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# NUMERICAL ANALYSIS OF FINITE HYPO-ELASTIC CYCLIC DEFORMATION WITH LARGE ROTATIONS

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**Abstract**. *Constitutive relations which describe engineering materials behaviour during the finite elastoplastic deformations are usually presented in the form of rates of stresses and strains. One of the possible approaches in the constitutive relations formulation is the additive decomposition of the total deformation rate into its elastic part and its plastic part. The elastic deformation rate contributes to any elastoplastic deformation at any stage. Hence, its exact and well-considered formulation is of particular importance and it has to be properly predicted by the corresponding material law. This is of great importance in particular when deformation cyclic processes are considered, in which case small errors may accumulate, even if the total deformation is small.*
*The implementation of the most frequently used corotational rates, i.e. the Jaumann rate and the Green-Naghdi rate, in the hypo-elastic constitutive relations regarding small and moderate rotations gives accurate results for low number of repeated deformation cycles. With increased number of cycles, however, the implementation of these rates results in different and physically non-admissible material responses. This instability in results is particularly observable during the cyclic deformations with large rotations, which is the main subject of this work. In contrast to the aforementioned objective rates, the results of the logarithmic rate implementation into the hypo-elastic constitutive relations for the case of pure elastic deformation describe a physically stable process.*

**Key words**: *hypo-elasticity, objective rate, logarithmic rate, finite cyclic deformation, ABAQUS, UMAT subroutine, large rotations*

## 1. INTRODUCTION

In the contemporary Eulerian formulation of finite elastoplasticity the elastic behaviour is often described by a grade zero hypo-elastic law that requires the implementation of an objective rate instead of material time derivative. By reviewing the corresponding literature, it

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may be observed that the objective Jaumann rate has been used by many researchers and has found a wide application in developing elastoplasticity theories. However, the shear oscillation phenomenon, firstly revealed by Lehmann (1972) and Dienes (1979), has questioned the correctness of application of the Jaumann rate in the constitutive relations for finite deformations. The work of Kojic & Bathe (1987), in which the authors showed that the application of the Jaumann stress rate produces residual stresses at the end of an elastic closed strain path, just confirmed the aforementioned conclusion on the inappropriateness of the Jaumann rate implementation in the constitutive relations even for the case of small deformations of cyclic nature. These findings have contributed to the development of numerous rates, corotational and non-corotational ones, such as the Truesdell rate, the Green-Naghdi rate, the Cotter-Rivlin rate, the Durban-Baruch rate. Their implementation, however, has not completely solved the existing problems. Additionally, in the work of Simo & Pister (1984), it was shown that for the case of pure elastic deformation, hypo-elastic rate equation, considering all then known objective rates, fails to be exactly integrable, and by that, is unable to define an elastic behaviour of the material realistically.

Although the theoretical studies have shown that the classical rates produce unstable solutions for finite deformations, some of them are still incorporated in widely used commercial finite element codes for structural analysis. For example, in the software ABAQUS, depending on the element type and constitutive model, the Jaumann rate or the Green-Naghdi rate are the available options to be selected by the solver (see ABAQUS documentations, 2013).

Recently, the logarithmic rate has been in a focus of a number of studies by various researchers, who have proved that application of this rate in hypo-elastic constitutive relations successfully solves the problems related to shear oscillation and residual stresses for a closed strain path. For details, the reader is referred to references Xiao et al. (1997a, 1997b) and Meyers et al. (2003, 2006).

The present study employs one distinctive numerical problem, performed with a view toward investigating the logarithmic rate implementation in the hypo-elastic constitutive relations for the case of pure elastic deformation and verifying that this rate obeys the Bernstein's integrability condition (see Bernstein, 1960) to give an elastic relation, thus meaning that path-dependent and dissipative processes are not detected. According to the results of a present study, the logarithmic rate is proved to be an appropriate solution for the aforementioned problems, opposed to formerly-used objective corotational and non-corotational rates.

## 2. Basic Relations

### 2.1. Kinematics

If $\mathbf{X}$ denotes the position of a material particle in the Lagrangian, or reference, configuration and $\mathbf{x}$ denotes its position in the Eulerian, or current, configuration, the particle displacement can be described by the deformation gradient $\mathbf{F}$ as follows:

$$\mathbf{F} = \frac{\partial \mathbf{x}}{\partial \mathbf{X}} \qquad \text{with} \qquad J = \det(\mathbf{F}) > 0, \tag{1}$$

where $J$ is the Jacobian determinant or shortly Jacobian. The velocity $\mathbf{v}$ of the same particle and the velocity gradient $\mathbf{L}$ are defined as:

$$\mathbf{v}(t) = \dot{\mathbf{x}} = \frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t}, \qquad \text{and} \qquad \mathbf{L} = \frac{\partial \mathbf{v}}{\partial \mathbf{x}} = \dot{\mathbf{F}} \cdot \mathbf{F}^{-1}. \tag{2}$$

In the previous relations a superposed dot denotes the material time derivative or ordinary time derivative when the variable is only a function of time. Applying the left polar decomposition theorem, the deformation gradient can be decomposed into a positive definite 2nd-order tensor **V**, named left stretch tensor, and an orthogonal 2nd-order rotation tensor:

$$\mathbf{F} = \mathbf{V} \cdot \mathbf{R}. \tag{3}$$

The square of the left stretch tensor, termed as the left Cauchy-Green tensor, is more convenient for numerical purpose than **V** and it is defined as:

$$\mathbf{B} = \mathbf{V}^2 = \mathbf{F} \cdot \mathbf{F}^{\mathrm{T}}. \tag{4}$$

The velocity gradient can be decomposed into its symmetric part, related to stretching, and skew-symmetric part, related to rotation:

$$\mathbf{L} = \mathbf{D} + \mathbf{W}. \tag{5}$$

The rate of deformation tensor, or the stretching tensor, **D** and the vorticity tensor, or the spin tensor, **W** are given respectively as:

$$\mathbf{D} = \frac{1}{2}(\mathbf{L} + \mathbf{L}^{\mathrm{T}}) \qquad \text{and} \qquad \mathbf{W} = \frac{1}{2}(\mathbf{L} - \mathbf{L}^{\mathrm{T}}). \tag{6}$$

### 2.2. Constitutive relations

In the case of finite deformation, constitutive relations of elasticity or elastoplasticity are usually given in rate form. The occurrence of moderate to large rotations in finite deformations requires the introduction of the objectivity concept. Since the material time derivative is not an objective quantity in the Eulerian description, adopted here, the objective rates have to be introduced in the constitutive relations. From the general hypo-elasticity model, introduced by Truesdell, the simplified form of the hypo-elastic equation of grade zero is given by the following expression:

$$\mathbf{D} = \underline{\mathbf{K}} : \overset{\circ}{\boldsymbol{\tau}} = \frac{1+\nu}{E}\, \overset{\circ}{\boldsymbol{\tau}} - \frac{\nu}{E}(\mathrm{tr}(\overset{\circ}{\boldsymbol{\tau}}))\mathbf{1}, \tag{7}$$

which represents the relation between the rate of deformation, i.e. the stretching tensor **D**, and the objective rate of the Kirchhoff stress tensor $\boldsymbol{\tau}$ via the constant and isotropic instantaneous elastic compliance 4th-order tensor $\underline{\mathbf{K}}$. Here, $\nu$ and $E$ stand for the elastic constants, i.e. the Poisson's ratio and the Young's modulus, respectively, and **1** represents the symmetric 2nd-order unit tensor. The Kirchhoff stress tensor, or weighted Cauchy stress tensor, $\boldsymbol{\tau}$ is related to the Cauchy stress, or true stress, $\boldsymbol{\sigma}$ as:

$$\boldsymbol{\tau} = J\, \boldsymbol{\sigma}. \tag{8}$$

The objective stress rate, given here in a general form by Eq. (9), implies the introduction of the so-called spin tensor $\boldsymbol{\Omega}$, whose choice divides the objective rates into two groups of corotational and non-corotational rates (for more details see Trajković-Milenković, 2016, and the references therein):

$$\overset{\circ}{\boldsymbol{\tau}} = \dot{\boldsymbol{\tau}} + \boldsymbol{\tau} \cdot \boldsymbol{\Omega} - \boldsymbol{\Omega} \cdot \boldsymbol{\tau}. \tag{9}$$

The corotational rates that are examined here are the Jaumann rate, the Green-Naghdi rate, and the logarithmic rate (the Log-rate), whereas the tested non-corotational rates are the Truesdell rate, the Oldroyd rate, and the Cotter-Rivlin rate. The spin tensors for the aforestated corotational rates are given as:

$$\boldsymbol{\Omega}^{\mathrm{J}} = \mathbf{W}; \qquad \text{for the Jaumann rate,} \tag{10}$$

$$\boldsymbol{\Omega}^{\mathrm{GN}} = \dot{\mathbf{R}} \cdot \mathbf{R}^{T} = \mathbf{W} + \sum_{i=1}^{m} \Big( \sum_{k=1, k \neq i}^{m} \frac{b_k - b_i}{b_k + b_i} \mathbf{B}_i \cdot \mathbf{D} \cdot \mathbf{B}_k \Big); \quad \text{for the Green - Naghdi rate,} \tag{11}$$

$$\boldsymbol{\Omega}^{\mathrm{Log}} = \mathbf{W} + \sum_{i=1}^{m} \Big( \sum_{k=1, k \neq i}^{m} \Big( \frac{b_k + b_i}{b_k - b_i} - \frac{1}{\ln b_k - \ln b_i} \Big) \mathbf{B}_i \cdot \mathbf{D} \cdot \mathbf{B}_k \Big); \quad \text{for the Log - rate.} \tag{12}$$

In Eqs. (10) - (12) $b_{i(k)}$ represent $m$ distinct eigenvalues of the left Cauchy-Green tensor, and $\boldsymbol{B}_{i(k)}$ denote the corresponding eigenprojections of the same tensor.

For the known spin tensors the corresponding objective rates are obtained from the general form given by Eq. (9):

$$\overset{\circ}{\boldsymbol{\tau}}{}^{\mathrm{J}} = \dot{\boldsymbol{\tau}} + \boldsymbol{\tau} \cdot \mathbf{W} - \mathbf{W} \cdot \boldsymbol{\tau}, \qquad \text{the Jaumann corotational rate,} \tag{13}$$

$$\overset{\circ}{\boldsymbol{\tau}}{}^{\mathrm{GN}} = \dot{\boldsymbol{\tau}} + \boldsymbol{\tau} \cdot \boldsymbol{\Omega}^{\mathrm{GN}} - \boldsymbol{\Omega}^{\mathrm{GN}} \cdot \boldsymbol{\tau}, \qquad \text{the Green-Naghdi corotational rate,} \tag{14}$$

$$\overset{\circ}{\boldsymbol{\tau}}{}^{\mathrm{Log}} = \dot{\boldsymbol{\tau}} + \boldsymbol{\tau} \cdot \boldsymbol{\Omega}^{\mathrm{Log}} - \boldsymbol{\Omega}^{\mathrm{Log}} \cdot \boldsymbol{\tau}, \qquad \text{the logarithmic corotational rate,} \tag{15}$$

$$\overset{\circ}{\boldsymbol{\tau}}{}^{\mathrm{Old}} = \dot{\boldsymbol{\tau}} - \mathbf{L} \cdot \boldsymbol{\tau} - \boldsymbol{\tau} \cdot \mathbf{L}^{\mathrm{T}}, \qquad \text{the Oldroyd non-corotational rate,} \tag{16}$$

$$\overset{\circ}{\boldsymbol{\tau}}{}^{\mathrm{CR}} = \dot{\boldsymbol{\tau}} + \mathbf{L}^{\mathrm{T}} \cdot \boldsymbol{\tau} + \boldsymbol{\tau} \cdot \mathbf{L}, \qquad \text{the Cotter-Rivlin non-corotational rate,} \tag{17}$$

$$\overset{\circ}{\boldsymbol{\sigma}}{}^{\mathrm{T}} = \dot{\boldsymbol{\sigma}} - \boldsymbol{\sigma} \cdot \mathbf{L}^{\mathrm{T}} - \mathbf{L} \cdot \boldsymbol{\sigma} + \boldsymbol{\sigma}\, \mathrm{tr}(\mathbf{D}), \qquad \text{the Truesdell non-corotational rate.} \tag{18}$$

The Truesdell rate is originally given for the Cauchy stress instead of the Kirchhoff stress in the form given in the last Equation.

## 3. CLOSED ELASTIC STRAIN PATH - HYPO-ELASTIC CYCLIC DEFORMATION

In the engineering practice strain cycles and cyclic loading can frequently occur as well if the agencies are repeated in a large number of cycles. It has been proved analytically that even for small but cyclic deformations the residual stress may be appreciable even after a single cycle and it may become of quite a high value with an increasing number of cycles (see Xiao et al., 1999). Accordingly, the analysis of cyclic deformation paths takes the important place in structural analysis. For the hypo-elastic law (Eq. 7), the aforementioned objective stress rates, i.e. the Jaumann, Green-Naghdi, and logarithmic rates, as corotational rates, and the Truesdell, Oldroyd, and Cotter-Rivlin rates, as non-corotational rates, have been compared in closed single parameter elastic deformation cycles.

In the numerical calculations the commercial software ABAQUS/Standard has been used in which material behaviour can be defined in terms of a built-in or a user-defined material model. In the latter case the actual material model is defined in the originally programmed code incorporated into ABAQUS via the user-defined subroutine UMAT. Here, Eqs. (13) - (18) have been incorporated in separate material models programmed in the user-defined subroutine UMAT. The outputs have been compared mutually, also with those obtained using ABAQUS built-in material model, as well as with the results from the relevant literature.

In the studies of Kojić & Bathe (1987) and Lin et al. (2003), a four-phase plain strain cycle was considered, which consists of extension, shear, compression, and return to original unstrained state. Here, the smooth strain cyclic deformation of the square element (see Fig. 1) has been considered. The square element of size $H$ is subjected to a combined lengthening and shearing process in the $\mathbf{e}_1$-$\mathbf{e}_2$ plane, such that the upper corners are moving along the ellipse with radii $a$ and $b$.

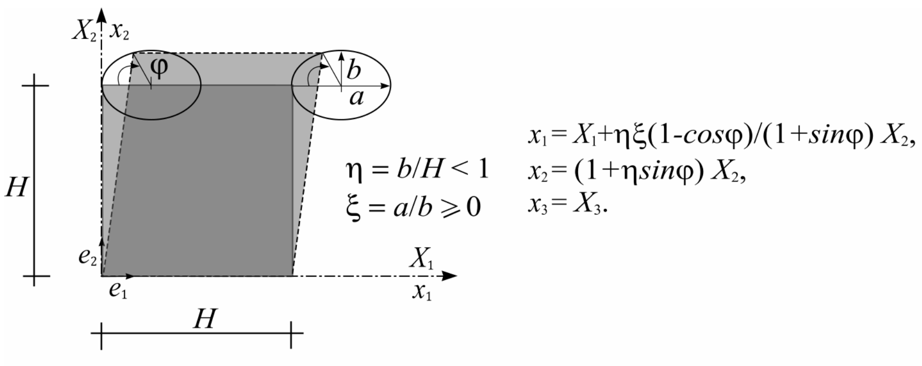



**Fig. 1** Model and equations of deformation

The deformation is described by equations given in Fig. 1, where $\eta$ and $\xi$ are dimensionless parameters. The parameter $\eta = b/H$ $(0 < \eta < 1)$ represents the measure of tension or compression, whereas $\xi = a/b$ is the measure of rotation to which the element is subjected. The parameter $\varphi$ in the last relation is the single parameter that describes the deformation of the square plate. The material of the plate has been considered as initially isotropic and stress free. The adopted values for the Young's modulus and Poisson's ratio are $E$ = 210 GPa and $\nu = 0.3$.

### 3.1. Large rotations

In this Section a stress response of the square element (presented in Fig. 1), subjected to the deformation consisting of the combined axial deformation and large rotation is examined. The extension in 2-2 direction will be in the range of 0 - 10%, whereas the shear deformation is predominant with relatively large values up to 50%, i.e. the parameters $\xi$ and $\eta$ are taking the values 5 and 0.1, respectively.

Development of the normal stresses $\tau_{11}$ and $\tau_{22}$ and the shear stress $\tau_{12}$ versus the deformation angle $\varphi$ for all the considered rates have been presented in Fig. 2 and 3, respectively. In Fig. 2, for the given rates, the normal stresses in 1-1 direction are marked with the dashed lines and the normal stresses $\tau_{22}$ are presented by the solid lines.

The diagrams over a single cycle can generally be divided into three characteristic parts. The first one is that where $\varphi / 2\pi$ is in the range of 0 - 0.1. These are the values that are usually met as elastic deformations in metals, for example in civil engineering structures and during metal forming. It can be seen that in this part for all rates the plots are almost congruent.

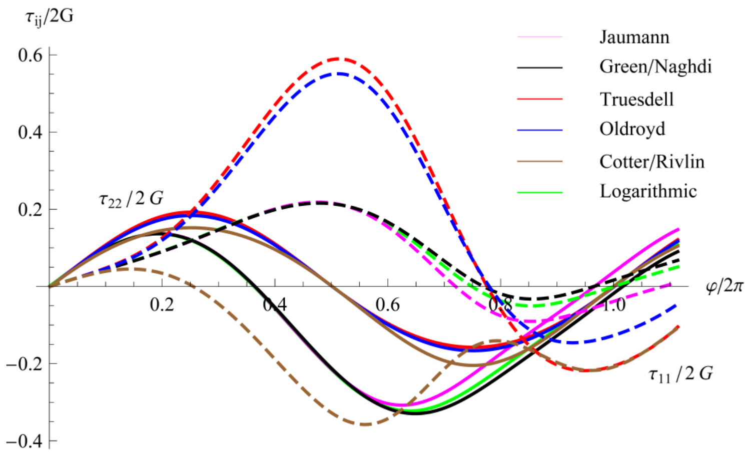



**Fig. 2** Normal stress $\tau_{11}$ and $\tau_{22}$ single cycle development, $\xi$ = 5, $\eta$ = 0.1

For rubber-like and composite materials and shape memory alloys, which can be subjected to very large elastic deformations, the second and the third part of diagrams are of great importance. The second part, where $\varphi / 2\pi$ is in the range of 0.1 - 0.53, is characterised with an almost identical stress response for the corotational rates, whereas the stress responses considering the non-corotational rates are drifting apart. Concerning the normal stress $\tau_{11}$, it is evident from Fig. 2 that the Oldroyd and Truesdell rates implementation in the hypo-elastic relation gives very high values of $\tau_{11,}$ while the reverse is observed for the Cotter-Rivlin rate that results in a rather low stress value. The third part of the diagrams is beyond the limit of 0.53 for $\varphi / 2\pi$, in which case the plots of shear and normal stresses even for the corotational rates are starting to drift apart.

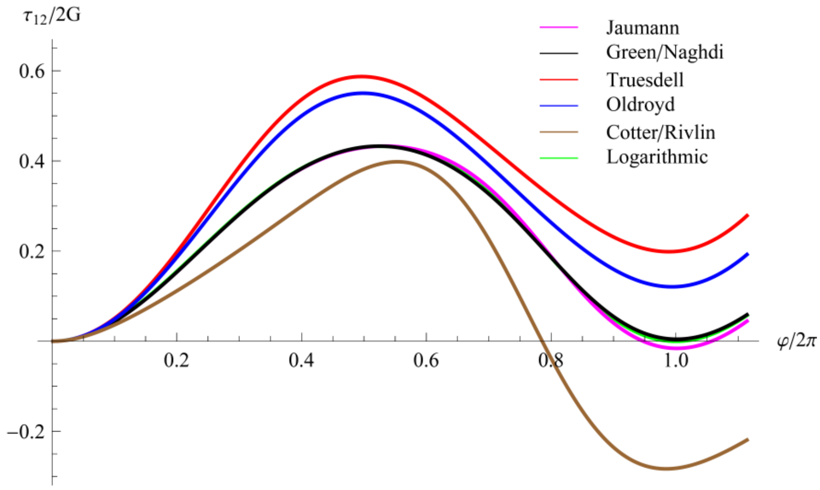



**Fig. 3** Shear stress $\tau_{12}$ single cycle development, $\xi = 5$, $\eta = 0.1$

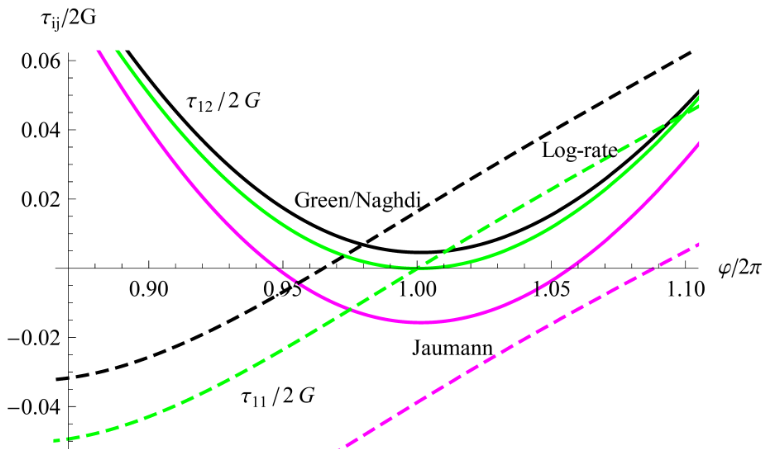



**Fig. 4** Enlarged representation of $\tau_{11}$ and $\tau_{12}$ at the end of the first cycle, $\xi = 5$, $\eta = 0.1$

At the end of the first cycle some residual normal and shear stresses for the Green-Naghdi and Jaumann rates occur, whereas the non-corotational rates produce extremely high values of residuals. Only the Log-rate gives zero stress values at the end of the elastic deformation cycle (see Fig. 4). Here again, the normal stresses in 1-1 direction are marked with the dashed lines and the shear stresses are presented by the solid lines.

If the continuum square element is subjected to a deformation that repeats cyclically, the stress response error is accumulating for all the considered rates except for the Log-rate. The development of normal and shear stresses during 10 and 100 cycles for the logarithmic rate has been presented in the upper and down part of Fig. 5, respectively. It can be seen that the development of all Kirchhoff stress components is regularly periodical with constant magnitudes and without any residuals at the end of each cycle.

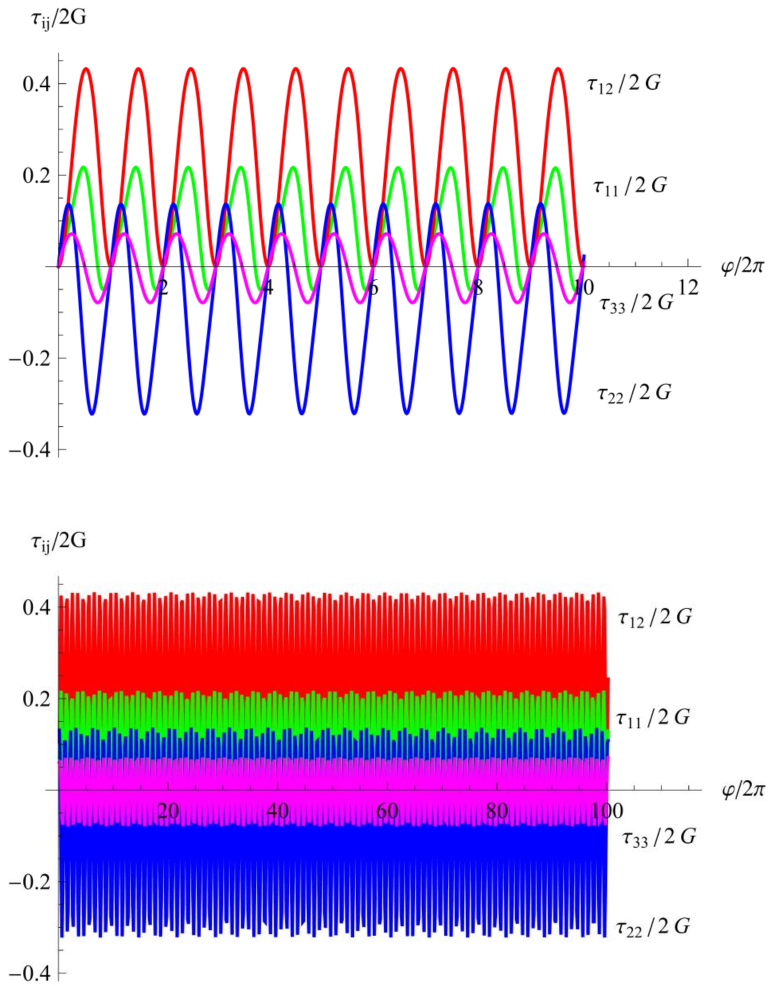
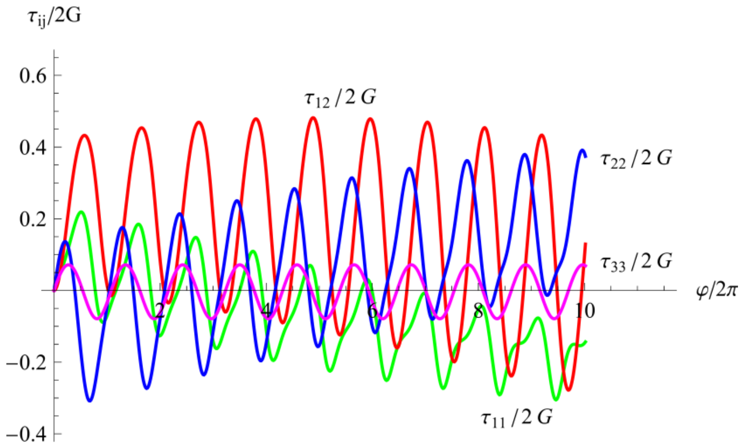



**Fig. 5** Ten and hundred cycle stress development for the Logarithmic rate, $\xi = 5$, $\eta = 0.1$

The stress developments obtained using the Jaumann rate in the user-defined UMAT subroutine and ABAQUS built-in subroutine for hypo-elastic constitutive model have been presented in Fig. 6. From these plots it can be observed that the results for both models are completely congruent.



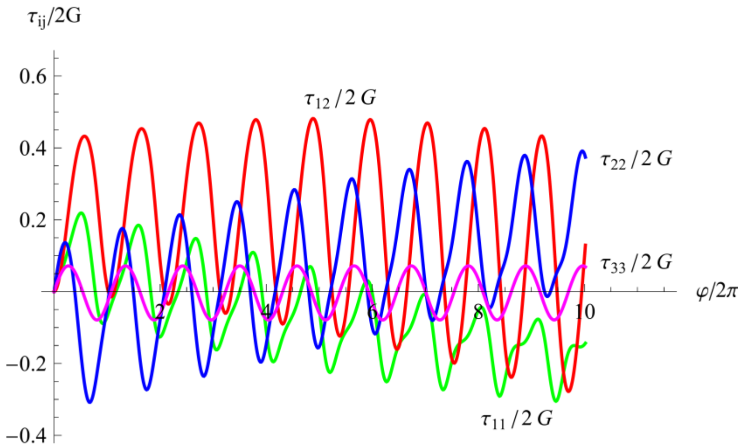




**Fig. 6** Ten cycle stress development for the Jaumann rate using UMAT (up) and ABAQUS (down) model, $\xi = 5$, $\eta = 0.1$

The development of normal and shear stresses using the Jaumann rate for 100 cycles has been presented in Fig. 7. It is obvious that the Jaumann formulation provides an oscillatory stress response for all stress components except for $\tau_{33,}$ with variable magnitude of $\tau_{11}$ and $\tau_{12}$. The residual stresses at the end of cycles are of non-negligible values and show an oscillating character as well (see Fig. 13). It can be concluded that the Jaumann rate gives results which are not in accordance with the physical behaviour of the materials. Therefore, this rate should not be implemented in the constitutive relations if large cyclic deformation occurs, ever for a low number of cycles (see Fig. 6).

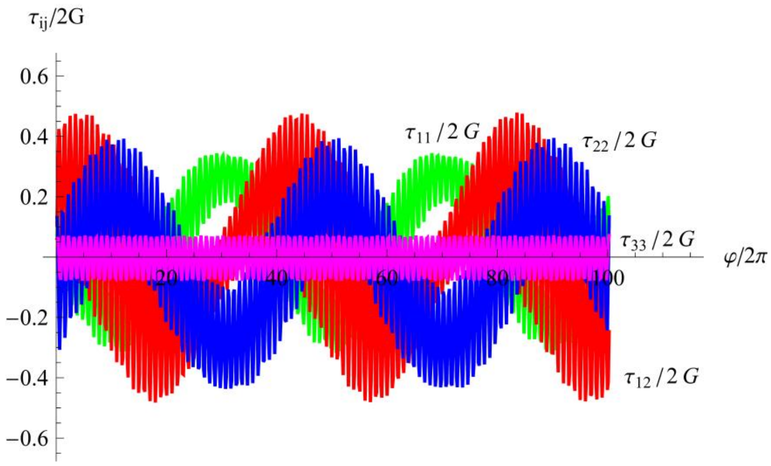



**Fig. 7** Hundred cycle stress development for the Jaumann rate, $\xi = 5$, $\eta = 0.1$

In the case of moderate rotations, which are presented in detail in reference Trajković-Milenković, (2016), the Green-Naghdi rate was a more reliable choice than the Jaumann rate. Here, it can be seen that very high values of residuals occur even after a single cycle and their values are monotonically increasing as it is depicted in Figures 8 and 9.

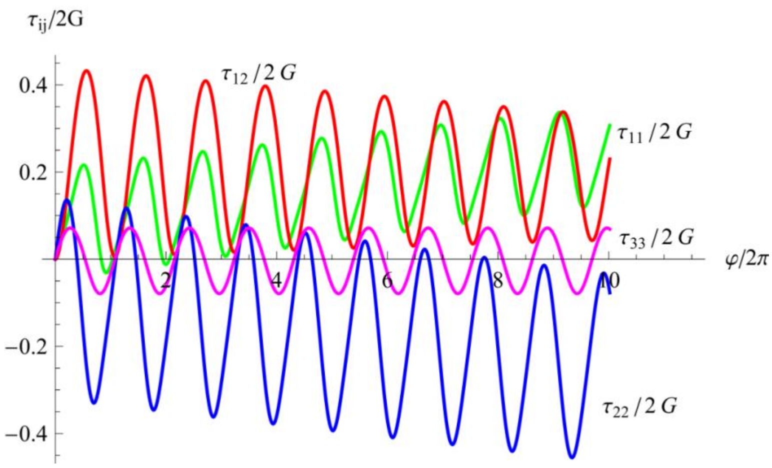



**Fig. 8** Ten cycle stress development for the Green-Naghdi rate, $\xi = 5$, $\eta = 0.1$

As in the case of moderate rotations (see reference Trajković-Milenković et al., 2017), the stress responses of the Green-Naghdi rate formulation show the feature that the magnitude of the shear stress decreases as the number of cycles increases and after approximately 30 cycles its values are zero, which is a totally unrealistic result. After that, the maximum shear stress is starting to increase monotonically. The plots for normal stresses $\tau_{11}$ and $\tau_{22}$ are drifting away and changing their magnitude with the number of cycles (see Fig. 9).

All the above stated remarks lead to the conclusion that for the case of cyclic elastic deformations with large rotations, the Green-Naghdi rate has to be excluded from the hypo-elastic formulations, except for only a few repeated cycles.

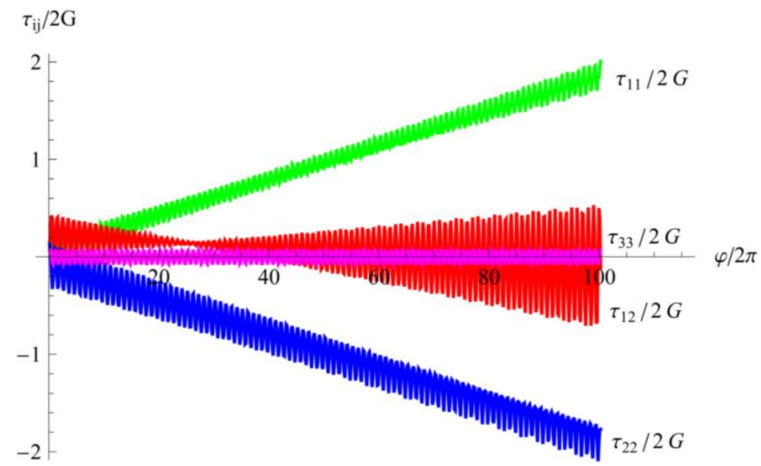



**Fig. 9** Hundred cycle stress development for the Green-Naghdi rate, $\xi = 5$, $\eta = 0.1$

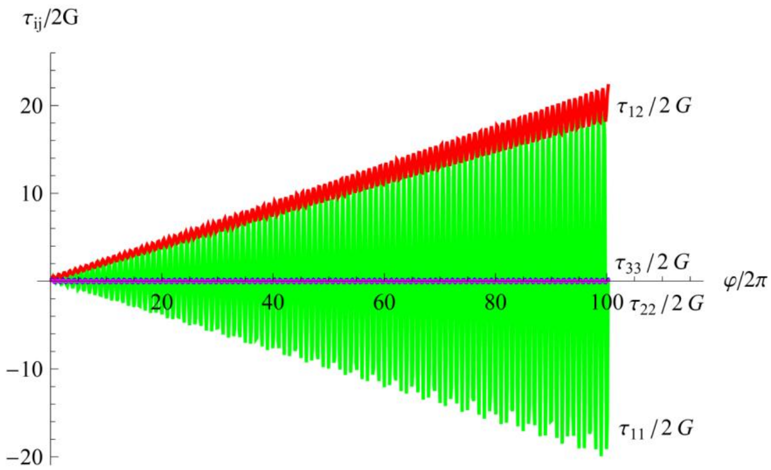



**Fig. 10** Hundred cycle stress development for the Truesdell rate, $\xi = 5$, $\eta = 0.1$

The Truesdell and the Oldroyd rates give similar stress responses as it is illustrated in Figures 10 and 11, where the developments of the shear stress drift apart and increase the magnitude to the unrealistic high values of 30 times for the Oldroyd rate and 40 times for the Truesdell rate compared with the shear modulus.

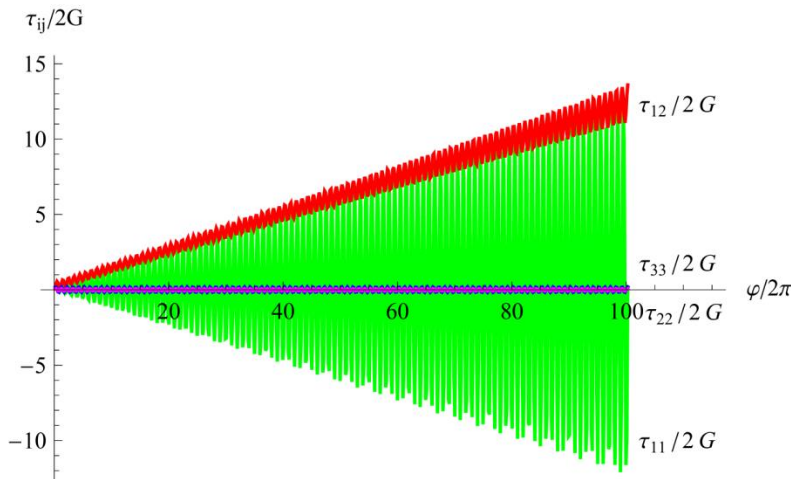



**Fig. 11** Hundred cycle stress development for the Oldroyd rate, $\xi = 5$, $\eta = 0.1$

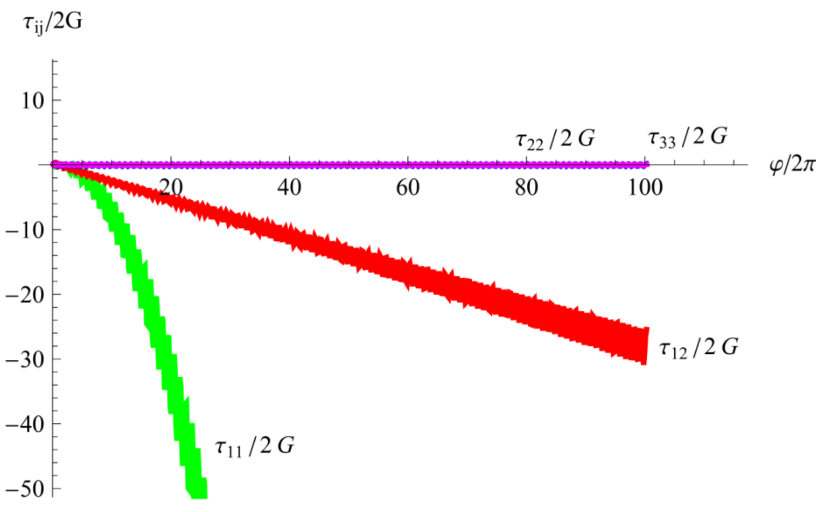



**Fig. 12** Hundred cycle stress development for the Cotter-Rivlin rate, $\xi = 5$, $\eta = 0.1$

The results for the Cotter-Rivlin rate show great deviations from realistic values as well (see Fig. 12), and therefore, this rate must be excluded from the constitutive relation construction, too.

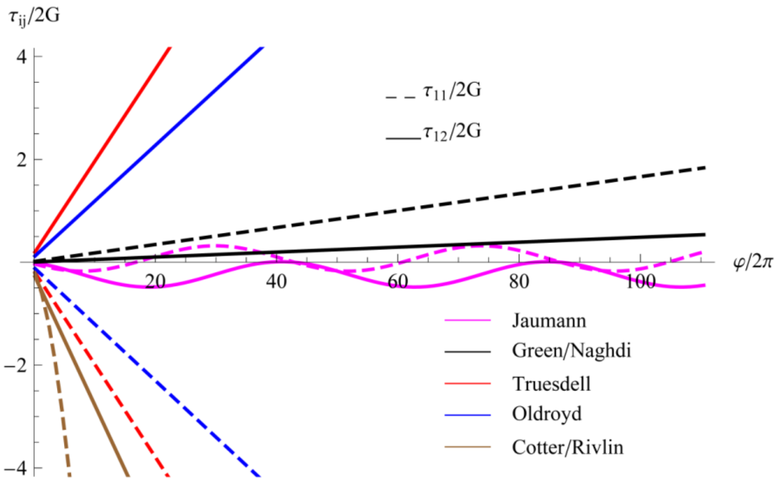



**Fig. 13** Residual normal and shear stresses for various rates, $\xi = 5$, $\eta = 0.1$

In Table 1 the normalised residual stresses after 10 cycles are given for all the examined rates. The applied elastic deformation is with very large rotations, i.e. $\eta$ =0.2 and $\xi$=10. It can be seen that except for the logarithmic rate the residual stresses considering the corotational rates are not negligible and for the non-corotational rates are of extremely high values.

**Table 1** Residual stresses after 10 cycles for $\xi = 10$, $\eta = 0.2$

| | Log-rate | Jaumann | Green/ Naghdi | Oldroyd | Truesdell | Cotter/Rivlin |
|---|---|---|---|---|---|---|
| $\tau_{11}/2G$ | 0.45732e-4 | 0.035174 | 2.8971 | -43.925 | -69.584 | -721.76 |
| $\tau_{12}/2G$ | -0.43250e-4 | -0.52436 | 3.1558 | 10.437 | 16.700 | -24.353 |

## 4. Conclusion

In the case of finite deformations, prediction of the material behaviour is usually given in the rate form. In modelling of the elastic part of the deformation rates the hypo-elastic constitutive relation of grade zero is frequently used. Since the material time derivative in the Eulerian description is not an objective quantity the objective rate has to be incorporated in the constitutive relation.

The main objective of this work is the comparison of the recently-discovered logarithmic rate with actually mostly used objective rates for the case of pure elastic cyclic deformations. The range of validity is determined for the corotational objective rates, namely the Jaumann rate, the Green-Naghdi rate and the logarithmic rate, and the non-corotational objective rates, namely the Truesdell rate, the Oldroyd rate and the Cotter-Rivlin rate, in the case of finite cyclic deformation with large rotations.

For that purpose, aforestated rates have been implemented into the commercial software ABAQUS/Standard using the user-defined subroutine UMAT.

Through the numerical procedure it is shown that the occurrence of finite rotations in total deformation significantly influences stress responses. For cyclic deformations with large rotations all the examined rates except the logarithmic rate produce high values of residual stresses at the end of the cycle, especially if the deformation is repeated in large number of cycles, which is the usual case.

Taking all into account, the results of the presented study reveal the distinguishing feature of the logarithmic rate from remaining objective rates for the case of elastic deformations with large rotations. This rate is superior to others, since it is the only rate that gives reliable and physically admissible results for this kind of deformation, i.e. the only elasticity-consistent hypo-elastic constitutive relation would be the one based on the logarithmic rate. Consequently, the only correct choice among all objective rates regarding elastic deformations with large rotations would be the logarithmic rate.

Therefore, our recommendation would be the implementation of the logarithmic rate into the hypo-elastic constitutive relations in modelling the large pure elastic deformation or the elastic part of the finite elastoplastic deformations, since it is the only one among all the examined objective rates that meet the requirement of the Bernstein's integrability condition, i.e. for whom path-dependent and dissipative processes are not detected even for a very large number of cycles.

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# NUMERIČKA ANALIZA KONAČNIH HIPOELASTIČNIH CIKLIČNIH DEFORMACIJA SA VELIKIM ROTACIJAMA

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*Konstitutivne relacije koje opisuju ponašanje materijala pri konačnim elastoplastičnim deformacijama su najčešće date u formi izvoda napona i deformacija. Jedan od mogućih pristupa u formulaciji ovih konstitutivnih relacija je aditivna dekompozicija ukupnog tenzora brzine deformacije na njegov elastični i plastični deo. Kako je doprinos elastične deformacije prisutan na svakom nivou ukupne elastoplastične deformacije, tačna i unapred dobro razmotrena formulacija elastičnog dela tenzora brzine deformacije je neophodna. Rešenje ovog problema je primena odgovarajućeg materijalnog zakona u kome glavnu ulogu imaju objektivni izvodi, koji u slučaju konačnih deformacija moraju zameniti materijalni izvod. Izbor odgovarajućeg objektivnog izvoda koji figuriše u konstitutivnoj relaciji ima ključnu ulogu i najvažniji je cilj ovog rada. Ovo može biti od posebne važnosti kada se razmatraju ciklične deformacije, čak i ukoliko su ukupne deformacije male.*

*U slučaju čiste elastične deformacije, implementacijom najčešće korišćenih korotacionih izvoda, t.j. Jaumanovog i Grin-Nagdijevog izvoda, u hipoelastičnim konstitutivnim relacijama pri malim i srednjim rotacijama dobijaju se tačni rezultati, dok je broj ponovljenih deformacionih ciklusa mali. Sa povećanjem broja ciklusa, međutim, implementacija ovih izvoda daje rezultate koji se medjusobno dosta razlikuju, a takođe često opisuju fizički nerealno ponašanje materijala. Ova nestabilnost u rezultatima je posebno uočljiva pri modeliranju cikličnih deformacija pri kojima se javljaju velike rotacije, što je glavni zadatak ovog rada. Suprotno predhodno pomenutim objektivnim izvodima, primena logaritamskog izvoda u hipoelastičnim konstitutivnim relacijama daje rezultate koji u slučaju čiste elastične deformacije opisuju fizički stabilan proces.*

Ključne reči: *hipoelastičnost, objektivni izvodi, logaritamski izvod, konačne ciklične deformacije, ABAQUS, UMAT podprogram, velike rotacije*