

SEISMIC ANALYSIS OF FRAMES WITH SEMI-RIGID CONNECTIONS IN ACCORDANCE WITH EC8

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Abstract. *Up to date research has pointed out that most of the structural connections of reinforced concrete (RC) frames, particularly precast ones, behave as semi-rigid. Therefore, it is of great importance to develop an analysis method which takes into account the connection rigidity. For that purpose matrix formulation of the deformation method is used in this paper, and the effect of rigidity of connections on the structure response is included by stiffness matrix for semi-rigidly connected member. The elements of this matrix are functions of the fixity factors at the ends of members. The proposed method is applied in seismic analysis of the precast RC frame structure of the existing industrial hall according to Eurocode 8 (EC8).*

Key words: *semi-rigid connection, stiffness matrix, seismic analysis, precast reinforced concrete system.*

1. INTRODUCTION

Connections form the vital part of precast concrete construction [1]. Up to date research has pointed out that structural connections in existing buildings, particularly in precast ones, behave neither as absolutely rigid nor perfectly pinned but as semi-rigid, which significantly influences the distribution of stresses and strains in the structure. Hence, there is a need to carry out the structural analysis and design taking into account the rigidity of connections. This is especially significant in earthquake engineering because seismic forces cause weakening of connections, i.e. even rigid ones become semi-rigid. This fact has not yet been adequately taken into account in structural analysis of RC structures. In practice the designers mostly tend to simplify dynamic actions of earthquake loads which directly results in structural systems with limited or poor seismic performances.

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In such a case, their seismic vulnerability and cumulative seismic risk appear very high. For example, due to the 1976 Friuli, Italy, earthquake most of the precast RC industrial buildings located in the affected area suffered extensive damage, or total collapse, particularly in the zone of connections [2].

Research on semi-rigid connections of structures has been carried out worldwide for about ninety years. The slope deflection and moment distribution methods were both applied to frames with semi-rigid connections in the 1930's by John F. Baker in England and J. Charles Rathbrun in the United States, [3]. Among other contemporary studies [4], [5], [6], the European project COST C1, Control of the Semi-Rigid Behavior of Civil Engineering Structural Connections [7], has significantly contributed in this field, but mainly in the field of steel structures, while there is less research on connections of precast RC structures.

Theoretical and experimental research on systems with semi-rigid connections has been going on at the Faculty of Civil Engineering and Architecture in Nis, Serbia, since 1980's [8]-[15]. Experimental tests have been performed on precast RC industrial hall structures and the obtained results related to connections have been a basis for the authors' theoretical work. A new simple design procedure for structures with semi-rigid connections has been developed using the matrix formulation of the deformation method, which is briefly presented in this paper. It is also shown how this procedure can be applied in seismic design according to Eurocode 8 (EC8) by use of an example of the existing precast RC industrial hall structure. The conclusions drawn about the influence of connection rigidity on seismic performances of the structure are significant for practical applications.

2 MATRIX ANALYSES OF PLANAR FRAMES WITH SEMI-RIGID CONNECTIONS USING THE DEFORMATION METHOD

2.1. Assumptions relating to semi-rigid connections introduced in classical formulation of the deformation method

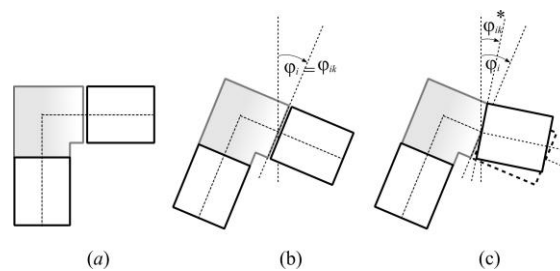


Fig. 1 a) Connection in the node i before deformation; b) Rotation φ_i of the node i in the case of rigid connection after deformation; c) Rotation φ_i of the node i and rotation φ_{ik}^* of the member end at i in the case of semi-rigid connection after deformation.

In this paper it is assumed that in the case of structures with semi-rigid (elastic) connections the node rotation is φ_i , i.e. φ_k , while rotation of the member end cross-section is φ_{ik}^* , i.e. φ_{ki}^* (Fig.1), so that the fixity factor in node i is designated as μ_{ik} , and in node k as μ_{ki} , [14], and they are defined as:

$$\mu_{ik} = \frac{\varphi_{ik}^*}{\varphi_i}, \quad \mu_{ki} = \frac{\varphi_{ki}^*}{\varphi_k} \quad (1)$$

In the classical formulation of the deformation method [16], the expressions for the bending moments at the ends of rigidly connected members are:

$$M_{ik} = a_{ik} \varphi_i + b_{ik} \varphi_k - c_{ik} \psi_{ik} + m_{ik}^{(o)} + m_{ik}^{(\Delta t)}, \quad (2)$$

$$M_{ki} = a_{ki} \varphi_k + b_{ki} \varphi_i - c_{ki} \psi_{ik} + m_{ki}^{(o)} + m_{ki}^{(\Delta t)}, \quad (3)$$

and for semi-rigidly connected members, in terms of the angles of rotation φ_{ik}^* and φ_{ki}^* of the end cross-sections, [14], they are:

$$M_{ik}^* = a_{ik}^* \varphi_{ik}^* + b_{ik}^* \varphi_{ki}^* - c_{ik}^* \psi_{ik} + m_{ik}^{(o)} + m_{ik}^{(\Delta t)}, \quad (4)$$

$$M_{ki}^* = b_{ki}^* \varphi_{ik}^* + a_{ki}^* \varphi_{ki}^* - c_{ki}^* \psi_{ik} + m_{ki}^{(o)} + m_{ki}^{(\Delta t)}, \quad (5)$$

or in terms of node rotations φ_i and φ_k :

$$M_{ik}^* = a_{ik}^* \varphi_i + b_{ik}^* \varphi_k - c_{ik}^* \psi_{ik} + m_{ik}^{(o)*} + m_{ik}^{(\Delta t)*} \quad (6)$$

$$M_{ki}^* = b_{ki}^* \varphi_i + a_{ki}^* \varphi_k - c_{ki}^* \psi_{ik} + m_{ki}^{(o)*} + m_{ki}^{(\Delta t)*} \quad (7)$$

For a member with rigid connections in nodes, introduced constants physically represent bending moments, so a_{ik} is the moment in node i due to unit rotation of node i , b_{ik} in node i due to unit rotation of node k , a_{ki} in node k due to unit rotation of node k , while c_{ik} is the moment in node i due to unit rotation of a member ik , Fig. 2a. Analogously, physical meaning of the corresponding constants for semi-rigidly connected members, which are marked by *, is the same, Fig. 2b [14].

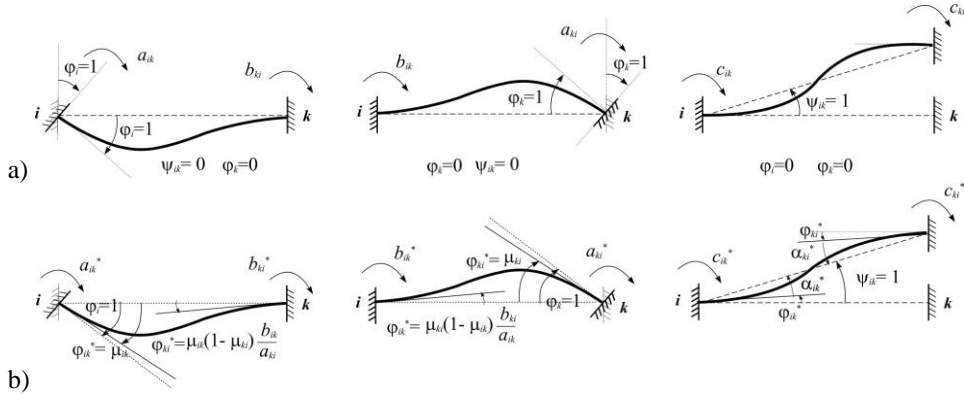


Fig. 2 Physical meaning of constants a_{ik} , b_{ik} , a_{ki} , b_{ki} , c_{ik} and c_{ki} in classical deformation method for a member: a) with rigid connections in nodes; b) semi-rigid connections in nodes

It follows from (1) that rotation of the left end is $\varphi_{ik}^* = \mu_{ik}$ due to rotation of the node i amounting to $\varphi_i = 1$. With that in mind, and knowing the physical meaning of the member constants a_{ik} and b_{ik} , the rotation of the right member end φ_{ki}^* can be assumed according to the principle of superposition in the following form:

$$\varphi_{ki}^* = \mu_{ik} (1 - \mu_{ki}) \frac{b_{ik}}{a_{ki}}. \quad (8)$$

Similarly, in the case of rotation $\varphi_k = 1$ of the node k , the rotation of the right member end is $\varphi_{ki}^* = \mu_{ki}$, and the angle φ_{ik}^* is:

$$\varphi_{ik}^* = \mu_{ki} (1 - \mu_{ik}) \frac{b_{ik}}{a_{ik}}, \quad (9)$$

while in the case of the member axis rotation $\psi_{ik} = 1$, the angles between chord of the member and tangents to the elastic line at the end cross sections are:

$$\alpha_{ik}^* = 1 - \varphi_{ik}^* = 1 - \mu_{ki} (1 - \mu_{ik}) \frac{b_{ik}}{a_{ik}}, \quad \alpha_{ki}^* = 1 - \varphi_{ki}^* = 1 - \mu_{ik} (1 - \mu_{ki}) \frac{b_{ik}}{a_{ki}} \quad (10)$$

2.2. Matrix formulation of the deformation method

In matrix analysis the model of a structure is discrete, composed of members (beams and columns) which are connected at discrete points - nodes [17].

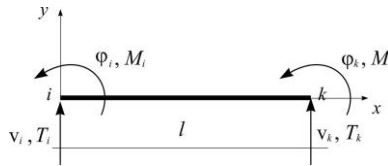


Fig. 3 Generalized displacements and forces at member ends

In structural analysis of line systems, which are composed only of beams and columns, the simplest member model is applied, that is a straight prismatic member at whose ends are the nodes of the structure, shown in Fig.3. Let the member be of length l , with a constant cross section, exposed to bending in the xOy plane of the local coordinate system. Its moment of inertia is I and the material modulus of elasticity is E . If the influence of axial forces on deformation of the member is neglected, the generalized displacements in nodes i and k (displacement parameters) are transversal displacements (v_i, v_k) and rotations (φ_i, φ_k) of the member ends, thus the element has four degrees of freedom, two at each end. Generalized forces are shear forces (T_i, T_k) and bending moments (M_i, M_k) at the ends i and k . Convention of positive directions of displacements and forces is shown in Fig 3.

The relation between the vector of generalized forces and the vector of generalized displacements is:

$$R = kq \quad (11)$$

where:

$$\mathbf{R}^T = [R_1 \quad R_2 \quad R_3 \quad R_4] = [T_i \quad M_i \quad T_k \quad M_k], \quad (12)$$

$$\mathbf{q}^T = [q_1 \quad q_2 \quad q_3 \quad q_4] = [v_i \quad \varphi_i \quad v_k \quad \varphi_k], \quad (13)$$

$$\mathbf{k} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix} \quad (14)$$

are generalized force vector, generalized displacement vector and member stiffness matrix, respectively.

Relation (11) applies to a member with both ideal (rigid and pinned) and semi-rigid connections if the elements of the stiffness matrix (14) are derived taking into account the fixity factor of connections, which is defined above. Herein the stiffness matrix for semi-rigidly connected member, and all of its elements, are marked by *, [14].

$$\mathbf{k}^* = \begin{bmatrix} k_{11}^* & k_{12}^* & k_{13}^* & k_{14}^* \\ k_{21}^* & k_{22}^* & k_{23}^* & k_{24}^* \\ k_{31}^* & k_{32}^* & k_{33}^* & k_{34}^* \\ k_{41}^* & k_{42}^* & k_{43}^* & k_{44}^* \end{bmatrix} \quad (15)$$

The stiffness matrix of the system is formed from stiffness matrices of all members, so determination of a member stiffness matrix is the most important for the solution of the considered problem.

When the axial forces effect on deformation is taken into account, the stiffness matrix of a semi-rigidly connected member can be written as follows:

$$\mathbf{k}^* = \begin{bmatrix} \frac{EA}{l} & 0 & 0 & \frac{EA}{l} & 0 & 0 \\ & k_{11}^* & k_{12}^* & 0 & k_{13}^* & k_{14}^* \\ & & k_{22}^* & 0 & k_{23}^* & k_{24}^* \\ & & & \frac{EA}{l} & 0 & 0 \\ & & & & k_{33}^* & k_{34}^* \\ \textit{simetrically} & & & & & k_{44}^* \end{bmatrix} \quad (16)$$

2.3. Stiffness matrix of a semi-rigidly connected member

It is known from literature that the elements of the stiffness matrix (14) of a rigidly connected member, based on the variation formulation of the problem of planar beam bending, can be represented in the form:

$$k_{mn} = EI \int_0^l N_m''(x) N_n''(x) dx \tag{17}$$

where $N_m(x)$ and $N_n(x)$ are interpolation functions defined in [17].

Analogously, the stiffness matrix elements k_{mn}^* for a member with semi-rigid connections at the ends can be determined using the expression:

$$k_{mn}^* = EI \int_0^l N_m^{*''}(x) N_n^{*''}(x) dx, \tag{18}$$

where $N_m^{*''}(x)$, i. e. $N_n^{*''}(x)$ are the second derivatives of the interpolation functions $N_m^*(x)$ and $N_n^*(x)$ for a semi-rigidly connected member [14]. The vector of interpolation functions can be shown in the form:

$$\mathbf{N}^* = [N_1^*(x) \quad N_2^*(x) \quad N_3^*(x) \quad N_4^*(x)], \tag{19}$$

where each interpolation function represents the elastic line of the semi-rigidly connected member at both ends due to the corresponding displacement parameter (generalized displacement) $q_m=1$, ($m=1,2,3,4$), while all other displacement parameters are $q_n=0$, $n \neq m$, Fig. 4.

When analyzing semi-rigid connections, in the case of applied unit translation $q_1=1$ at the end i of a member or unit translation $q_3=1$ at the end k of a member, while all other generalized displacements are equal to zero, the angles between chord of the member and tangents on the end cross sections after deformation, Fig.4, can be expressed according to (10) and the fact that they are small angles (for which it is $\text{tg}\alpha \sim \alpha$), as follows:

$$\alpha_{ik}^* = \left[1 - \mu_{ki} (1 - \mu_{ik}) \frac{b_{ik}}{a_{ik}} \right] \frac{1}{l}, \quad \alpha_{ki}^* = \left[1 - \mu_{ik} (1 - \mu_{ki}) \frac{b_{ik}}{a_{ki}} \right] \frac{1}{l} \tag{20}$$

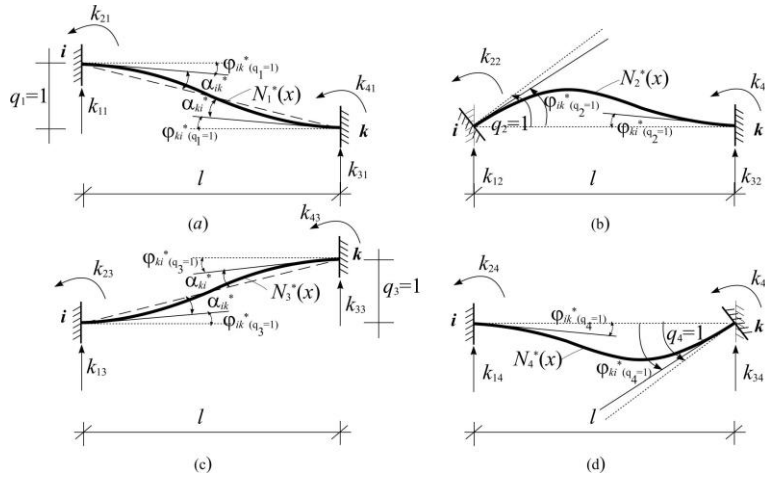


Fig. 4 Physical meaning of interpolation functions and the elements of stiffness matrix of a semi-rigidly connected member

Derivation of the expressions for interpolation functions (19) is presented in [15]. The elements of stiffness matrix are obtained in the following form:

$$\begin{aligned}
k_{11}^* &= \frac{4EI}{\ell} [\alpha_{ik}^{*2} + \alpha_{ik}^* \alpha_{ki}^* + \alpha_{ki}^{*2}] \\
k_{12}^* &= \frac{2EI}{\ell} [2(\alpha_{ik}^* \mu_{ik} + \alpha_{ki}^{*2} \ell - \alpha_{ki}^* \mu_{ki}) - \alpha_{ik}^* \mu_{ki} + \alpha_{ik}^* \alpha_{ki}^* \ell + \alpha_{ki}^* \mu_{ik}] = k_{21}^* \\
k_{13}^* &= -\frac{4EI}{\ell} [\alpha_{ik}^{*2} + \alpha_{ik}^* \alpha_{ki}^* + \alpha_{ki}^{*2}] = k_{31}^* \\
k_{14}^* &= \frac{2EI}{\ell} [2(\alpha_{ik}^* \mu_{ik} - \alpha_{ki}^{*2} \ell + \alpha_{ki}^* \mu_{ki}) + \alpha_{ik}^* \mu_{ki} - \alpha_{ik}^* \alpha_{ki}^* \ell + \alpha_{ki}^* \mu_{ik}] = k_{41}^* \\
k_{22}^* &= \frac{4EI}{\ell} [\mu_{ik}^2 - \mu_{ik} \mu_{ki} + \mu_{ki}^2 + \alpha_{ki}^* \mu_{ik} \ell - 2\alpha_{ki}^* \mu_{ki} \ell + \alpha_{ki}^{*2} \ell^2] \\
k_{23}^* &= -\frac{2EI}{\ell} [2(\alpha_{ik}^* \mu_{ik} + \alpha_{ki}^{*2} \ell - \alpha_{ki}^* \mu_{ki}) - \alpha_{ik}^* \mu_{ki} + \alpha_{ik}^* \alpha_{ki}^* \ell + \alpha_{ki}^* \mu_{ik}] = k_{32}^* \\
k_{24}^* &= \frac{2EI}{\ell} [2(\mu_{ik}^2 - \alpha_{ik}^* \mu_{ik} \ell - \mu_{ki}^2 + \alpha_{ki}^* \mu_{ki} \ell) + \alpha_{ik}^* \mu_{ki} \ell + \alpha_{ki}^* \mu_{ik} \ell - \alpha_{ik}^* \alpha_{ki}^* \ell^2] = k_{42}^* \\
k_{33}^* &= \frac{4EI}{\ell} [\alpha_{ik}^{*2} + \alpha_{ik}^* \alpha_{ki}^* + \alpha_{ki}^{*2}] \\
k_{34}^* &= -\frac{2EI}{\ell} [2(\alpha_{ik}^* \mu_{ik} - \alpha_{ki}^{*2} \ell + \alpha_{ki}^* \mu_{ki}) + \alpha_{ik}^* \mu_{ki} - \alpha_{ik}^* \alpha_{ki}^* \ell + \alpha_{ki}^* \mu_{ik}] = k_{43}^* \\
k_{44}^* &= \frac{4EI}{\ell} [\mu_{ik}^2 + \mu_{ik} \mu_{ki} + \mu_{ki}^2 - 2\alpha_{ik}^* \mu_{ik} \ell - \alpha_{ik}^* \mu_{ki} \ell + \alpha_{ik}^{*2} \ell^2]
\end{aligned} \tag{21}$$

3. SEISMIC DESIGN ACCORDING TO THE EUROCODE 8

According to the European standard Eurocode 8 (EN 1998-1:2004) [18], when it comes to the design of buildings in seismic regions, depending on the structural characteristics of the building, one of two types of linear-elastic analysis can be used: *lateral force method* or *modal response spectrum analysis*. As an alternative to the linear approach, non-linear methods can be used, such as *non-linear static (pushover) analysis* and *non-linear time history (dynamic) analysis*. For buildings conforming to the criteria for regularity in plan, or with the conditions presented in provisions (4.2.3.2) and 4.3.3.1(8) of Eurocode 8, the analysis may be performed using two planar models, one for each main direction. In seismic design of such buildings, the above presented proposed method, which includes the influence of the connection rigidity on the response of the structure by stiffness matrix in the form (15) or (16), can be applied.

Lateral force method of analysis may be applied to buildings which can be analyzed by the use of two planar models, and hence it is suitable for the implementation of the proposed procedure. The condition that structure response is not significantly affected by contributions from modes of vibration higher than the fundamental mode in each principal direction has to be met. It is fulfilled if a building has fundamental periods of vibration T_1 in the two main directions which are smaller than the following values:

$$T_1 \leq \begin{cases} 4T_c \\ 2,0 \text{ s} \end{cases} \tag{22}$$

where T_c is defined depending on earthquake action, and meet appropriate criteria for regularity in elevation.

For the calculation of the fundamental period T_1 of free vibration of the building well known methods of structural dynamics can be applied [19].

The seismic base shear force F_b for each horizontal direction in which the building is analyzed, is determined using the following expression:

$$F_b = S_d(T_1) m \lambda \quad (23)$$

where:

- $S_d(T_1)$ – is the ordinate of the design spectrum (see 3.2.2.5, [18]) at period T_1 ;
- T_1 – is the fundamental period of vibration of the building for lateral motion in the direction considered;
- λ is the correction factor, the value of which is equal to: $\lambda=0,85$ if $T_1 \leq 2T_c$ and the building has more than two stories, or $\lambda=1,0$ otherwise;
- m - is the total mass of the building, above the foundation or above the top of a rigid basement, computed in accordance with 3.2.4(2), [18]:

$$m = \sum G_{k,i} "+" \sum \Psi_{E,i} Q_{k,i} \quad (24)$$

where:

- Σ means „combination of effects of“;
- $G_{k,i}$ is characteristic value of permanent action i ;
- “+” denotes „in combination with“;
- $Q_{k,i}$ is characteristic value of variable action i ;
- $\Psi_{E,i}$ - is the combination coefficient for variable action i .

For the horizontal components of seismic action the design spectrum $S_d(T)$ is defined by the following expressions:

$$0 \leq T \leq T_B : S_d(T) = a_g \cdot S \left[\frac{2}{3} + \frac{T}{T_B} \left(\frac{2.5}{q} - \frac{2}{3} \right) \right] \quad (25)$$

$$T_B < T \leq T_C : S_d(T) = a_g \cdot S \cdot \frac{2.5}{q} \quad (26)$$

$$T_C < T \leq T_D : S_d(T) \begin{cases} = a_g \cdot S \cdot \frac{2.5}{q} \cdot \left[\frac{T_C}{T} \right] \\ \geq \beta \cdot a_g \end{cases} \quad (27)$$

$$T_D < T : S_d(T) \begin{cases} a_g \cdot S \cdot \frac{2.5}{q} \cdot \left[\frac{T_C \cdot T_D}{T^2} \right] \\ \geq \beta \cdot a_g \end{cases} \quad (28)$$

where:

- a_g, S, T_B, T_C, T_D – are values for the elastic response spectrum (Table 3.2 and Table 3.3 EN 1998-1:2004);
- $S_d(t)$ the value of the design spectrum;
- q is the behavior factor, depending on material and structure type;
- β is the lower bound factor for the horizontal design spectrum; the recommended value for β is 0,2.

The seismic action effects are to be determined by applying, to the two planar models, horizontal forces F_i to all stories:

$$F_i = F_b \frac{s_i m_i}{\sum s_j m_j} \quad (29)$$

where:

- F_i is the horizontal force acting on the store i ;
- F_b is the seismic base shear in accordance with the expression (23);
- s_i and s_j are the displacements of masses m_i and m_j in the fundamental mode;
- m_i and m_j are the stories masses computed in accordance with 3.2.4(2) of EN 1998-1:2004.

The displacements induced by the design seismic action are to be calculated on the basis of the elastic deformations of the structural system by means of the following simplified expression:

$$d_s = q_d d_e, \quad (30)$$

where:

- d_s is the displacement of a point of the structural system induced by the design seismic action;
- q_d is the displacement behavior factor, assumed equal to q unless otherwise specified;
- d_e is the displacement of the same point of the structural system, as determined by a linear analysis based on the design response spectrum in accordance with 3.2.2.5 EN 1998-1:2004.

4. NUMERICAL EXAMPLE

The structure of the existing industrial hall constructed in precast RC structural system AMONT, developed in Serbia, is chosen for the illustration of proposed design method which takes into account the rigidity of connections.

This building meets criteria for regularity in plan and therefore can be analyzed by two planar frames, according to the statement 4.3. 1(5) of standard EN 1998-1:2004.

Laboratory investigation of bearing capacity and deformability of full scale models of chosen characteristic connections of the precast RC industrial hall, shown in Fig.5, has been carried out in the Institute for Earthquake Engineering and Engineering Seismology (IZIIS), Skopje, Macedonia, [22]. Connections have been tested under simulated adequate load to the failure, and both linear and nonlinear analyses of the connections behavior have been carried out.

Based on the results of the tests it is observed that the most of the connections behave as semi-rigid. Frames which are analyzed in two orthogonal directions are both symmetrical, therefore elements in one half of the structure are marked in the Fig.6. Based on the test results connection column-to-foundation pocket can be considered as almost absolutely rigid (fixed) and because of that in longitudinal direction the fixity factor μ_{ik} in nodes 1, 2, 3, 4, 5, as well as in nodes 5 and 6 in transversal direction, is adopted as $\mu_{1-6}=\mu_{2-9}=\mu_{3-12}=\mu_{4-15}=\mu_{5-18}=1$. Beam-to-column connections at the roof level behave as pinned, and therefore nodes 19, 22, 25 are modeled as pinned. Columns denoted as S_1 and S_2 are precast as one-piece, so in nodes 6, 12, 18 on these columns of longitudinal frame, as well as in node 7 of transversal frame,

connection is rigid with $\mu_{ik}=1$. The fixity factor of the remainder of the connections was varied from $\mu_{ik}=0$ to $\mu_{ik}=1$ for the purpose of computing dynamic characteristics, seismic forces and internal forces due to them, as well as displacements of the structure, depending on the connections rigidity.

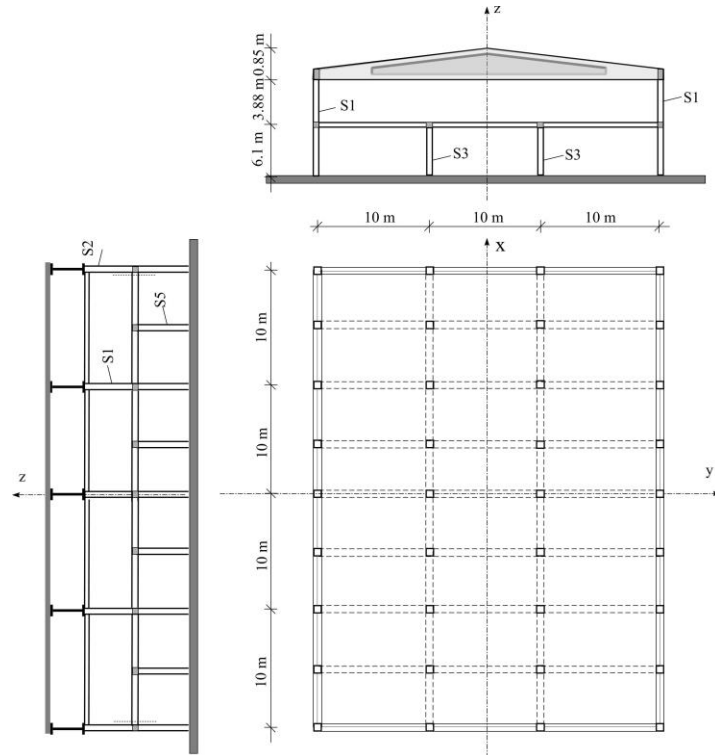


Fig. 5 Ground floor layout, longitudinal and transversal section of the industrial hall

At the Faculty of Civil Engineering and Architecture in Nis, Serbia, software has been developed, which is intended for seismic analysis of frame structures and which facilitates the calculation of basic dynamical characteristics and seismic forces and influences due to these forces [21]. It can be applied for structures with both classical connections and semi-rigid connections. Seismic design of considered structure is carried out by use of this software in accordance with provisions of Eurocode 8. Having in mind that above proposed and described method can be applied only for a planar frame, the structure is modeled by two orthogonal planar frames.

Seismic base shear forces F_b are calculated using equation (23). The values S_d are calculated according to the formula (27). The design ground acceleration is taken $a_g=0.1$ g for seismic zone VII, $a_g=0.2$ g for zone VIII and $a_g=0.4$ g for IX zone. In our case, for Type 2 elastic response spectrum and ground type B, S is 1.35 and T_c is 0.25. Total mass of the transversal middle frame is $361.39 \text{ kNs}^2 \text{ m}^{-1}$, the longitudinal end frame $517.606 \text{ kNs}^2 \text{ m}^{-1}$, while the correction factor is $\lambda=1.0$. The behavior factor is adopted as $q=3.9$ according to EN 1998-1: 2004, 5.2.2.2, for multistory and multi-bay frame and middle ductility (DCM) [18].

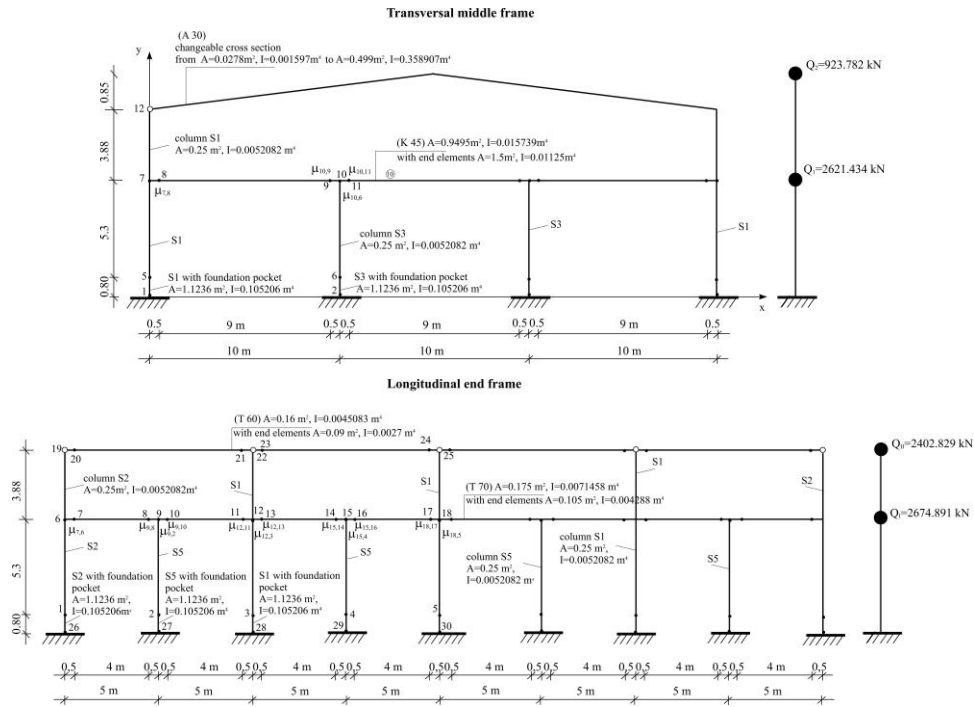


Fig. 6 Mathematical models for linear-elastic analysis taking into account the connection rigidity of the tested RC precast structure

Some of the results of the performed seismic analysis using proposed method are shown in the diagrams (Fig. 7), Table 1. and Table 2.

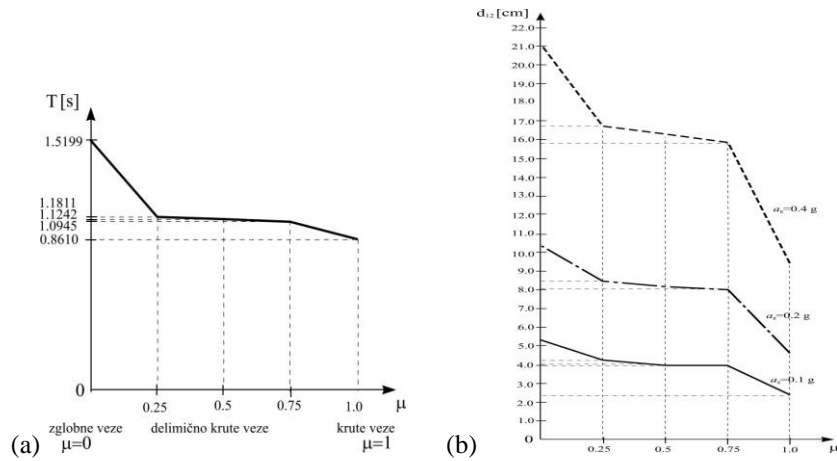


Fig. 7 Dependence on the fixity factors μ_{ik} : (a) Frame fundamental vibration period T_1 -transversal direction; (b) horizontal displacement d_s of the frame top due to design seismic action calculated according to EN 1998-1:2004- transversal direction.

4.1. Discussion of obtained results

Based on the results obtained from the *lateral force method* and taking into account the rigidity of connections, it can be concluded that fixity factor significantly affects the redistribution of influences, what is shown in Table 1 and Table 2, where values of displacements of the characteristic points in the first floor level and the top of the building are given for the longitudinal and transversal direction, depending on the assumed connection rigidity for different intensity of seismic action.

Table 1 Displacements d_e [m] according to linear analysis and displacements d_s [m] due to design seismic action calculated according EC8 for transversal direction

		pinned connections		semi-rigid connections				rigid connections			
				$\mu=0$	$\mu=0.25$	$\mu=0.5$	$\mu=0.75$	$\mu=1$			
				$T_1=1.5119$ s	$T_1=1.1811$ s	$T_1=1.1242$ s	$T_1=1.0945$ s	$T_1=0.8610$ s			
VII zone $a_g=0.1g$	d_7 (m)	$F_B=22.51$ kN $S_{d,7}=0.0127$ g	$d_{e,7}=0.0052$ $d_{s,7}=0.0203$	$F_B=28.86$ kN $S_{d,7}=0.0162$ g	$d_{e,7}=0.0040$ $d_{s,7}=0.0156$	$F_B=30.31$ kN $S_{d,7}=0.0171$ g	$d_{e,7}=0.0038$ $d_{s,7}=0.0148$	$F_B=31.14$ kN $S_{d,7}=0.0176$ g	$d_{e,7}=0.0036$ $d_{s,7}=0.0140$	$F_B=39.53$ kN $S_{d,7}=0.0223$ g	$d_{e,7}=0.0035$ $d_{s,7}=0.0136$
	d_{12} (m)		$d_{e,12}=0.0135$ $d_{s,12}=0.0527$		$d_{e,12}=0.0107$ $d_{s,12}=0.0417$		$d_{e,12}=0.0104$ $d_{s,12}=0.0406$		$d_{e,12}=0.0102$ $d_{s,12}=0.0398$		$d_{e,12}=0.0060$ $d_{s,12}=0.0234$
VIII zone $a_g=0.2g$	d_7 (m)	$F_B=45.02$ kN $S_{d,7}=0.0127$ g	$d_{e,7}=0.0104$ $d_{s,7}=0.0406$	$F_B=57.72$ kN $S_{d,7}=0.0162$ g	$d_{e,7}=0.0080$ $d_{s,7}=0.0312$	$F_B=60.62$ kN $S_{d,7}=0.0171$ g	$d_{e,7}=0.0076$ $d_{s,7}=0.0296$	$F_B=62.28$ kN $S_{d,7}=0.0176$ g	$d_{e,7}=0.0072$ $d_{s,7}=0.0281$	$F_B=79.06$ kN $S_{d,7}=0.0223$ g	$d_{e,7}=0.0070$ $d_{s,7}=0.0272$
	d_{12} (m)		$d_{e,12}=0.027$ $d_{s,12}=0.1053$		$d_{e,12}=0.0214$ $d_{s,12}=0.0835$		$d_{e,12}=0.0208$ $d_{s,12}=0.0811$		$d_{e,12}=0.0204$ $d_{s,12}=0.0796$		$d_{e,12}=0.0120$ $d_{s,12}=0.0468$
IX zone $a_g=0.4g$	d_7 (m)	$F_B=90.04$ kN $S_{d,7}=0.0127$ g	$d_{e,7}=0.0208$ $d_{s,7}=0.08011$	$F_B=115.44$ kN $S_{d,7}=0.0162$ g	$d_{e,7}=0.0160$ $d_{s,7}=0.0624$	$F_B=121.24$ kN $S_{d,7}=0.0171$ g	$d_{e,7}=0.0152$ $d_{s,7}=0.0593$	$F_B=124.56$ kN $S_{d,7}=0.0176$ g	$d_{e,7}=0.0144$ $d_{s,7}=0.0562$	$F_B=158.12$ kN $S_{d,7}=0.0223$ g	$d_{e,7}=0.0140$ $d_{s,7}=0.0544$
	d_{12} (m)		$d_{e,12}=0.0540$ $d_{s,12}=0.2106$		$d_{e,12}=0.0428$ $d_{s,12}=0.1669$		$d_{e,12}=0.0416$ $d_{s,12}=0.1622$		$d_{e,12}=0.0408$ $d_{s,12}=0.1591$		$d_{e,12}=0.0240$ $d_{s,12}=0.0936$

Table 2 Displacements d_e [m] according to linear analysis and displacements d_s [m] due to design seismic action calculated according EC8 for longitudinal direction

		pinned connections		semi-rigid connections				rigid connections			
				$\mu=0$	$\mu=0.25$	$\mu=0.5$	$\mu=0.75$	$\mu=1$			
				$T_1=1.4646$ s	$T_1=0.8629$ s	$T_1=0.798$ s	$T_1=0.7633$ s	$T_1=0.697$ s			
VII zone $a_g=0.1g$	d_6 (m)	$F_B=33.356$ kN $S_{d,6}=0.0130$ g	$d_{e,6}=0.0040$ $d_{s,6}=0.0156$	$F_B=56.62$ kN $S_{d,6}=0.0223$ g	$d_{e,6}=0.0023$ $d_{s,6}=0.0090$	$F_B=61.18$ kN $S_{d,6}=0.0171$ g	$d_{e,6}=0.0021$ $d_{s,6}=0.0082$	$F_B=63.97$ kN $S_{d,6}=0.0252$ g	$d_{e,6}=0.0019$ $d_{s,6}=0.0074$	$F_B=82.513$ kN $S_{d,6}=0.0325$ g	$d_{e,6}=0.0020$ $d_{s,6}=0.0078$
	d_{19} (m)		$d_{e,19}=0.0106$ $d_{s,19}=0.0413$		$d_{e,19}=0.0063$ $d_{s,19}=0.0246$		$d_{e,19}=0.0059$ $d_{s,19}=0.0230$		$d_{e,19}=0.0058$ $d_{s,19}=0.0226$		$d_{e,19}=0.0039$ $d_{s,19}=0.0152$
VIII zone $a_g=0.2g$	d_6 (m)	$F_B=66.712$ kN $S_{d,6}=0.0260$ g	$d_{e,6}=0.0080$ $d_{s,6}=0.0312$	$F_B=113.24$ kN $S_{d,6}=0.0446$ g	$d_{e,6}=0.0046$ $d_{s,6}=0.0179$	$F_B=122.36$ kN $S_{d,6}=0.0171$ g	$d_{e,6}=0.0042$ $d_{s,6}=0.0164$	$F_B=127.94$ kN $S_{d,6}=0.0504$ g	$d_{e,6}=0.0038$ $d_{s,6}=0.0148$	$F_B=165.026$ kN $S_{d,6}=0.0650$ g	$d_{e,6}=0.0040$ $d_{s,6}=0.0156$
	d_{19} (m)		$d_{e,19}=0.0212$ $d_{s,19}=0.0827$		$d_{e,19}=0.0126$ $d_{s,19}=0.0491$		$d_{e,19}=0.0116$ $d_{s,19}=0.0460$		$d_{e,19}=0.0452$ $d_{s,19}=0.0796$		$d_{e,19}=0.0078$ $d_{s,19}=0.0304$
IX zone $a_g=0.4g$	d_6 (m)	$F_B=133.424$ kN $S_{d,6}=0.0520$ g	$d_{e,6}=0.0160$ $d_{s,6}=0.0624$	$F_B=226.48$ kN $S_{d,6}=0.0892$ g	$d_{e,6}=0.0092$ $d_{s,6}=0.0359$	$F_B=244.72$ kN $S_{d,6}=0.0171$ g	$d_{e,6}=0.0084$ $d_{s,6}=0.0328$	$F_B=255.88$ kN $S_{d,6}=0.1008$ g	$d_{e,6}=0.0076$ $d_{s,6}=0.0296$	$F_B=330.052$ kN $S_{d,6}=0.1300$ g	$d_{e,6}=0.0080$ $d_{s,6}=0.00312$
	d_{19} (m)		$d_{e,19}=0.0424$ $d_{s,19}=0.1654$		$d_{e,19}=0.0252$ $d_{s,19}=0.0983$		$d_{e,19}=0.0236$ $d_{s,19}=0.0920$		$d_{e,19}=0.0232$ $d_{s,19}=0.0905$		$d_{e,19}=0.0156$ $d_{s,19}=0.0608$

Even a small change in rigidity of connection significantly affects the displacements, which is especially noticeable when one compares pinned with connections with small rigidity. For example, displacement δ_{19} of the top of the longitudinal frame with pinned connections ($\mu = 0$) is 67 % greater than in the case of the frame with the fixity factor $\mu = 0.25$. (Table 2).

Fundamental periods also depend on connections rigidity, as can be seen from the tables. For example, the fundamental period of the longitudinal frame is $T_1 = 1.4646$ s for $\mu = 0$, and it is 69 % greater than in the case of $\mu = 0.25$, when it is $T_1 = 0.8629$ s, (Table 2). Hence, it can be concluded that even small rigidity of connection effects favorably on redistribution of influences in the structure, as well as on the basic dynamic characteristics.

5. CONCLUSIONS

A method which takes into account the rigidity of connections, based on matrix formulation of the deformation method, for calculation of dynamic properties of a frame structure, as well as influences due to design seismic forces according EC8 is proposed in this paper. The elements of stiffness matrix of semi-rigidly connected members are functions of the fixity factors which are introduced for the purpose of simulating the real connection behavior in the structural design. Fixity factors can be determined either experimentally or assumed, and ranges from 0 (pinned connection) to 1 (rigid connection). The frame structure of the existing precast RC industrial hall, as an example of a frame with semi-rigid connections is chosen for illustration of the proposed method.

The following conclusions are drawn:

- Up to date research has shown that absolutely rigid connection is difficult to achieve in RC precast structures, but at the other hand there is always some rigidity in each connection.
- Significant difference regarding the influences in a structure is observed comparing pinned and connection with a small rigidity. Even small rigidity of connection effects favorably on redistribution of influences in the structure, as well as on the basic dynamic characteristics.
- If the real rigidity is ignored and pinned connections are assumed, as per normal practice for RC precast structures, the structure dimensions would be over designed, i.e. the solution would be uneconomical. On the other hand, if assumed full restraint is not realized, negative consequences regarding the distribution of stresses in structure would arise. It is therefore of utmost importance in optimal dimensioning of the structure to take into account the real fixity factor of connections, particularly in the case of seismic design of precast RC structures.

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SEIZMIČKA ANALIZA RAMOVA SA POLUKRUTIM VEZAMA U SKLADU SA EC8

Dosadašnja istraživanja su pokazala da se većina konstruktivnih veza armiranobetonskih (AB) ramova, posebno montažnih, ponaša kao delimično krute. Zbog toga je od velike važnosti razviti metod analize koji uzima u obzir krutost veze. U ovom radu je za to korišćena matična formulacija metode deformacije, a uticaj krutosti veza na odziv konstrukcije obuhvaćen je matricom krutosti za delimično kruto vezani štap. Elementi ove matrice su funkcije stepena uklještenja na krajevima štapova. Predložena metoda je primenjena u seizmičkoj analizi prefabrikovane AB ramovske konstrukcije postojeće industrijske hale u skladu sa Evrokodom 8 (EC8).

Ključne reči: delimično-krute veze, matrica krutosti, seizmički proračun, montažni armirano-betonski sistem.