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EXPERIMENTAL AND NUMERICAL IDENTIFICATION OF STRUCTURAL MODES FOR ENGINEERING EDUCATION

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Abstract. *Development of simple classroom demonstration device and software for visualization of structural normal modes is presented. Device is made of parts of old speaker, controlled with personal computer, where the harmonic motion of solenoid is used as an excitation for beam and plate models. Simple code for finite element free vibration analysis of plates is written in Wolfram Mathematica. Good agreement of results and attractive visual patterns of normal modes attracted attention of students. Results are confirmed using modern modal testing methods. Presented approach is complementary to standard teaching of structural dynamics.*

Key words: *engineering education, normal modes, Chladni plate, modal testing, finite element method*

1. INTRODUCTION

Teaching students of meaning and nature of structure dynamical characteristics is a crucial step in structural engineering education. Since all processes in structure life-cycle are of dynamical nature, knowledge of main parameters which determine structural dynamical behaviour is essential. There are three well-known quantities which completely define structure in theoretical approach: eigenfrequencies, eigendampings and eigenshapes. Every set of these three quantities represents one *normal mode* (*eigenmode*) of structure. Each structure has infinite number of normal modes while only few of them are of interest in design process. Eigenshapes of the system model can be visualized and allow a direct physical interpretation.

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From the experimental point of view, instead of the eigenmodes, the term *modal parameters* is commonly used. The modal model describes the structure's dynamic through modal parameters: resonant frequencies, modal damping and mode shapes. Using modal models, we can improve the design and optimize the dynamic behaviour of the structure.

Modal analysis is a process of determining all modal parameters which are sufficient for the formulation of the mathematical dynamic model. The modal representation of a mechanical structure can be determined analytically for only few simply cases. For all the other structures, numerical approximation is necessary. The most versatile contemporary procedure is the Finite element method (FEM) which discretizes the structure into finite number of simple elements connected at nodes. Although highly applicable for vast range of mechanical problems, this numerical approach has some limitations, such as poor continuity of the model, high computing costs, modelling of complex materials, etc.

Present-day software for structural analysis using FEM is often "black boxes" for inexperienced users, such as undergraduate students. Also, the high price of these packages reduces their availability in developing countries. It is evident that the development of open-source codes can help in understanding of the basic principles on which this software is based on. This should finally lead to more efficient usage of commercial software in practice.

Besides numerical analysis, there is an experimental approach, known as Experimental modal analysis (EMA), which is commonly used for modal analysis of mechanical systems. EMA involves modal testing and modal parameters identification procedure. This method has become a standard engineering tool in solving many structural dynamic problems.

Contemporary instruments for EMA allow efficient detection of dynamical characteristics of mechanical systems. Standard equipment consists of transducers (typically accelerometers and load cells), an analogue-to-digital converter front-end and a host computer. If the excitation and response of the system are measured simultaneously, it is possible to determine frequency response functions which contain dynamical parameters of the system. Unfortunately, this kind of measurement equipment is rather expensive and is not readily available in schools, especially in developing countries.

The aim of this paper is to present development of simple device and software for detection (*visualisation*) and calculation of normal modes of elementary types of structures. An opportunity to see electromagnetic shaker and source-code of FEM software increases student's interest for subject. This should lead to well-educated engineers which are ready to cope with design of complex structures subjected to dynamic load.

The brief description of modal testing methods is given in the following section. Development of the simple electromagnetic shaker and the open-sourced FEM code are presented afterwards, while the two illustrative examples of beam and plate model vibration analysis are given at the end.

2. Modal Testing Methods

To accurately determine a structure′s response to a dynamic loading, it is necessary to perform some form of structural dynamic testing such as modal testing. Modal testing represents the way of vibration testing for modal analysis. Two fundamentally different methods of modal testing are used for testing structures. These methods are referred to as the Normal mode method (NMM) and Transfer function or Frequency response function method (FRFM). NMM is the more traditional of these two, and has been used since the 1950s, mostly in the airspace industry for testing large spacecraft and aircraft structures. Modern modal testing methods are based on measurement of transfer functions or frequency response functions (FRF), and became popular soon after the discovery of the Fast Fourier transformation (FFT) algorithm in the late 1960s, [1].

2.1. Normal mode method (traditional method)

Modal testing has traditionally been done using sine wave excitation. This traditional method for determination and visualisation of modal shapes is based on the discovery of German physicist Chladni [2]. Investigating the vibration of plates, he noticed that, when the plate is excited to vibrate with resonant frequency, sand on the plate will concentrate in such a way to build attractive patterns, today known as *Chladni figures*, Fig. 1. These patterns are actually nodal lines of a plate which have zero displacements when plate vibrates in one of its normal modes, and the sand is gathered there. Chladni's technique for exciting a plate was to draw a bow over a piece of metal whose surface was lightly covered with sand. The plate was bowed until it reached resonance.

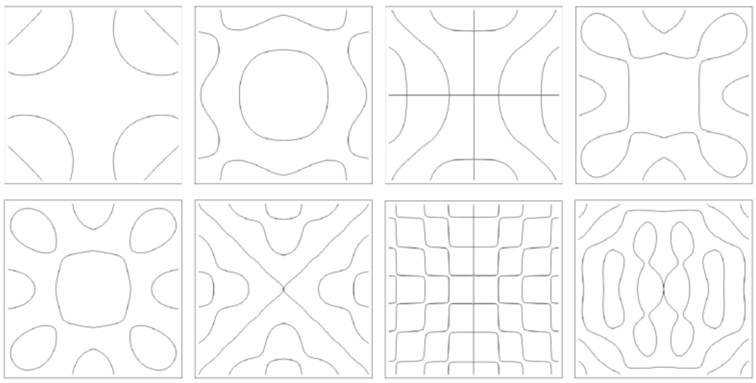

Fig. 1 Some of Chladni figures on a rectangular plate fixed at center

Based on this approach, the NMM was developed, with primary objective to excite the undamped (or normal) modes of a structure, one at a time. Today, exciting normal modes is typically done by attaching several shakers to the structure and driving them with a sinusoidal signal equal in frequency to the natural frequency of the mode to be excited. The amplitudes and phases of the sinusoidal drive signals are adjusted so that the predominant motion of the structure is due to the desired mode of vibration. The process of adjusting the amplitude, phase and frequency of the shaker to excide a normal mode is called modal tuning. By doing this, modal separation is achieved physically rather than mathematically and each mode is viewed in its purest possible form. Once a mode is properly excited its amplitudes of vibration at many points on the structure are measured, and taken as the mode shape. This condition is referred to as modal dwell. Once the mode is physically isolated, modal parameters and modal shapes are extracted real-time.

At the very beginning, modal test was performed with an eccentric shake table and a strobe light. The test structure was mounted on the shake table and the unbalanced rotating mass of the shake table caused it to shake with a sinusoidal motion. The strobe light was illuminating the test structure with flashes of light at the same frequency as the rotational speed of the shaker. Testing was done in the room with lights off, so that the test structure could be clearly seen. The strobe light made the vibrating structure to stand still for a brief moment, so one could view its shape.

There is a number of problems which make the NMM testing difficult, time consuming and expensive to implement: it is difficult to know where to locate shakers on the structure without some forehand knowledge of the modes of vibration; it is often extremely difficult to excite closely coupled modes (that is close in frequency with heavy damping) one at time; as all the mode shape data is collected during modal dwell, the structure must be completely instrumented with enough transducers and signal conditioning equipment so that amplitudes for all the desired degrees-of-freedom can be measured at once.

2.2. The FRF or transfer function methods (modern modal testing methods)

Soon after the discovery of the FFT algorithm in the late 1960s, it was implemented in computer-based laboratory test instrument, which is FFT analyser. In the early 1970s, a lot of new modal testing methods based on the use of FFT analysers were developed.

A fundamental measurement of any multichannel FFT based data acquisition system is the tri-spectrum averaging, which can be done on two simultaneously measured signals. In averaging process, auto-spectral function of each signal and cross-spectral function between these signals are calculated. From these three functions, the FRF is calculated. The FRF describes the input-output relationship between two points on a structure as a function of frequency. That is, the FRF is a ratio of the response of the structure (expressed as displacement, velocity or acceleration) at an output point due to an applied excitation force at an input point. Every peak in the measured FRF indicates a natural frequency of the structure, the width of the modal peak is related to the damping of the mode, while mode shape is obtained by assembling the peak values at the same frequency from all measurements.

With FRF measurement, rather than excite a structure one frequency at time with sine wave, the structure can be excited at many frequencies using broadband signal, which include impulses, random signals and rapidly swept sine signals (chirps). This is the major difference between the two modal testing methods – traditional and modern.

The FRF measurement is much easier and faster than NMM, since a structure can be excited using simple device, such as an impulse hammer, instead of shakers attached to structure, required for the NMM testing.

To obtain the mode shapes for a structure, a minimum set of FRF measurements must be done either between a single (fixed) input and many outputs, or between a single (fixed) output and many inputs.

3. DEVELOPMENT OF DEVICE

The aim of this part of research was to develop simple device for NMM investigation in a classroom environment. Besides Chladni plate experiment, objective was to construct a device able to test a beam model, too. The built device is named MK-20, Fig. 2, and its structure is briefly presented in this section.

It should be emphasized that the device was constructed by the 4th and 5th authors, who were students at that time. They also developed one shaking table [4]. Due to lack of financial resources, cheap, used parts are utilized as components for build-up of the devices.

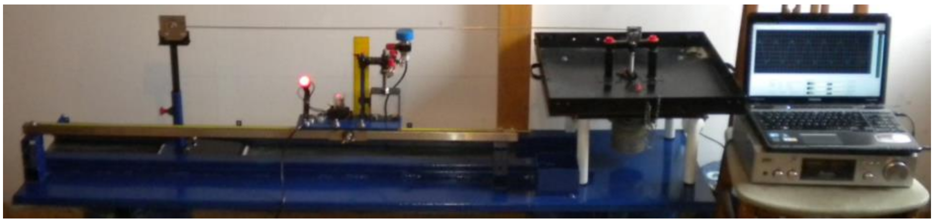

Fig. 2 Electromagnetic shaker for beam and plate models MK-20

MK-20 consists of four main components:

- the main base plate with sliding rail;
- electromagnetic shaker on carriers which excites vibrations in one support;
- fixation of other support;
- fleximeter with carrier rail.

4.1 Electromagnetic shaker

The main part of the MK-20 is an electromagnetic shaker, which consists of a permanent magnet and solenoid, taken from the old speaker, and vertical shaft which is guided through two fixators in order to prevent its horizontal movement.

The permanent magnet is attached to the main base plate with the steel plate. The dimensions of the plate are 300×300×3 mm. This plate is fixed to the main plate with four supports. Solenoid is carefully connected with vertical axle, Fig. 3. Aluminum shaft is used, in order to minimize load from the models and shaft itself on the limiting membrane.

Using the simple frequency generator software, digital signal of desired frequency is produced on a personal computer. After the amplification, it creates a variable magnetic field in the solenoid which starts to oscillate vertically with a given frequency. This mechanical vibration is transmitted through the aluminum shaft, which is plastically welded to the solenoid.

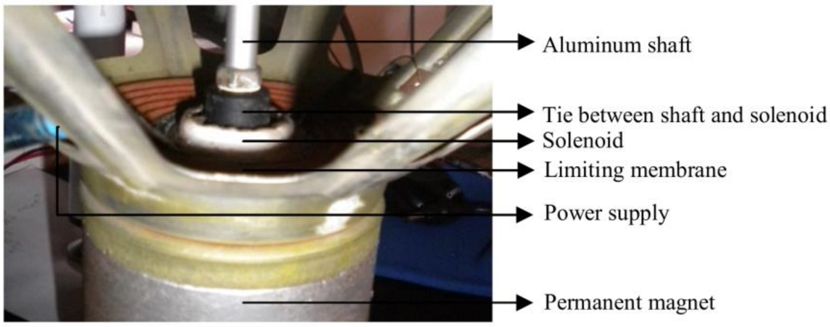

Fig. 3 Detail of connection between shaft and solenoid

4.2. Fleximeter with carrier rail

With fleximeter, it is possible to measure the static and dynamic deflection of the excited beam model, using floating-inductive sensor with an accuracy of 0.5 mm.

Fleximeter works on the principle of detecting metal that disturbs the magnetic field of sensor, which is 8 mm wide. Inductive sensor registers the disturbance of the magnetic field caused by the moving beam. This signal is observed by a signal lamp that is connected via relay inductive sensor. Inductive sensor is located on a rod attached to a series of joints which allows it to move in all directions, Fig 4. This allows adjustment of the sensor to the desired position and the desired angle. The dynamic deflection is measured as the sum of the calculated static deflection and relative position difference.

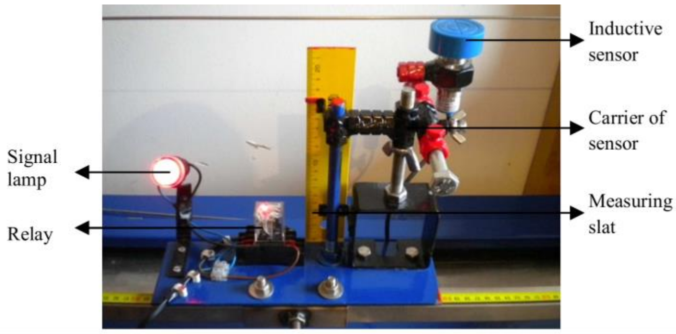

Fig. 4 Fleximeter with measuring slat

4.3. Types of supports

Two types of beam supports can be modeled with presented device: fixed end and pinned. Pinned support which restrains translations but allows free rotation is achieved by means of a ball bearing. Bearing with axle and stops is fixed by bolt to the carrier head and support. The connection between beam and support is achieved by screwing beam into a ring around the bearing and by tightening the nut, Fig. 5.

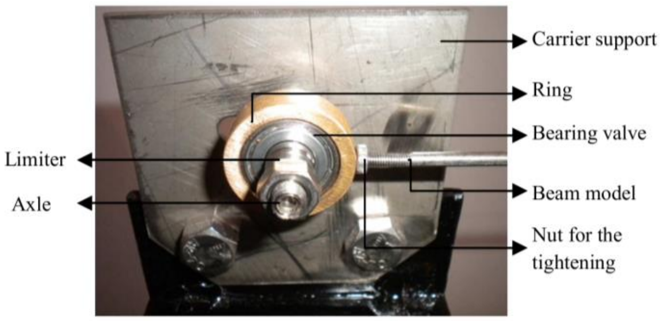

Fig. 5 Detail of pinned support

Fixed end support that prevents all translations and rotations is achieved by usage of metal connector with openings for connections with the head at the end of the movable shaft. Dimensions of this connector are 22×22×8 mm. In order to totally restrain all degrees of freedom, even those caused by high-frequency vibration, a hole is pierced and carved through the connector so that the beam can be screwed 20 mm into steel plate. Afterwards, the nut is tighten, which gives solid connection, Fig. 6.

Fixed plate support is achieved using the two nuts which tighten the plate so that all degrees of freedom are restrained. The first version of the support did not have a U-cross section fixator, and the axle was connected to the solenoid and passed through a steel plate fixator with the thickness of 3 mm. Since the movement of the solenoid is limited by the elastic membrane, the shaft had the freedom to rotate around the crossing point at the steel plate fixator. To prevent rotation of the shaft and the horizontal movement of its tip, it was fixed in two points. Currently, the device has the shaft elastically wedged in solenoid and horizontally fixed in the base plate fixator and on the top with the guide which is welded to the rigid U-cross section.

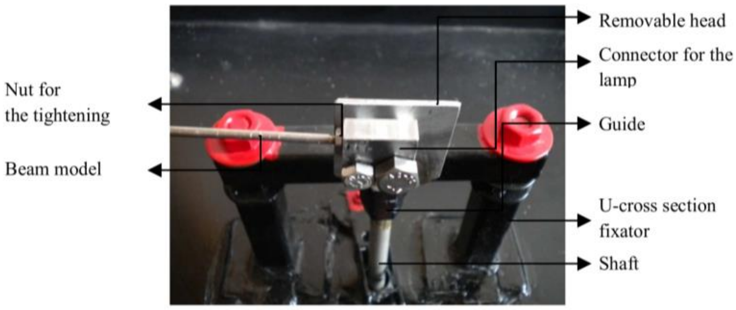

Fig. 6 Detail of fixed support on the moving shaft

4. Development of Software

In order to carry out numerical analysis, a simple code for free vibration analysis of plates is developed. Classical rectangular finite element with 12 degrees of freedom per node is used [3], Fig 7. Code is written in Wolfram Mathematica using its highly efficient built-in functions. The most useful ones for dynamical analysis are analytical integration and solution of eigenvalue problem. Visualization of three shape functions is given in Fig. 8.

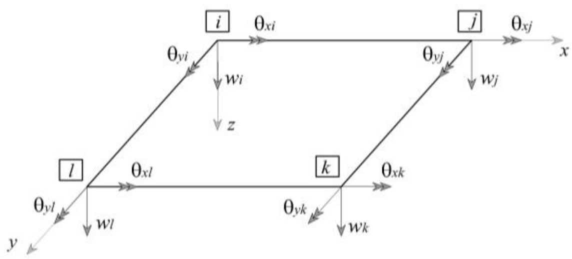

Fig. 7 Rectangular finite element with 12 degrees of freedom

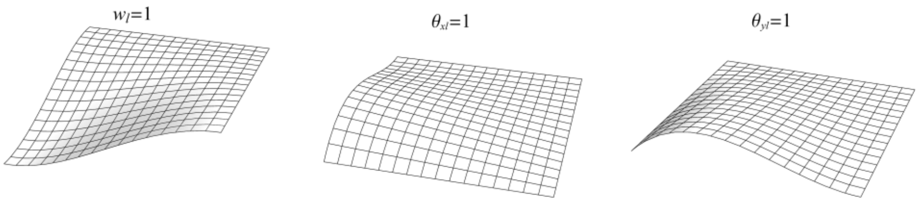

Fig. 8 Shape functions for unit valued generalized displacements of node *l*

Developed code is named "*MKE ploca vibracije*" and it is freely available at [5]. It is part of a greater project for development of open-source software for engineering education which runs at the University of Banjaluka for some time [6]. During years, many students worked on this project which proved as excellent method for increasing their motivation and knowledge.

4.1. Eigenvalue problem

A short overview for eigenvalue problem in dynamical analysis using the FEM is given below.

Applying the Hamilton's principle, the equations of motion of finite element are obtained. Stiffness and mass matrices follow from strain and kinetic energy of mechanical system, respectively [7]. If the system is free of damping and load, then the problem of free vibrations can be reduced to following homogenous matrix equation

$$(\mathbf{M}+\lambda\mathbf{K})\mathbf{q}=0 \tag{1}$$

where **K** is the stiffness matrix, **M** is the mass matrix and **q** is the vector of generalized nodal diplacements. λ is a scaling factor related to the eigenfrequencies of the system. This equation represents a well-known eigenvalue problem. Its solutions are eigenfrequencies and eigenvectors of vibration of the discrete system.

5. Examples and Discussion

In this section, one beam and one plate model are analysed using analytical, numerical and experimental approaches.

5.1. Example 1

Simple beam model is considered first. Its geometric characteristics are: R=3 mm and L=1200 mm, while the material is structural steel with following properties: E=210 GPa, ρ=7.86 t/m^3 and ν=0.3.

Excitation could not be applied at the end supports of a simple beam, because it is applied at only one point. It this way, boundary condition would be violated. Only way to excite simple beam in one point was to use conditions of symmetry, and apply excitation at the end with free vertical displacement.

In Fig. 9, fifth mode shape for one half of a simple beam is presented. It occurs for excitation frequency of 26 Hz. This result is in excellent agreement with well-known analytical solution for Bernoulli-Euler beam

$$\omega_5 = \frac{5^2\pi^2}{L^2}\sqrt{\frac{EI}{\mu}} = \frac{5^2\pi^2}{2.4^2}\sqrt{\frac{2.1\cdot 10^8 \, \frac{0.003^4\pi}{64}}{\frac{0.003^2\pi}{4}\cdot 7.86}} = 166.07\, rad/s = 26.43\, Hz. \quad (2)$$

FRF analysis is not peformed on this model because of its geometrical characteristics, since it resembles wire more than a beam, and accelerometer would significantly change its dynamical properties.

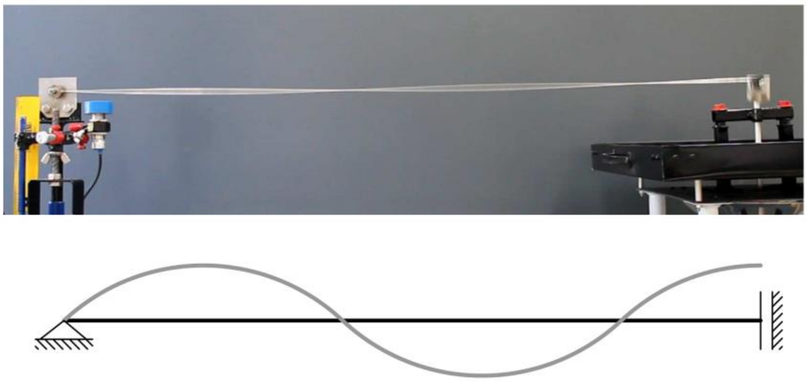

Fig. 9 Fifth normal mode of simple beam using conditions of symmetry

5.2. Example 2

Square plate with dimensions of 300×300×0.77 mm, fixed at the centre, is analysed using three methods: NMM, FRF method and FEM. Plate is made of the same material as the beam in previous example.

NMM analysis is performed using MK-20, where the sinusoidal vertical displacement excitement of support is applied. Five characteristic mode shape patterns are given in Fig. 10, and compared with the ones obtained from the developed FEM code. Convergences of some eigenfrequencies are given in Fig. 11.

Excellent agreement of the results is observed. The differences between the experimental and numerical models are small and occur due to many reasons. Some of them are: stress resulting from mounting beam systems and surface mount, imperfections of the device, numerical nature of FEM calculation, mass of the salt on the plate etc.

Good convergence properties of adopted finite element are observed in Fig. 11. For convergence of higher frequencies, denser meshes are needed.

Modal testing, that is FRF measurement, is done by using the Portable Pulse 3560C analyser and modal accelerometer type 4506, both by B&K, and ENDEVCO modal hammer. Measurement equipment is shown in Fig. 12. Plate is impulse excited applying modal hammer vertically on the screw at the plate centre. Testing is performed for two frequency ranges, 0-800 Hz and 0-3200 Hz. The resulting auto-spectrums of excitation and response signals are given in Fig. 13, while resulting FRF is given in Fig. 14. In Fig. 14, all frequencies presented in Fig. 10 are clearly noticeable and designated.

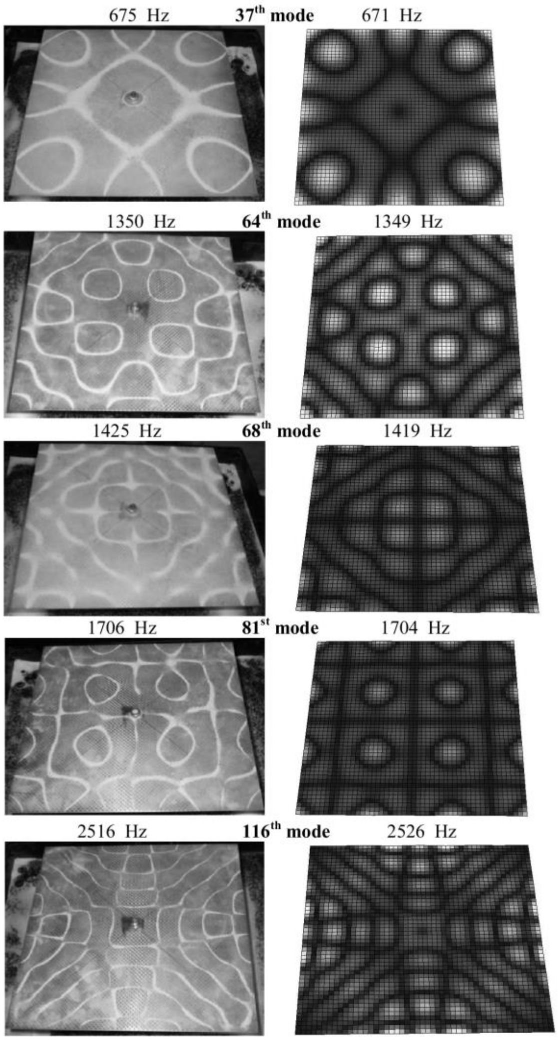

Fig. 10 Five normal modes of a rectangular plate; NNM - left, FEM – right

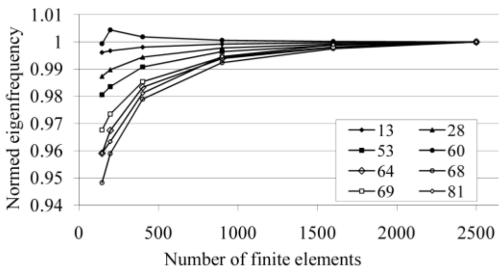

Fig. 11 Convergence of eigenfrequencies vs. number of finite elements

Fig. 12 Measurement equipment used for FRF testing

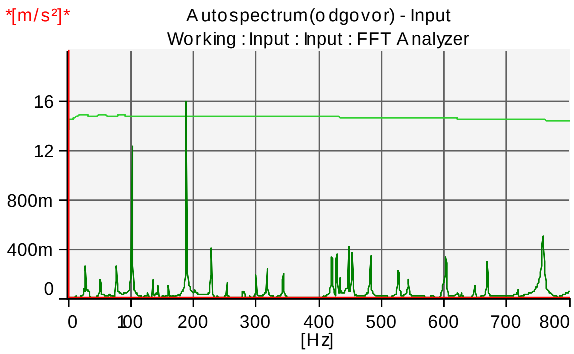

Fig. 13 Auto-spectrums of excitation and response signals

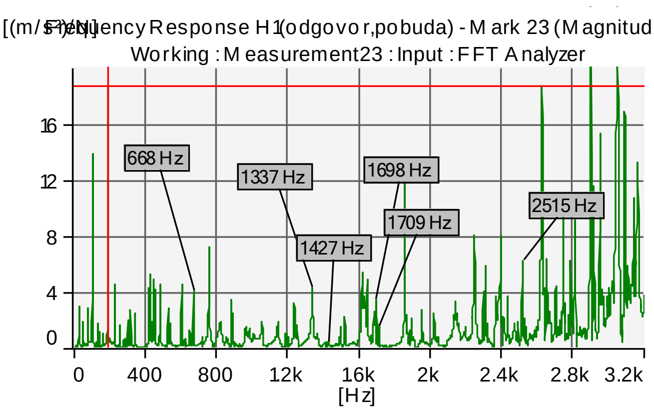

Fig. 14 FRF of square plate for frequency range 0-3200 Hz

7. CONCLUSIONS

Normal modes are fundamental quantities in dynamical analysis of structures and their determination is of crucial importance for engineering education. Experienced engineers are usually familiar with these concepts, but for students, these terms are very difficult to understand with standard, purely mathematical, approach and without experiment.

Presented analysis clearly shows that normal modes are inherent to structure and independent on type of analysis.

Adopted approach should allow students better understanding of normal modes and their experimental and numerical analysis. Teacher's task is to raise their motivation and get them enthusiastic about learning structural dynamics. This is especially feasible if they are included in development of experimental device and numerical codes, or at least, if they have free access to them.

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EKSPERIMENTALNO I NUMERIČKO IDENTIFIKOVANJE MODOVA KONSTRUKCIJE U CILJU EDUKACIJE INŽENJERA

Predstavljen je razvoj jednostavnog pokaznog školskog uređaja i softvera za vizuelizaciju normalnih modova konstrukcija. Uređaj je napravljen od dijelova starog zvučnika kontrolisanog od strane računara, dok je harmonijsko kretanje solenoida iskorišteno kao pobuda za modele greda i ploča. Jednostavan kod za analizu konačnim elementima je razvijen u paketu Wolfram Mathematica. Dobro poklapanje rezultata i atraktivne šare normalnih modova su privukli pažnju studenata. Rezultati su potvrđeni primjenom modernih metoda modalnog testiranja. Predstavljeni pristup se uklapa u standardnu metodologiju nastave iz dinamike konstrukcija.

Ključne reči: *obrazovanje inženjera, normalni modovi, Hladnijeva ploča, modalno testiranje, metod konačnih elemenata*