DEFORMATION INDETERMINACY IN GLOBAL ANALYSIS OF STEEL STRUCTURES WITH SEMI-RIGID JOINTS

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Abstract. *This paper shows the procedure for determination of deformation indeterminacy within global analysis of steel constructions with semi-rigid joints, as well as the analysis of deformation indeterminacy within the approximate deformation method. In this paper there has been a detailed comparative analysis with criteria of cinematic system stability and division on steel frame with semi-rigid connections to systems with movable and immovable joints.*

Key words: *steel constructions, deformation model, semi-rigid connections.*

1. INTRODUCTION

The extension of classical deformation method that has been applied for global analysis of steel constructions with semi-rigid joints has been conducted in paper [1]. The process of calculation of steel linear systems with semi-rigid joints in function of the rotational rigidity of joints as a realistic parameter for determination of the stress field of both the joint itself and the construction as a whole has been given. Expressions for determination of bending moments at the ends of semi-rigidly connected elements within steel constructions and conditional equations for determination of deformation indetermined values at an approximate deformation method under static load of the First order theory have been given for introduced rotation rigidities of realistic connections.

This paper shows the procedures for determination of deformation indeterminacy within the global analysis of steel constructions with semi-rigid connections as well as the analysis of deformation indeterminacy of steel constructions within the approximate deformation method.

2. CONNECTIONS BETWEEN THE COMPONENTS OF KNOTS MOVING WITH SEMI-RIGID JOINTS AND BARS LENGTH SHIFTING

Relations between displacement components of nodes i and k - (u_i, v_i, u_k, v_k) and the length change (δl_{ik}) of the member ik that connects the two nodes are derived by comparing these two nodes and the length of the member before deformation, l_{ik}, and after deformation, $l_{ik} + \delta l_{ik}$, (Fig. 1).

The coordinates of the nodes i and k are denoted by (x_i, y_i) and (x_k, y_k), and the angle between the longitudinal axis of the nondeformed member ik and the axis O_x of the arbitrary coordinate system O_{xy} (valid for all members) is denoted by α_{ik} (Fig. 1). After the nodes i and $k,$ as the result of the member deformation, change their position to i' $(x_i + u_i, y_i + v_i)$ and k' $(x_k + u_k, y_k + v_k$), the member length would change from δl_{ik} to $l_{ik} + \delta l_{ik}$, and angle α_{ik} would change by $\delta \alpha_{ik} = \psi_{ik}$ and become $\alpha_{ik} + \psi_{ik}$. Also, the member would rotate for the angle ψ_{ik}. The node displacement components of nodes i and k are denoted by u_i, v_i, u_k, v_k . Figure 1 shows that:

$$(x_k + u_k) - (x_i + u_i) = (l_{ik} + \delta l_{ik})cos(\alpha_{ik} + \psi_{ik}), \qquad (1)$$
$$(y_k + v_k) - (y_i + v_i) = (l_{ik} + \delta l_{ik})sin(\alpha_{ik} + \psi_{ik}). \qquad (2)$$

In case of small displacements:

$$cos(\alpha_{ik} + \psi_{ik}) \approx cos\,\alpha_{ik} - \psi_{ik}sin\,\alpha_{ik},$$
$$sin(\alpha_{ik} + \psi_{ik}) \approx sin\,\alpha_{ik} + \psi_{ik}cos\,\alpha_{ik},$$

which means that equations (1) and (2), disregarding small values of the higher order, can be written as:

$$u_k - u_i = \delta l_{ik} \cos \alpha_{ik} - l_{ik}\psi_{ik} \sin \alpha_{ik}, \qquad (3)$$
$$v_k - v_i = \delta l_{ik} \sin \alpha_{ik} + l_{ik}\psi_{ik} \cos \alpha_{ik}. \qquad (4)$$
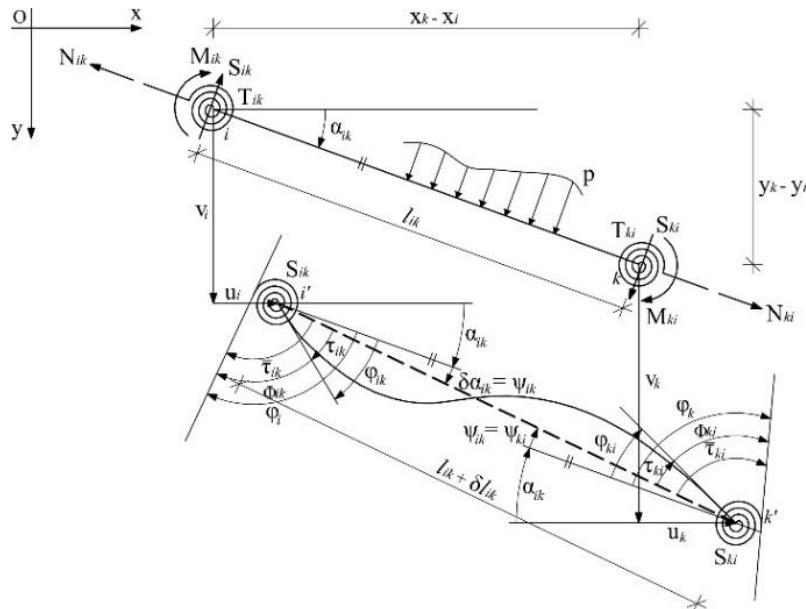

Fig. 1 Straight semi-rigidly connected member at its ends (before and after deformation)

By solving these equations with respect to δl_{ik} and ψ_{ik} we get:

$$\delta l_{ik} = (u_k - u_i) \cos \alpha_{ik} + (v_k - v_i) \sin \alpha_{ik}, \tag{5}$$

$$\psi_{ik} = \frac{v_k - v_i}{l_{ik}} \cos \alpha_{ik} - \frac{u_k - u_i}{l_{ik}} \sin \alpha_{ik}. \tag{6}$$

The change of the member length δl_{ik} is purely deformation value which is equal to zero when the bar is undeformed. Thus, the equation (5) shows the relation between nodes displacement components and the deformation value of bar δl_{ik}. The total number of these equations for the entire system is equal to the number of members within the system, i.e., z_s.

The rotation angle of the member ψ_{ik} is not a purely deformational value and can exist even when the member is undeformed. Thus, equation (6) does not represent the relation between the displacement components of nodes and deformation value of the member.

3. Deformation Unspecifity within Global Analysis of Steel Constructions with Semi-rigid Connections of Approximate Deformation Method

Papers [2], [3], [4], [5], [6] and [7] show that the influence of axial forces on the deformation of frames that are widely applied in steel constructions, do not significantly affect the internal force values at cross-sections of the structural system, thus can be neglected. This significantly reduces the number of deformational indetermined values [5], [6] and [7] which makes this method very convenient for global analysis of steel constructions with semi-rigid connections.

Once the influence of the axial forces on deformations are neglected, the changes of the lengths of the members will only depend on temperature oscillations. The relationships between the component displacements of nodes and the changes lengths of members (5) are as follows:

$$\delta l_{ik,t} = (u_k - u_i) \cos \alpha_{ik} + (v_k - v_i) \sin \alpha_{ik}, \tag{7}$$

Where, according to [6], $\delta l_{ik,t}$ is given by the equation:

$$\delta l_{ik,t} = \int_i^k \alpha_t t^o dl. \tag{8}$$

If t^o is constant on the entire member length, which is actually very common [7], then:

$$\delta l_{ik,t} = \alpha_t t^o l_{ik}. \tag{9}$$

Displacement components u_i, v_i, u_k, v_k, beside from z_o boundary condition of supports [7], now have to satisfy z_s compatibility conditions of node displacements according to equation (7), denoted by $z_o + z_s$:

$$n = 2k - (z_o + z_s). \tag{10}$$

The number of unknown angles φ of node rotation within the approximate deformation method is the same as within the accurate deformation method [7], i.e., equal to m – a number that represents the number of nodes with is at least one semi-rigid connection [1], hence the total number of deformation indeterminates of the structural system within the approximate deformation method *(d)* equals to:

$$d = m + n = m + [2k - (z_o + z_s)]. \tag{11}$$

4. CONCLUSION

In the analysis of equation (11) taking in consideration whether the number of mutually independent conditions [5], [7] and (7) is equal, smaller or larger than the number of displacement components of the nodes u_i, v_i, u_k, v_k, three different cases can occur:

1. Case where:

$$z_o + z_s = 2k \Longrightarrow n = 0 \rightarrow d = m, \qquad (12)$$

i.e., the number of compatibility conditions of nodes displacements $(z_o + z_s)$ is equal to the number of nodes displacement components and if all the conditions are mutually independent, the displacements of all of the nodes can be determined only from deformation conditions and thus be expressed by the function of temperature oscillations and supports displacements, i.e., rotation of clamping as:

$$u_i = u_{i,t} + u_{i,c}, \qquad (13)$$
$$v_i = v_{i,t} + v_{i,c}, \qquad (14)$$

where $u_{i,t}$ and $v_{i,t}$ are the node displacement components due to temperature oscillations and $u_{i,c}$ and $v_{i,c}$ are node displacement components due to supports displacements, i.e., rotation of clamping. Once there are no temperature oscillations and the supports do not move, when there is no clamping rotation, the conditions (7) are homogenous, thus the displacement components of all nodes are equal to zero. These frame-like structural systems of steel constructions with semi-rigid connections are called systems with immovable nodes. In these cases, the deformation indeterminate values are only the rotation angles of nodes (φ). Hence, the number of deformation indeterminates (*d*) of these systems is equal to *m*.

2. Case where:

$$z_o + z_s > 2k \Longrightarrow n < 0, \qquad (15)$$

if the number of conditions for displacement compatibility of nodes $(z_o + z_s)$ is larger than the number of displacement components and if there is $2k$ mutually independent conditions, the system also has immovable knots. When the supports do not move, i.e., the clampings do not rotate and the temperature oscillations are nonexistent, the displacements u_i, v_i, u_k, v_k of all the nodes of the system must equal to zero. Here as well, those deformation indeterminate values are only the rotation angles of nodes (φ) and the of deformation indeterminacy is equal to *m*. However, [7] shows the difference among systems with ideal connections at nodes, where the analyzed systems where those where the condition number (7) is equal to the number of displacements, and those where the number of conditions overcomes the number of displacements. When it comes to steel frame systems with semi-rigid joints, where the number of conditions (7) is equal to the number of displacements, i.e., when the relation (12) is valid, the displacement components due to temperature oscillations and support movements – clamping rotation, can be specifically determined from condition (7) and expressed through equations (13) and (14). If the number of conditions (7) overcomes the number of displacements, in general case there are no displacements that will satisfy all the conditions. Neglecting the influence of axial forces on deformation, the incompatible deformation conditions are

derived. In order to determine displacements of nodes due to temperature oscillations and support displacement – clamping rotation for these systems, the influence of axial forces on deformations must be taken into consideration [7].

3. Case where:

$$z_o + z_s < 2k \Rightarrow n > 0, \tag{16}$$

if the number of conditions of displacement compatibility of nodes $(z_o + z_s)$ is smaller than the number of displacement components, some displacements can be chosen arbitrarily, with keeping all of the conditions satisfied [7]. When it comes to steel frame constructions with semi-rigid joints, the nodes can move even when there are no temperature oscillations and when supports are not moving – the clamping's are not rotating. Hence, these systems are called the systems with moving nodes. Apart from m angle rotations, deformation indetermined values are:

$$n = 2k - (z_o + z_s), \tag{17}$$

which represent the independent displacement components of nodes (u and v).

For the systems with movable nodes, the number of independent equations [7] is smaller than the number of indeterminate displacement components of u and v by the value of n. In order to express displacements of nodes with equations (7), n more equations must be added:

$$F_j(u_i, v_i) = \Delta_j, j = 1, 2, \ldots, n. \tag{18}$$

The system of equations (7) and (18) can be solved if functions $F_j(u_i, v_i)$ are mutually independent and independent from functions of displacement of nodes at the right side of equations (7). Movements at nodes are thus dependent on support displacements and temperature oscillations that are considered in equations (7) and values $\Delta_j, j = 1, 2, \ldots, n$ (equal 18), which are referred to as displacement parameters in [6] and [7].

Once the comparative analysis of previously presented cases and systems stability criteria is conducted [7], it can be seen that the criteria by which the structural systems are divided into systems with movable and immovable nodes are nothing more than criteria to determine whether the truss created by z_s members and z_o supports, obtained by removing all the clamping's and semi-rigid joints and replacing them with pinned joints, is cinematically stable or unstable. Trusses formed in this way from the systems with immovable nodes are cinematically stable (simple or multiple), whilst the systems with movable nodes are those which have a labile system truss.

Division of systems with immovable nodes systems where relation (12) can be applied and those on which relation (15) applies corresponds to division of cinematically stable system trusses to simply cinematically stable and multiple cinematically stable, i.e., to statically determined and statically undetermined trusses.

It is known that, when it comes to statically determined truss systems, support displacements, i.e., the node movements and temperature oscillations do not cause internal forces, thus determination of node displacement is purely geometrical and needs to be solved by construction of Williot's displacement diagram.

When it comes to statically undetermined truss systems, support displacements, clamping rotation and temperature oscillations cause internal forces that affect the displacements, hence it is impossible to determine node displacement while neglecting the influence of axial forces on the deformation of the system. Node displacements of these systems are determined by well-known methods for determination of statical displacement of undetermined structural systems.

Determination of node displacement and member rotation of cinematically labile system truss is based on the determination of mechanism of movement, while the number of displacement parameters $n = 2k - (z_o + z_s)$ represents the number of degrees of freedom of truss system which is equal to minimal number of elements that needs to be added in order to turn the truss into a stable system.

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DEFORMACIJSKA NEODREĐENOST U GLOBALNOJ ANALIZI ČELIČNIH KONSTRUKCIJA SA POLU-KRUTIM VEZAMA

U ovom radu prikazan je postupak za određivanje deformacijske neodređenosti u globalnoj analizi čeličnih konstrukcija sa polu-krutim vezama kao i analiza deformacijske neodređenosti u približnoj metodi deformacija. U radu je detaljno izvršena komparativna analiza sa kriterijumima o kinematičkoj stabilnosti sistema i podeli na čelične ramovske nosače sa polu-krutim vezama na nosače sa pomerljivim i nepomerljivim čvorovima.

Ključne reči: *čelične konstrukcije, metoda deformacije, polu-krute veze.*