and architectural point of view, while the sag of the first case is too large, and the forces in the last two are too high. The membrane prestress is 1 kN/m in both directions, in all cases.

4 kN 8 kN 12 kN 16 kN 20 kN

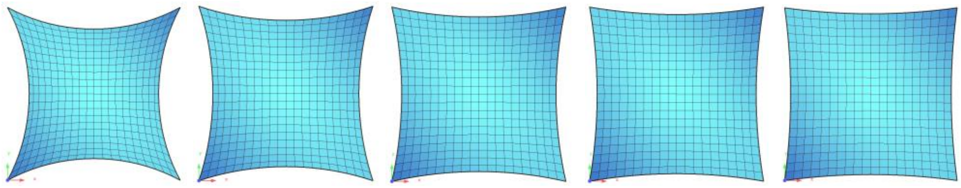

Fig. 7 Change of membrane form depending on the cable prestress intensity

The expected result was that the increase in cable prestress will lead to the overall increase of stiffness of the structure, and will thus result in reduction of the displacements under point loads. However, the results favored the opposite case. The results of this part of the research are shown in Table 1.

Table 1 Maximal displacements in mm, depending on the cable prestress intensity

Cable prestress	4 kN	8 kN	12 kN	16 kN	20 kN
Point load 1	82.7	85.1	86.9	88.0	88.6
Point load 2	78.2	80.3	82.3	84.2	85.9
Point load 3	81.6	84.0	86.2	87.9	89.3
Point load 4	85.7	86.5	87.2	88.2	89.2

The assumptions about the effects of the increase of the cable prestress on the displacements under point loads turned out to be wrong, since the increase of cable prestress does not lead to the reduction of the maximal displacements. To the contrary, it results in a slight increase of displacements. The second unexpected result is how small the changes in displacements are when the prestress of the cables is varied.

After carefully analyzing the obtained results, the following conclusions were drawn. Increase in cable prestress makes the cables more straight, thus the distance between the point loads and the cables increases. The membrane strains close to the position of the point load. Because of the localized displacements, analyzed point loads have very low impact on the cables. Cables provide more stiffness compared to the membrane, and should help in reducing the displacements, but the farther they are the less they can affect the concentrated deflections. Even the point loads 2, 3 and 4 which are closer to cables show the same behavior. We can conclude that increase in cable prestress while maintaining constant membrane prestress in the formfinding phase will lead to the increase in displacements under point loads, which is not desirable. Both lower cable forces and lower displacements in membrane structures are preferable from the structural point of view.

4. CONCLUSION

The research investigating the relation between the prestress values and the displacements of the membrane was done as a numerical experiment in software Sofistik. A total of 60 different combinations of prestress intensities of the membrane and the cables combined with the four positions of the point loads were analyzed. The prestress of the membrane and the prestress of the cables were varied independently from each other. The membrane prestress was varied on a model with fixed edge supports to avoid the interfering of the edge cable prestress with the results. The second model with different cable prestress values was used to find the connections between the cable prestress and the maximal displacements of the structure. This model has a constant membrane prestress. To achieve this, formfinding had to be done for every different cable prestress value.

The results of the conducted numerical experiment were substantially different from the expected results. The experiment showed that the change of the membrane prestress has a much larger influence on membrane displacements under point loads compared to the cable prestress. While varying the membrane prestress in the rational range can change the displacements up to 50%, the varying of cable prestress leads to changes in displacements of well below 10%. As expected, the increase of membrane prestress will result in the reduction of displacements, since the structure increases its stiffness. However, the increase in cable prestress in the formfinding stage has a negative effect on the displacements, making them increase. This is explained by change of the form of the membrane during formfinding, which increases the distance between the point loads and the cables. Cables with higher prestress provide more support for the membrane, but are further away, and because of the large strain of the membrane the point where the load is applied suffers a greater change in geometry. These conclusions apply to all analyzed positions of the point loads. Four different analyzed positions actually provide results for nine different points on the structure because the structure is doubly symmetric to its diagonals. These nine positions give a good representation of the overall structural behavior of the membrane.

Finally we can conclude that the increase of membrane prestress is a choice for reducing the excessively large membrane displacements under point loads. However, when using this approach special care should be taken not to exceed the allowed forces in the membrane. This requirement should be carefully checked for every designed load case and not just for the one where the maximal displacements are present. On the other hand, the increase of cable prestress in the formfinding phase does not have a positive effect on reducing the membrane displacements under point loads. Thus this approach should be abandoned. Instead, above-mentioned recommendations for the optimal cable prestress force and sag should be respected. These conclusions should help structural engineers designing membrane structures to better understand, to predict and to be able to reduce the displacements under point loads.

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UTICAJ PROMENE INTENZITETA PREDNAPREZANJA NA POMERANJA MEMBRANSKIH KONSTRUKCIJA POD DEJSTVOM KONCENTRISANE SILE

Membranske konstrukcije imaju velika pomeranja pod dejstvom spoljašnjeg opterećenja. Ova pomeranja su posledica male krutosti membrane. Jedan on načina za povećanje krutosti konstrukcije je i povećanje sile prednaprezanja u membrani ili u kablovima. U ovom radu predstavljeno je istraživanje uticaja intenziteta prednaprezanja na pomeranja pod dejstvom koncentrisane sile, budući da koncentrisane sile izazivaju velika pomeranja membranskih konstrukcija. Uticaji intenziteta prednaprezanja u membrani i u kablovima su istraživani zasebno. Analizirano je deset različitih intenziteta sile prednaprezanja u membrani i pet intenziteta sila prednaprezanja u kablovima, u kombinaciji sa četiri različite pozicije dejstva sile. Intenzitet sile nije variran tokom ispitivanja. Istraživanje je sprovedeno na numeričkim modelima u softveru Sofistik. Pomeranja pod zadatim parametrima konstrukcije i opterećenja su beležena, i maksimalna pomeranja su međusobno upoređivana. Dobijeni rezultati pružaju bolji uvid u odnos između intenziteta prednaprezanja i pomeranja membranskih konstrukcija sedlastog oblika. Rezultati istraživanja mogu pomoći boljem predviđanju i smanjivanju deformacije membrane pod dejstvom koncentrisane sile.

Ključne reči: *membranske konstrukcije, koncentrisana sila, prednaprezanje, deformacije, dvostruka zakrivljenost, zategnute konstrukcije*