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SENSITIVITY ANALYSIS OF TIMBER-CONCRETE COMPOSITE STRUCTURES

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Abstract. *The sensitivity analysis could be defined as a study of how the variability of the output parameter of the considered model can be distributed to its sources, actually, on the variability of the various input model parameters. It helps to identify the most important design parameters of a particular structure and to focus on them during the design and optimization process. This paper is focused on the application of stochastic sensitivity analysis of maximum equivalent stress and maximum mid-span deflection of timber-concrete composite beam. All input parameters were considered to be random variables. Latin Hypercube Sampling numerical simulation method was employed. The estimation of the sensitivity was derived from Spearman rank-order correlation coefficient.*

Key words: *sensitivity analysis, timber-concrete composite, simulation method, Spearman rank-order correlation*

1. INTRODUCTION

The design of building structures is a comprehensive process in which the most favorable design solution should be proposed. The design parameters that describe the proposed geometry of the structure, the material properties and the all influences on the structures should be considered during the design process. In order to more efficiently develop design proposals it is a great benefit to be able to identify the most important input parameters that have the greatest impact on a particular output parameter, such as, load-carrying capacity or deformation.

The sensitivity analysis could be defined as a study of how the variability of the output parameter of the considered model can be distributed to its sources, actually, on the variability of the various input model parameters [1]. It provides us with the ability to

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identify the most important design parameters of a particular structure and helps to focus on them during the design and optimization process. It is desirable to perform the sensitivity analysis at an earlier design stage, when it is possible to change the most important design parameters.

The sensitivity analysis can generally be divided into two basic categories: the deterministic and the stochastic sensitivity analysis. The deterministic sensitivity analysis is often referred to as design sensitivity, because it is often used in the design of structures. It uses a design model that allows a successive change in the value of the input parameters of the design model, considering the effect of their change on the desired output parameter. The basic principle of this analysis is that the estimation of the variability of the output design parameter is based on the variation of only one input parameter, while the other design parameters have a constant value. This study represents the simplest way of considering the relative impact of different input design parameters that vary within its range by comparing the obtained results from the observed output parameter in each calculation step. Although this study is easy to apply in practice and it provides a quick overview of the performance of the design model, it neglects the correlation among the input design parameters that certainly exist in real conditions. Also, it cannot perceive significant information about the character of the variability of the input design parameters. However, the stochastic sensitivity analysis provides much more complex information about the design parameters because they are considered as the random variables with a certain distribution. Unlike the deterministic sensitivity analysis, the variability of a particular output parameter due to a single input parameter is evaluated by varying all other design parameters at the same time. The influence of other design parameters is relevant for consideration, because in real conditions overall characteristics of structure depend on the correlation of all design parameters.

2. STOCHASTIC SENSITIVITY ANALYSIS BASICS

The methodology of conducting a stochastic sensitivity analysis is the same for different fields of its application, but the difference is in defining the basic steps that may vary from case to case. Based on the review and analysis of available literature [2,3,4,5], we have defined the basic steps in the stochastic sensitivity analysis:

1) Defining the design model of the structure based on the selected input design parameters that we want to include in the analysis as well as defining the output parameters taking into account the questions that should be answered by this sensitivity analysis
2) Characterization of the input design parameters defining the distribution and characteristics of each random variable
3) Generating a sample of the input design parameters through the use of an appropriate random sampling method
4) Conducting a series of numerical simulations over a design model in order to calculate the output parameter
5) Evaluation of the impact and relative importance of each input design parameter on the observed output parameter

In order to conduct the stochastic sensitivity analysis, it is necessary to use numerical simulation methods. They are mainly based on calculating a deterministic problem several times, each time with a different set of input data. The generated samples of the input design parameters present input data sets for numerical simulations and can be represented by the matrix *N*x*M*. Each row in this matrix represents the input data of the independent numerical simulation. Matrix element x_{ij} represents the individual sample of each input design parameter.

$$\begin{bmatrix} x_{11} & x_{12} & \dots & x_{1m} \\ x_{21} & x_{22} & \dots & x_{2m} \\ \dots & \dots & \dots & \dots \\ x_{n1} & x_{n2} & \dots & x_{nm} \end{bmatrix} \rightarrow \begin{bmatrix} y_1 \\ y_2 \\ \dots \\ y_n \end{bmatrix} \tag{1}$$

The Monte Carlo sampling method is the most common method for the generation of such a matrix. The Latin Hypercube Sampling (LHS) method [6], which is applied in this paper, represents advanced and more efficient form of the Monte Carlo sampling method. This method is very popular, because its efficient method of spreading domains allows obtaining a large amount of information on variability and sensitivity based on a relatively small sample. The LHS method generates samples of input design parameters, taking into account the predetermined probability distribution for each of the input parameter. This method ensures that each input parameter has full coverage of its range.

In general, we can distinguish two types of stochastic sensitivity analysis: methods based on variance analysis and methods based on regression and correlation. Methods based on variance analysis perform the distribution of the total variance of the output parameter to the input parameter variance, while the methods based on regression and correlation use the regression of the output parameter to the input parameters [7].

In this paper, the estimation of the sensitivity will be derived from Spearman rank-order correlation coefficient (ρ) [8] which is defined as follows:

$$\rho_i = 1 - \frac{6\sum d_i^{\ 2}}{n(n^2 - 1)} \tag{2}$$

where d_i is a difference between each pair of ranges of the corresponding variables X_j and *Y*, and n denotes the number of pairs of samples.

Spearman rank-order correlation coefficient ρ_i indicates the strength of a monotone connection between the variables and can have a value of +1 to -1. The value +1 indicates the perfect link of ranks, while the value -1 shows the perfect negative link of ranks. On the other hand, when the value of the coefficient ρ_i is equal to zero, this indicates the absence of a connection between the ranks.

3. Application of Sensitivity Analysis in the Design of Timber-concrete Composite Beams

Timber-concrete composite (TCC) structure is a structural system in which the timber beam is connected to the concrete flange using different types of shear connectors. In this paper we will consider a simply supported TCC beam of span 4m. The concrete flange is made of concrete strength class C25/30 and the timber beam is made of Spruce, sawn soft-wood strength class C27. The dimensions of the considered beam have been adopted to present the usual dimensions of TCC floors (Figure 1). Based on the literature review about TCC floors, we have adopted the width of the concrete slab (b_c) to be 70 cm, while its height (h_c) is 7 cm. The width of the timber beam (b_t) is 10 cm, and its height (h_t) is 20 cm.

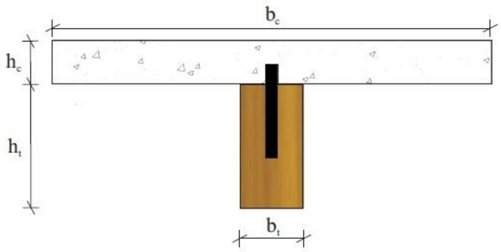

Fig. 1 Timber-concrete composite cross section

The observed TCC beam is loaded with imposed load of 3 kN/m^2 which is recommended for residential and office buildings, based on Eurocode 1 [9]. In our case, the connection between the timber and concrete part was achieved by the glued-in steel rods ϕ20/150 *mm*, made of the steel grade S235 and placed on every 24 cm. Steel rods are installed at right angles to the beam in the previously drilled holes and coated with an epoxy resin [10].

3.1. Numerical model of timber-concrete composite beam

In recent years, numerical models of structures are increasingly used in design process to simulate their behavior. Generation of the numerical model of the TCC beam was carried out using the finite element method in the software package ANSYS Workbench 15.0. This analysis uses the symmetry of the beam and the load. Therefore, the 3D model of the half of the beam is generated, which is shown in Figure 2. The concrete is modeled using the finite elements SOLID 258. The timber is modeled as an orthotropic material, using SOLID158 elements. The steel used for shear connectors is modeled using SOLID158 elements, as well.

The TCC beam is modeled considering the different friction coefficients between the contact surfaces of the constituent elements. The following friction coefficients between the materials were used: timber-concrete 0.5 [11], timber-steel 0.57 [12] and steel-concrete 0.9 [13]. The contact elements TARGE170 and CONTA174 were used to model the contact between the different parts of this TCC beam.

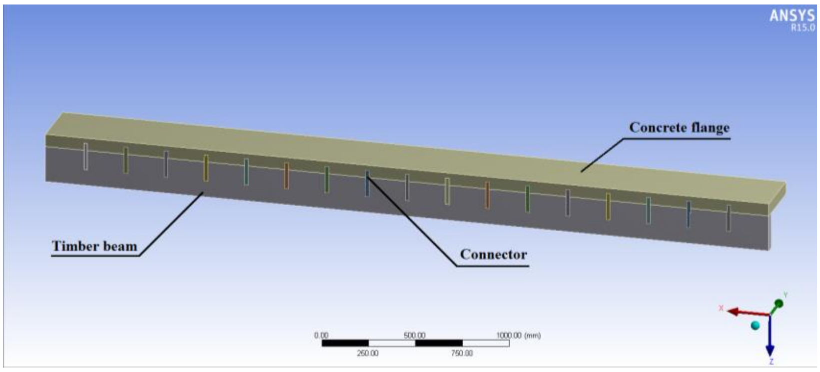

Fig. 2 3D model of the half of TCC beam

3.2. Selection and characterization of the input design parameters

Taking into account all the parameters that make the previously generated numerical model, would make the sensitivity analysis too cumbersome. Therefore, we have selected a number of design parameters that are shown in Table 1. The characterization of the selected input parameters will be determined based on the JCSS Probabilistic Model Code [14]. Table 1 gives an overview of the selected probability distribution, mean value and coefficients of variation for each of the selected input parameters involved in this sensitivity analysis.

Table 1 Input design parameters involved in this analysis

Design parameter	Quantity	Dimension	Distribution	Mean	Coefficient of variation
Concrete flange width	b_c	*mm*	Normal	700	0.02
Concrete flange height	h_c	*mm*	Normal	70	0.02
Timber beam width	b_t	*mm*	Normal	100	0.01
Timber beam height	h_t	*mm*	Normal	200	0.01
Diameter of the connector	d_s	*mm*	Normal	20	0.02
Number of connectors	n_s	*kom*	Uniform	17	
Concrete Young's modulus	E_c	*MPa*	Lognormal	31000	0.05
Density of the concrete	ρ_c	kg/m^3	Normal	2300	0.04
Timber Young's modulus	E_{tx}	*MPa*	Lognormal	12000	0.13
Density of the timber	ρ_t	kg/m^3	Normal	450	0.1
Steel Young's modulus	E_s	*MPa*	Lognormal	210000	0.13

3.3. Sensitivity analysis of the output parameter

The results of the conducted sensitivity analysis can be presented in different ways, but in engineering the common way is using the bar chart. Input parameters with a higher correlation coefficient have a greater influence on the observed output parameter, while the input parameters with a correlation coefficient whose value is close to zero have a low impact on the observed output parameter. The positive sensitivity of an input parameter indicates that as the value increases, the value of the observed output parameter increases as well. However, the negative sensitivity shows that with the increase in its value, the value of the considered output parameter decreases.

3.3.1. Sensitivity analysis of maximum equivalent stress

Firstly, we will present the results of the stochastic sensitivity analysis of the maximum equivalent stress σ_e, given in Table 2.

Table 2 Spearman rank-order correlation coefficients of the maximum equivalent stress

	b_t	d_s	n_s	h_c	h_t	b_c	E_c	E_s	E_t	ρ_c	ρ_t
ρ	0.05	-0.49	-0.57	-0.06	-0.12	0.2	-0.01	0.47	0.04	0.21	-0.01

Also, these results could be presented using the bar chart, as it shown in figure 3.

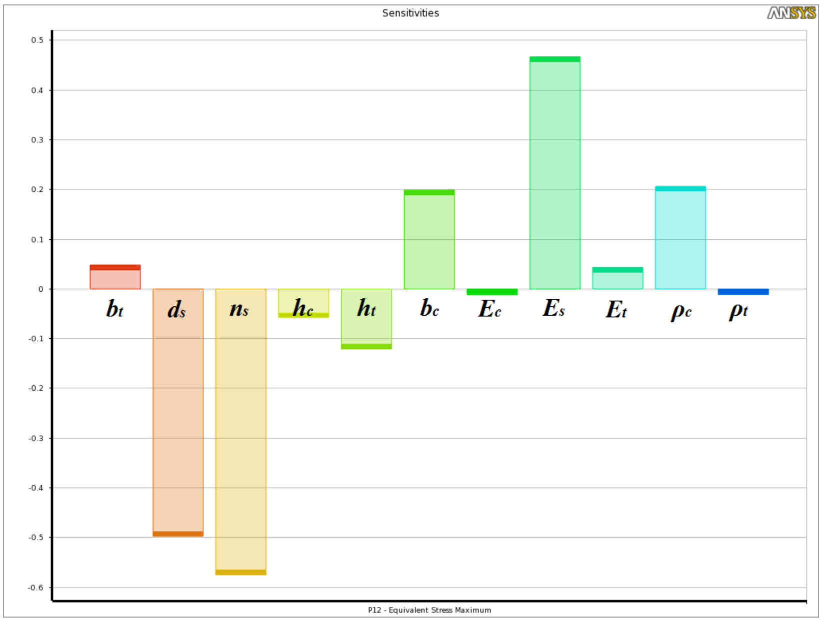

Fig. 3 Sensitivities of the maximum equivalent stress

Considering the results of the conducted stochastic sensitivity, it is noticeable that the parameters which describe shear connectors, such as the number of rods (n_s), the diameter of connector (d_s) and the Young's modulus of the steel (E_s) have the dominant influence on the maximum equivalent stress. Also, from the given results, it can be concluded that the increase in the values of the input design parameters b_t, b_c, E_s, E_t and ρ_c directly affects the increase in the maximum equivalent stress of the observed TCC beam. Also, as the values of the design parameters (d_s, n_s, h_c, h_t, E_c and ρ_t) increase, the maximum equivalent stress of the observed TCC beam reduces.

3.3.2. Sensitivity analysis of the maximum mid-span deflection

The results of the stochastic sensitivity analysis of the maximum deflection of the observed TCC beam, given in Table 3.

Table 3 Spearman rank-order correlation coefficients of maximum mid-span deflection

	b_t	d_s	n_s	h_c	h_t	b_c	E_c	E_s	E_t	ρ_c	ρ_t
ρ	-0.03	-0.04	-0.22	-0.24	-0.42	0.31	-0.28	-0.07	-0.61	0.29	0.07

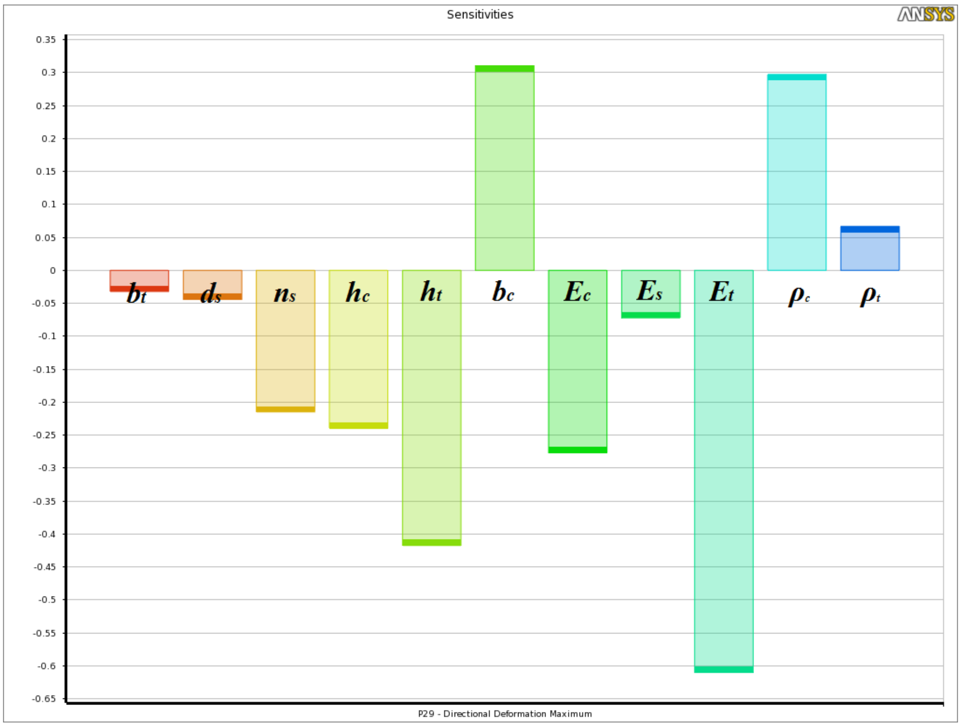

Fig. 4 Sensitivities of the maximum mid-span deflection

In Figure 4, we graphically presented the results of the stochastic sensitivity analysis of the maximum mid-span deflection of the TCC beam. It is interesting to note that the parameters which describe shear connectors, the diameter of the rod (d_s), the number of connectors (n_s) and the Young's modulus of the steel (E_s) do not have a dominant influence on the maximum mid-span deflection of the TCC beam. Based on the results of conducted sensitivity analysis, it can be concluded that the increase in the values of the design input parameters b_c, ρ_c and ρ_t directly affects the increase in the maximum mid-span deflection of the beam. Also, as the value of the design input parameters (b_t, d_s, n_s, h_c, h_t, E_c, E_s and E_t) increases, the maximum deflection of the observed beam decreases.

3.4. Response surface method

In order to better understand the correlation among the input parameters and the output parameter the Response surface method (RSM) is employed. This method provides a more precise description of correlation among design parameters [15].

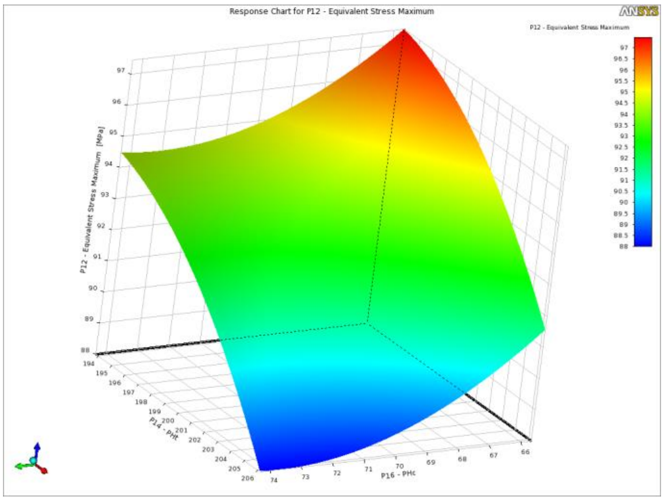

Fig. 5 Response surface for maximum equivalent stress

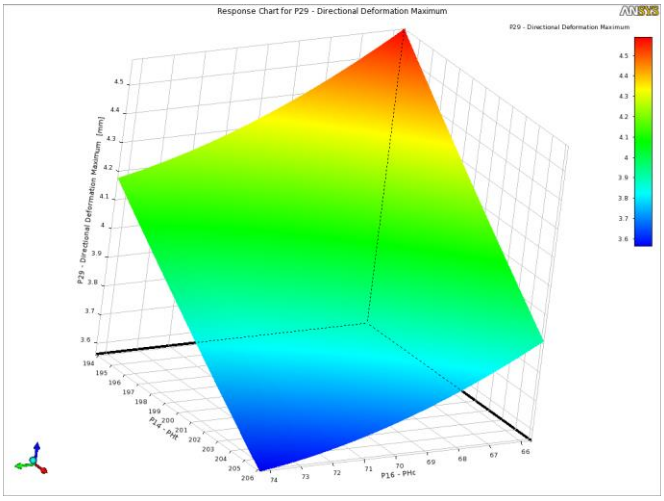

Fig. 6 Response surface for maximum mid-span deflection

RSM is based on the basic assumption that the influence of the input parameters on the output parameter can be approximated by the polynomial function. In the most cases, square function is used:

$$Y = c_0 + \sum_{i=1}^{n} c_i X_i + \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} X_i \cdot X_j \tag{3}$$

where c_0 is constant, c_i is linear coefficient and c_{ij} is a square coefficient. Regression is usually applied for the evaluation of these coefficients using the least square method.

4. CONCLUSION

The sensitivity analysis can generally be divided into two basic categories: the deterministic and the stochastic sensitivity analysis. Although the deterministic sensitivity analysis is easy to apply in practice and it provides a quick overview of the performance of the design model, it neglects the correlation among the input design parameters that certainly exist in real conditions. However, the stochastic sensitivity analysis provides much more complex information about the design parameters because they are considered as random variables with a certain distribution. The characterization of the selected input parameters was determined based on the JCSS Probabilistic Model Code. Unlike the deterministic sensitivity analysis, the variability of a particular output parameter due to a single input parameter is evaluated by varying all other design parameters at the same time. This paper was focused on the application of the stochastic sensitivity analysis on the maximum equivalent stress and the maximum mid-span deflection of the timber-concrete composite beam. The generation of the numerical model of the TCC beam was carried out using the finite element method in the software package ANSYS Workbench 15.0. The conducted analysis used the symmetry of the beam and the load. The sensitivity of considered output parameter was derived from Spearman rank-order correlation coefficient. Considering the results of the conducted stochastic sensitivity analysis, it is noticeable that the parameters which describe shear connectors, the number of rods (n_s), the diameter of connector (d_s) and the Young's modulus of the steel (E_s) have the dominant influence on the maximum equivalent stress, but do not have a dominant influence on the maximum mid-span deflection of the TCC beam. Based on the results of the conducted sensitivity analysis of the mid-span deflection, it can be concluded that the increase in the values of the design input parameters b_c, ρ_c and ρ_t directly affects the increase in the maximum mid-span deflection of the beam. Also, as the value of the design input parameters (b_t, d_s, n_s, h_c, h_t, E_c, E_s and E_t) increases, the maximum deflection of the observed beam decreases. In order to better understand the correlation among the input parameters and the output parameter, the Response surface method was employed.

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ANALIZA OSETLJIVOSTI SPREGNUTIH KONSTRUKCIJA TIPA DRVO-BETON

Analiza osetljivosti se može definisati kao studija o tome kako varijabilnost izlaznog parametra modela može biti raspodeljena na njene izvore, odnosno na varijabilnosti različitih ulaznih parametara tog modela. Ona pomaže u identifikaciji najvažnih projektnih parametara određene konstrukcije, kako bi se projektovanje i optimizacija konstrukcije fokusirala na njih. Ovaj rad predstavlja primenu stohastičke analize osetljivosti maksimalnog ekvivalentnog napona i maksimalnog ugiba u sredini raspona spregnute grede tipa drvo-beton. Svi ulazni parametri su posmatrani kao slučajne promenljive. U radu je korišćena Latin Hypercube Sampling numerička simulaciona metoda. Ocena osteljivosti je izvršena pomoću Spirmanovog koeficijenta korelacije rangova.

Ključne reči: *analiza osetljivosti, spregnute konstrukcije tipa drvo-beton, simulaciona metoda, Spirmanov koeficijent korelacije rangova*