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Survey Paper

A REVIEW OF DRILL STRING DYNAMICS AND MODELING TECHNIQUES

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Abstract. *The complex dynamics of drill strings under various vibration modes during drilling operations present significant challenges to the oil and gas industry due to the high costs associated with oil well drilling. This analytical study aims to provide a comprehensive overview of the most important and widely used mathematical modeling tools and techniques for describing the behaviors of drill strings under various conditions. The study highlights significant scientific contributions and research papers that have utilized or addressed these methods, including finite element, lumped mass, partial differential, wave equation, and Cosserat theory. This emphasis is meant to be partial but reflects the frequent use of these methods in drill string modeling. It also sheds light on the evolution and classification of different mathematical models used, divided into two main categories: static models and dynamics models, and how they have been involved with advancements in drilling technologies. Finally, this review underscores the potential for significant cost savings in the oil and gas industry by incorporating artificial intelligence and modern technologies into well drilling and drilling operations control, which is closely tied to improving the productivity and efficiency of oil and gas fields.*

Key words: *Drilling operation, vibration mode, finite element, mathematical models, modern technologies, static models.*

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1. INTRODUCTION

Drill string vibrations significantly influence the efficiency and performance of drilling operations, and understanding these dynamics is crucial for improving drilling processes. Various vibration forms, including torsional, axial, and lateral vibrations, lead to challenges such as stick-slip, bit-bounce, and wear-related issues[1].

This review aims to consolidate and synthesize insights from extensive scholarly investigations over the past two decades, focusing on modeling and describing the complex dynamics of drill strings under different vibration modes.

This work presents a novel and original tool for understanding the mathematical modeling of drill string behaviors under different environments and the main characteristics of these models. This scientific contribution analyzed many research papers from the last two decades.

The introductory section of this review delves into the harmful dynamic modes that emerge during well-drilling operations, presenting the biggest challenges for engineers and researchers to navigate in these research topics.

The second section exhaustively examines prior investigations concerning the modeling, quantification, and analytical understanding of drill string dynamics in the context of detrimental oscillations. We emphasize the procedural intricacies involved in modeling drill string behavior.

The third section offers a primary classification of existing modeling approaches used to describe BHA and discusses their development with advancements in drilling technologies.

Finally, the fourth section underscores the importance of integrating advanced technologies, such as edge computing and artificial intelligence, into drill string control operations. It provides a concise review of their applications.

2. DRILL STRING VIBRATIONS AND SHOCKS HARMFUL FORMS

The intricate dynamics of a drill string give rise to three distinct and severe vibratory modes, each with its own unique characteristics. These vibrational phenomena, aligned with harmful mechanical oscillations, have the potential to induce subtle perturbations that serve as early warning signs of wear and tear within the heterogeneous array of constituent elements. In the subsequent discourse, a comprehensive elucidation of these severe vibratory modes is presented, highlighting their mechanical attributes and the potential implications they carry [2].

2.1. Axial dynamic and Bit-Bounce vibration mode

This particular vibration mode, characterized by irregular movements of drilling components along their longitudinal axis, induces a phenomenon known as bit bounce.
This results in a rough and aggressive drilling behavior that can lead to excessive wear and damage to critical components. The drill bit, for instance, may suffer accelerated deterioration, and the Bottom Hole Assembly (BHA) can sustain damage, contributing to increased total drilling time. Additionally, downhole coupling mechanisms come into play, exacerbating the situation by exciting lateral displacements of the drill string. The complex motion patterns induced by this vibration mode are often observed when employing a roller-cone drill bit, also, referred to as a tricone bit due to its distinctive multiple-lobe structure. In this case, the erratic interaction of the bit with the well bottom can cause the bit to

momentarily lose contact with the rock formation, leading to an uneven and unpredictable drilling performance. These vibrations, including the bit bounce pattern, can propagate upwards and be detected at the surface, providing valuable insights into the drilling dynamics and underscoring the importance of comprehensive vibration analysis in drilling operations to mitigate detrimental effects on equipment and overall project timelines [3].

2.2. Torsional dynamic and Stick-Slip vibration mode

Downhole measurements reveal an intriguing phenomenon: the rotational motion of the drill bit does not always mirror the constant rotary speed applied at the surface. In fact, data show that during a significant portion of drilling time, the downhole torsional speed fluctuates wildly, deviating from a steady state. This self-excited rotational motion, known as stick-slip, arises from the nonlinear relationship between torque and angular velocity at the bit. The drilling assembly's torsional flexibility further amplifies these fluctuations, resulting in a nonuniform oscillatory behavior. Consequently, the drill bit can experience rotational speeds up to ten times higher than the nominal rotary table speed or, conversely, come to a complete standstill. These torsional vibrations inflict fatigue on drill collar connections, lead to drill bit damage, and slow down the drilling operation, thereby extending the overall drilling process [4].

2.3. Lateral dynamic and Whirling vibration mode

The detection of lateral vibrations is now possible due to the development of acceleration measurements at the bottom of the hole during drilling operations. According to research by Spanos et al. (2003), this type of vibration is caused by incorrect alignment of the drill string relative to the drill axis, its interaction with the drill walls, and weight variations on the tool. These vibrations cause drill pipe fatigue, failure, and premature tool wear. Repeated impacts between the drill string and the wellbore wall can lead to local enlargement of the hole, explaining the irregularity in the drill’s whole diameter. It is important to note that the discovery of lateral vibrations is relatively recent compared to the understanding of axial and torsional vibrations. These lateral vibrations are quickly attenuated and do not propagate at the surface].

In the following table and figure, different harmful dynamics are presented with their severe forms cases [5].

Table 1 Harmful dynamic mode and sever forms

Vibration Mode	Severe Form	Direction of appearance	Main reason of appearance
Axial Mode	Bit-Bounce	Along axis of drill string	Component misalignment
Torsional Mode	Stick-Slip	Entire drill string	Elastic deformation and recovery of drill sting
Lateral Mode	Whirl, and Bit-Whirl	Perpendicular to drill string axis	Center eccentricity rod, Buckling

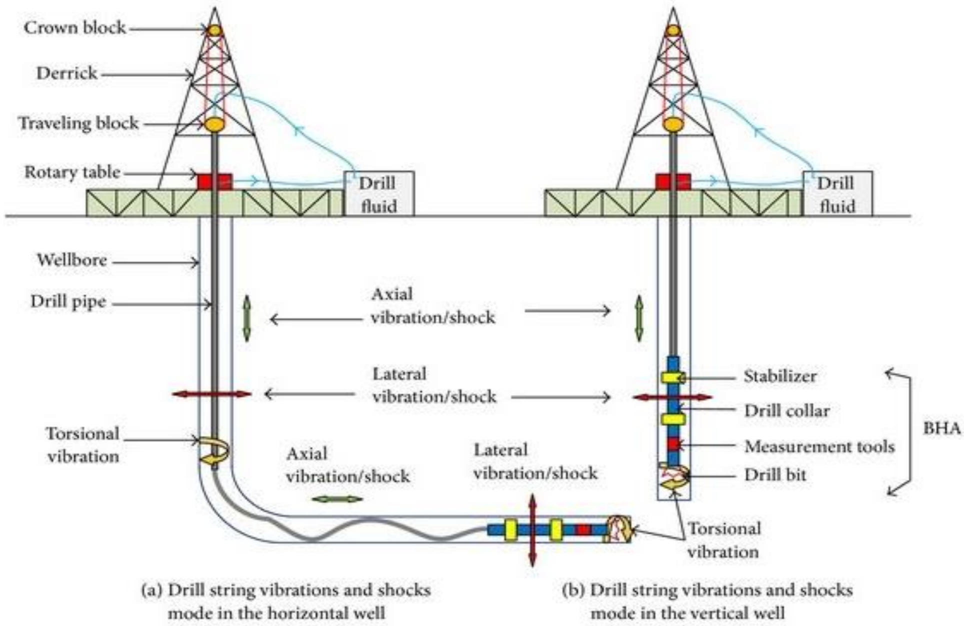

Fig. 1 Drill string vibrations and shocks in typical well

3. The Review Study

In this context, we propose a comprehensive review that unifies various research inquiries about the mathematical representation of drill string behavior. This review integrates several fundamental methodologies, including lumped theory, the finite element method, wave equations, partial differential equations, and the application of Cosserat modeling, to provide a holistic understanding of drill string dynamic. By analyzing these diverse approaches, we aim to offer a synthesis that contributes to a deeper insight into the complex behavior of drill strings during drilling operations.

3.1. Application of Finite Element (FEM) in Modeling

Finite Element Analysis (FEA) has revolutionized the understanding of drill string behavior within drilling systems, offering an invaluable and intricate lens into their complex dynamics. The versatility and sophistication of FEA have attracted researchers and investigators alike, providing a powerful tool to explore the diverse facets of drill string behavior. Over the years, many studies have employed FEA to unravel the mysteries of drill string interactions and phenomena during drilling operations. This analytical technique has proven indispensable in the quest to comprehend and optimize the performance of drilling systems, contributing significantly to advancements in both theoretical and practical aspects [6].

Trindade and Sampaio's 2005 investigation employed an advanced finite element model to explore axial-torsional coupled vibrations and their impact on drill string rotational speed regulation. Their work shed light on the complex dynamics of the stick-slip phenomenon, a challenge in drilling operations [7].

In their paper [8], Y.A. Khulief and H. Al-Naser aimed to enhance the understanding and mitigation of drill string vibrations. They developed a dynamic model using the Lagrangian approach and the finite-element method to capture the complex vibrational behavior of drill strings. The model accounted for crucial factors such as gyroscopic effects, torsional-bending inertia coupling, and the influence of axial gravity.

In 2010 [9], Compton presented a proven approach to enhancing deep-water drilling efficiency. Their main objective was to optimize the combination of whole enlargement and drilling operations while mitigating damaging vibrations. They employed the advanced finite element dynamic formulation, to analyze the interactions between bottom hole assembly (BHA) components and identify the root causes of vibrations.

In 2015 [10], a novel experimental rig was designed to mimic various types of drill-string vibrations, including stick-slip oscillations and whirling. The nonlinear dynamics of the drill-string were described using the Finite Element Method (FEM), and the results verified experimentally.

Vromen et al. [11] Used a finite element method with eighteenth degree of freedom (DOF), to describe the structure of a jack-up drilling rig, to mitigate the Stick-Slip severe form of torsional vibration.

An evaluative investigation has determined the impact of two different formulations on the reliability and precision of the finite element method in large-diameter drilling operations. The updated Lagrangian formulation with dynamic re-meshing (UL-DR) in DEFORM 3D yielded the most accurate results. It provided the best predictions for drilling thrust force, torque, and chip thickness when compared to experimental data [12].

Mingjie Cai and colleagues in [13], developed a beam finite element (FE) model to analyze drill string dynamics in curved wells. They investigated the effects of various parameters on lateral vibration, including rotation speed, WOB, COF, and STB. While the model reproduced field data, it did not consider drill string-wellbore wall contact, a critical aspect of curved well drilling.

In the contribution in 2023 [14], the authors developed a rigid-flexible coupled model to analyze and optimize the vibration of a rock drilling arm. Their study showed that this model better describes the propelling beam's vibration, leading to reduced offset values and smaller vibration amplitudes at a steady state.

A finite element analysis coupled with The Taguchi method have used in the investigation conducted by A.L. Muthuveerappa et al. to determine the optimum drilling parameters with CNN drill material and specify cutting speeds and drill points angles recommended to minimize deformations and stress[15].

The work in [16] proposed two novel controller techniques relying on simple linear relations of measured signals to mitigate torsional vibrations. The proposed controllers were based on finite element theory with non-regularized dry friction, which proved their adaptation to these two novel techniques.

The study in [17] presented a significant advancement in understanding drill-string dynamics by employing a three-dimensional finite element model that considers dynamic friction and buckling effects. Validated by laboratory experiments, the model offers enhanced computational accuracy in predicting sliding force and vibration load transfer.

A key strength of the research lies in identifying the critical factors that control vibration propagation along the drill string. This leads to the establishment of a prediction model for vibration propagation distance, providing a valuable tool for optimizing axial vibration parameters.

3.2. Application of Lumped Mass as a model

This section explores the use of lumped-parameter models as a powerful tool for analyzing and controlling various dynamic phenomena in drilling operations.

The application of lumped-parameter models offers a simplified and practical approach to studying the dynamic behavior of drill strings, providing a comprehensive framework for analyzing and addressing the challenges posed by various vibration modes and oscillations [18].

In 2010, authors undertook a semi-analytical investigation aimed at analyzing stick-slip oscillations within drilling operations. They employed a lumped mass methodology featuring singular inertia along the axial direction; the researchers successfully computed the precise limit cycle and optimized parameters hitherto unknown [19].

The research by Kamel and Yigit (2014), offered a comprehensive analysis of severe bit-bounce and stick-slip vibrations experienced by oil well drill strings with drag or PDC bits. Central to their work is introducing a meticulously crafted, two-degrees-of-freedom lumped parameter model, derived using the Lagrangian approach, which proves instrumental in deciphering the intricate nature of these vibrations [20].

The author's paper in, introduced a distributed-lumped modeling technique to simulate oil well drilling systems. The study aimed to mitigate vibrations at the BHA, focusing on drill string effective length. The distributed-lumped model found to be more precise than the lumped model, especially for long drill strings [21].

The paper published in 2020 focuses on the nonlinear dynamics of a drill string in a horizontal well. They introduced a novel and comprehensive six-degrees-of-freedom (6DoF) lumped mass model specifically designed for horizontal wells. The dynamic model accounts for the drill string's longitudinal, lateral, and torsional motions [22].

Idir Kessai et al. used a lumped mass model with multiple degrees of freedom to look at how drill bits change shape when there are large-amplitude stick-slip vibrations in rotary drilling systems. The results were validated using MWD data from real wells, providing insights into the dynamic behavior of drill bits during stick-slip phenomenon [23].

The thesis in [24] contributed to the development of advanced modeling and control systems for offshore drilling. Two new lumped-parameter models were proposed and analyzed for the stability of drill string dynamics. The first model was derived from Lagrangian mechanics and structures using the Bond Graph methodology, and the second one used Kane's methods based on the Newton-Euler formulations.

The 2022 study by Lelya A. Khajiyeva, focused on analyzing the nonlinear dynamics of drill strings using the lumped-parameter method (LPM). Their primary objective was to investigate lateral vibrations in a vertical drill string influenced by supersonic gas flow. LPM exhibited superior convergence and stability compared to its linear counterpart [25].

Laib et al.'s study proposed a hybrid interval type-2 fuzzy PID controller for a multi-DOF Lumped parameters oil well drill-string system. The system was modeled using a four-degree-of-freedom drill string model, incorporating nonlinear bit-rock interactions. The hybrid controller combined fuzzy logic (IT2FC) with conventional PID control, resulting in faster response times. Simulations and experimental testing validated the controller's performance, and proved its effectiveness compared to other controllers as sliding mode controllers [26].

To eliminate the stick-slip phenomenon Rian et al. designed an observer-based H∞ control, in collaboration with observer-based LQG controller using a lumped parameter model with ten degrees of freedom. This modeling approach has shown its adaptation with the control technique used in this investigation [27].

The main objective of the study in [28] was to develop a comprehensive drill string dynamics model based on the lumped parameter model (LPM). The developed model showed its capability to describe the fully coupled axial, torsional, and lateral vibrations of drill string.

The study in [29]explored lateral vibration in rotor-stator rubbing systems. Through experimental and numerical methods, authors investigated the effects of mass unbalance and radial clearance on system response. The study revealed significant impacts of these parameters on dynamic characteristics, including periodic and chaotic responses. A simple test rig and a two-degree-of-freedom lumped mass model were used, with numerical analysis employing the Runge-Kutta technique.

3.3. Application of Wave equations and PDE as models

Drill string vibrations can have significant negative consequences for the drilling industry, affecting efficiency and equipment integrity and compromising borehole stability. To address these concerns effectively, it is essential to utilize a wave equation model that accurately depicts the behavior of the drill string under various vibration modes. By adopting this modeling approach, engineers and researchers can gain valuable insights into the intricate dynamics of the drill string. The wave equation model considers the elastic nature of the drill string, the forces exerted on it, and the relevant boundary conditions [30].

Improving mud-pulse telemetry techniques to improve while-drilling data transmission was the aim of the study in 2000. The main objective was to develop a comprehensive model that accurately represents the complex behavior of wave propagation in drill strings, with the goal of improving data transmission speeds and mud-pulse telemetry [31].

Miroslav Krstic's 2013 study focused on developing an adaptive control strategy for anti-stable wave PDE systems, specifically for applications in oil drilling systems. The research proposed a tailored approach to address the challenges of wave equations, offering a stable and efficient control strategy for these complex systems [32].

The scholarly work presented by Boussaada et al, 2013 in [33]centers on investigating control strategies targeting the intricate interplay of axial and torsional vibrations encountered in drilling operations. The authors employ a neutral delay differential equations (NNDE) model, which stems from a systematic reduction of more complex partial differential equations (PDE) models. To address the control objectives, the study advocates the application of PID controllers and delayed feedback control mechanisms.

The authors in [34] adopt a coupled modeling framework that integrates wave and ordinary differential equations (ODE) to capture the system dynamics comprehensively.

The investigation conducted in [35], presents a novel observer-based boundary condition in which coupled ordinary differential equations (ODEs) and partial differential equations (PDEs) are used as a basic model, which proved its effectiveness in designing the controller and the novel observer.

He Zhang developed a high-dimensional coupled model of PDE (partial differential equation) and ODEs (ordinary differential equations) to analyze self-excited axial and torsional vibrations in rotary drilling systems. The model combines a multi-degrees-of-freedom representation with a rate-independent bit-rock interface law, capturing the regenerative effect through a PDE and ODEs. The study includes a stability analysis and a parametric study to understand torsional stick-slip occurrences [36].

The contribution in [37] aimed to develop a more improved and comprehensive model for drill string dynamics. They used delay-differential equations to simulate the cutting process. The adopted equation avoided the delays and allowed for more continuous cutting during the drilling operations.

In their paper, Jean Auriol addressed a critical aspect of the drilling industry: improving the estimation of drill-string dynamics and rock properties during directional drilling. The drill-string dynamics was represented by a coupled dynamical model wave equations with an ordinary differential equation at the downhole boundary (bit-rock interaction) [38].

To design a hybrid full-state feedback controller and a state observer for a drilling system, the researchers combined the backstopping methodology and frequency analysis based on a linear 2 dimension hyperbolic PDE system coupled with proximal ODEs and distal Load ODEs [39].

Toumi's research delves into the complex dynamics of rotary drilling systems, proposing innovative control strategies. The study employs nonlinear models to identify critical factors contributing to drill string vibrations. Two distinct control approaches are introduced, using PDEs and ODEs as modeling tools, to mitigate these vibrations and ensure operational stability [40].

3.4. Application of Cosserat theory as a modeling technique

The Cosserat rod theory has attracted much interest lately as a solid and adaptable framework for simulating the complex behaviors of drill strings. These studies have presented and confirmed drill string dynamic models that are comprehensive and based on the Cosserat theory.

Marcos Silviers’s paper presented an analysis of stick-slip oscillations in drill strings using a modified Cosserat rod element method. The integrated model captures the complex dynamical and geometrical behavior, including general deformations like flexure, extension, torsion, and shear. This approach offered computational advantages compared to Finite Element methods, enabling the simulation of stick-slip oscillations and drill-string-borehole wall contact [41].

To analyze the complex dynamics of the drill string, the authors in [42]employed a Cosserat rod theory, taking into account the slenderness of the structure and the geometrical aspects. This modeling approach gave rise to adaptations with the stochastic analysis technique used in the investigation.

The study conducted by Tucker and Wang in 2022 presents a novel approach to understanding the vibrational dynamics experienced by the functional components of a drilling assembly. The study focuses on creating a comprehensive mathematical framework based on the ideas of Cosserat's theory [43].

Fan Yu, Genlu Huang, and colleagues in 2023, present a comprehensive model for analyzing the lateral vibration of a bottom hole assembly (BHA) during drilling operations. The Cosserat theory, which treats the BHA as an elastic rod, was utilized to develop a more sophisticated drill string dynamics model. This model incorporates various factors encountered in practical drilling, such as wellbore constraints, deviation angle, friction, torque, centrifugal force, and material viscosity [44].

Hector Eduardo in his thesis developed a deterministic structural model using Cosserat rods to study the dynamics of drill strings in curved oil wells. The model considers drill pipes and the bottom whole assembly as a one-dimensional Cosserat structure, offering a strategy to address lateral contact in curvilinear well configurations [45].

To address the challenges in drilling complex oil and gas wells, Zambetti et al. developed a 3D geometrical dynamic model based on the Cosserat rod theory. The mathematical model demonstrated its advantages in simulating real-time scenarios [46].

In the study in [47] , researchers developed a comprehensive continuous Cosserat rod model to study the dynamics of a drill-string in arbitrary well geometries. The model simulates 3D dynamics, including lateral, axial, and torsional motion, and incorporates lateral contact, offering a versatile tool for drill string's dynamics analysis.

3.5. Modelling techniques comparative study

The prior modeling approaches are summed up in the following table, which compares their benefits and drawbacks in explaining drill string dynamics.

Table 2 Comparison table of the most used mathematical techniques in drill string modelling

Year	Authors	Drill string model	Control strategy	Counting /considered parameters	Testing	Results
2000	J.M. Carcione et.al [31]	Wave equation model	/	MWD tool / Data transmuions speed	Simulation	Developed model proved its accurately represents the complex behavior of wave propagation in drill strings
2005	Trindade et.al [7]	Advanced finite element model	Active control strategy	Speed rotation	Simulation/ Experimental	Drill string vibration mitigation using finite element methodology modelling review
2005	Y.A. Khulief et.al [8]	Finite element analysis with 12DOF	/	Gyroscopic effect, Gravity factor Torsional/ bending inertia coupling	Simulation	The model accounts for the gyroscopic, as well as the bending/torsional inertia coupling. In addition, the axial gravitational filed effect on the drillstring
2010	Compton et.al [9]	Advanced finite element dynamic formulation	/	Drill bits / reamers drive systems/ MWD/LWD tools	Simulation / Experimental	A detail analysis of interactions between BHA components Identified root cause of vibrations
2013	Miroslav Krstic's [32]	Wave equations system	Adaptive control strategy	Rotary speed WOB Top Drive torque	Simulation	Modeling and control strategies were offered a stable and efficient understanding for the complex BHA dynamics

2013	Boussaada et al[33]	Neutral delay differential equations (NNDE) model	PID controllers Delayed feedback control mechanisms	Axial vibration trajectory	Simulation	The PID controller gain and the feedback gain are established the same to compare their regulations results
2013	Marcos Silviers et al[41]	Cosserat rod element method	/	Flexure deformation Torsion, Extension and shear deflections	Simulation	The proposed model captures the complex dynamics and geometrical behavior of drill string
2014	Kamel and Yigit [20]	Lumped parameter model	/	Rotary and translational motions of the drillstring Bit/rock formation interaction	Experimental	The model and the proper choice of operational parameters, may be possible to minimize the effects of stick-slip and bit-bounce and increase the ROP
2015	M. Kapitaniak et.al [10]	Finite element method	/	RPM/ WOB Friction factor	Experimental /numerical	Experimental rig to reproduce all modes of vibrations was investigated Great predictive capabilities of the developed model especially for stick-slip vibration
2016	B. Saldivar et al [34]	wave and ordinary differential equations (ODE)	Feedback controllers The soft-torque system Flatness-based control	Axial velocity Rotational velocity	Simulation	The proposed control systems could eliminate the stick-slip and the bit-bounce vibrations.
2017	H. Ibrahim Basturk et al [35]	Ordinary differential equations (ODEs) partial differential equations (PDEs)	Active control strategy	The angular velocity	Numerical Simulations.	The proposed control was effectively suppressed stick-slip oscillation.
2017	T. Vromen *et al* [11]	Finite element model	Robust control based on skewed-μ DK iteration	Top drive velocity Top drive torque Drill bit and well interaction	Simulation	The controller offered: only surface measurements are employed no need for down-hole measurements multimodal drill-string dynamics are dealt with it.

2020	P. Athanasius et.al [21]	Lumped parameter model Distributed Lumped modelling	/	Drill string effective length	Simulation	The D–L model was found to be more precise, particularly for long strings.
2020	D. Xie et.al [22]	6 DOF lump mass model	/	The friction and cutting effects	Simulation	The dynamic model established tested where the friction and cutting effects are gradually switched on. It analyzed the complex whirling of a horizontal drill-string.
2020	Idir Kessai et.al [23]	Several DOF mass-spring-damper model	/	Torque variation WOB / Drill bit radius	Simulation	hat the stick-slip vibrations are very damaging to the drill bit even if it is controlled by parametric variation because the delay between its appearance and the intervention.
2021	N. K. Tengesdal et al [24]	LPM based Lagrangian mechanics and Bond Graph LPM based used Kane's methods	Nonlinear predictive controller and Kalman filter Multivariable control PID controller	Rotational velocity The nonlinear frictional pressure forces	Simulation	The proposed advanced controllers proved effectiveness robust to mitigate vibrations in offshore drilling operations.
2021	J. Priest et al [12]	3D finite element model Update Lagrangian	/	Torque Chip thickness values	Simulation/ experimental	The update Lagrangian is not a computationally viable for large diameter drill string Combine the updated-Lagrangian with dynamic re-meshing methodology is the most suitable with large diameters.
2021	J. Auriol et.al [38]	Wave equations Ordinary differential equation	/	Rock proprieties	Simulation	The results was improving the estimation of drill string dynamics and rock properties during directional drilling.
2021	H. E. Goicoechea et al [42]	Cosserat model	/	The slenderness of the structure The geometrical proprieties	Simulation	The used modeling approach gave rise adaptations with the stochastic analysis technique of drill string dynamics

2022	L. A. Khajiyeva [25]	Lumped parameter method	/	Angular velocity Lateral displacements	Simulation / Numerical	The proposed model was effectively solve the nonlinear problems drill string dynamics
2022	H. Zhang and E. Detournay [36]	Partial differential equation ordinary differential equations	/	Rate of penetration Bit-rock interaction	Simulation	A stability analysis and parametric analysis of stick-slip and bit-bounce phenomenon
2022	Tucker and Wang et.al [43]	The Cosserat theory	/	The geometrical aspects of the structure	Simulation	The novel approach presented a high understanding level of BHA dynamics
2023	Neto et al [16]	Finite element method	Active control Based state feed back control (LQR) approach	Rotary velocity signals	Simulation	The proposed model can define which signals are relevant for state feedback for ensuring asymptotic stability.
2022	H. E. Goicoechea et al [37]	A delay-differential equations	/	The angular speed The bit-rock interaction	Simulation	The proposed equation could avoid the delays and allowed for more continuous cutting during the drilling operations.
2022	Fan Yu et.al [44]	The Cosserat theory	/	WOB / TOB / rotary speed	Simulation / Laboratory experiment	The Cosserat theory was used to develop more sophisticated drill string dynamics model
2023	Weili Liu et al [17]	3D FEM dynamic model	/	Axial force friction model Buckling effect	Laboratory experiment	The exciting force amplitude, drilling fluid density, axial force, friction coefficient, and drill-string dimension have significant effects on the friction reduction performance and effective
2023	A. L. Muthuveerappan et al [15]	Finite element analysis	/	Drilling bit cutting speed drill point angle	Simulation	Optimum drilling parameters for deformation were found for real value of rotational velocity
2023	J. Redaud et al [39]	2DOF hyperbolic PDE system Proximal ODEs	A hybrid full-state feedback Back-stepping strategy and frequency analysis	Rotational velocity	Simulation / Experimental	An adequate feedback control was designed in frequency domain and a robust tracking output stat observer was proposed.

2023	Zambetti et.al [46]	3D Cosserat rod theory in Euclidean strands	/	Contact points drill string-wellbore walls. The geometry of the rod	Simulation/ experimental	The mathematical model demonstrated its advantages in simulating real-time scenarios
2023	Hector Eduardo [45]	Cosserat rods model	/	The geometrical rod proprieties	Simulation	The proposed model was offering a strategy to address lateral contact in curvilinear well configurations
2024	Y. Zhang et. al [28]	Lumped parameter model (LPM).	Oriented control	Angular velocity Top drive torque	Simulation	The developed model showed its capability to describe the fully coupled axial, torsional, and lateral vibrations
2024	S. Toumi and R. Mlayeh, [40]	Wave equation PDE coupled with ODEs	Feedback controller based on Lyapunov approach	The velocity at the bottom extremity The damping factor od BHA	Simulation	The proposed control effectively suppressed the harmful vibrations
2024	H. E. Goicoechea et.al [47]	3D Cosserat rod model	//	Lateral contact points Drill cutters-well contact	Simulation	The proposed modelling offered a versatile tool for drill string's dynamics analysis

Table 3 A summary of modelling strategies and theories to describe drill string dynamics.

Modelling technique	Description	Advantages	Drawbacks
Finite Element method	Numerical method, divide complex structures' behavior into smaller parts and solving mathematical equation	Using multiple approaches to analyse complex structures and systems	Need a powerful calculating process
The Lumped Mass Approach	This method simplifies complex structures by modelling them as masses and springs, aiding dynamic behaviours analysis.	It simplifies the complex structure to easier mathematical problems	Missing of important parts of systems , and ignore many structure’s details
Wave Equation and PDE methods	The Wave Equation and Partial Differential Equations (PDEs) are essential tools in engineering. They describe wave propagation and changes in physical quantities.	Easy mathematical tools, They could adapt with many system’s modelling	They represent analytical techniques can be away from the real process
The Cosserat Method	This technique in continuum mechanics considers microstructure and material rotations within deformable bodies	It is valuable for accurately modeling materials with micro scale deformations. Very precise method	It needs knowledge of the microscopic structure of systems which is not easy for many systems

4. STATIC AND DYNAMICS MODELS

The vibration phenomenon in drilling processes is complex due to the coupling of different modes and numerous parameters. Interactions between the drill string and the formation further complicate modeling efforts. Initially, static models focused on calculating stresses, deformations, and directional behavior. Dynamic models introduced, as a temporal and frequency approaches to predict stress evolution over time and identifying critical rotation speeds to prevent resonance. Both static and dynamic models are validated through site measurements or laboratory experiments, acknowledging the complexity of the complete mathematical problems [48].

4.1. Static models (Torque and Drag)

Static models are developed to calculate deformations, axial forces, and the twisting of models are called also Torque and drag models. They divide into two mains categories: smooth models, which neglect the rigidity of the filling, and rigid models, which consider it [49] .

In their pioneering work, Johancsik and Friesen set out to tackle the challenging issue of predicting torque and drag forces in directional wells. Their innovative computer model, developed in 1984, revolutionized the industry by providing a powerful tool to forecast these critical parameters accurately [50].

In their 1998 paper, M.S. Aston et al. addressed the growing importance of torque and drag minimization in drilling operations. They present a comprehensive review of the diverse techniques available in the industry to tackle these challenges [51].

In their 2003 work, D. Stuart introduced an innovative drilling technology to address torque and drag issues. By combining an extended-gauge bit design with a steerable motor or rotary steerable system, they achieved remarkable reductions in friction factors and wellbore cline [52].

The analysis in [53] aimed to enhance the accuracy and the practicality of torque and drag analysis in drilling operations through the application of the finite element method (FEM), leading to improved decision-making in well drilling operations.

In the context of offshore well operations, the torque and drag analysis using the 3D model to account for the friction forces was the object of the Master's thesis realized by Knut Tveitan, where the correspondence between the 3-dimensional friction model and field data was examined [54].

In the thesis of Mohammad Fazaelizadeh delved into the critical aspects of torque and drag calculations in directional drilling. The thesis introduces novel torque and drag models, considering wellbore geometry and drill string interactions while accounting for drill string stiffness and partial wellbore contacts [55].

To improve Torque and Drag modeling using traditional and machine learning methods, Mayowa realized a full PhD research, using a soft-string model for torque and drag analysis. This involved addressing frictional losses, considering the well-path trajectory, and solving coupled differential equations [56].

Da Silva's study introduced real-time monitoring software, utilizing torque and drag models. This technology automatically diagnoses drilling problems, analyzing drill string loads and borehole contact. The model aims to optimize RAM actuation, minimizing risks [57].

Zou et al.'s research introduced a novel controllable hybrid steering drilling system CHSDS, aiming to minimize drag and torque challenges in extended-reach wells. Based on the stiff-string drag torque model combined with the friction-reducing performance of the CHSDS, the study revealed the system's efficacy, particularly in inclined sections, with rotary speed and fluid density enhancing friction reduction [58].

4.2. Dynamic models

The development of dynamic models linked to the evolution of surface and downhole measurements. There are two essential classes: temporal and frequency models. The first dynamic models appeared in the middle of the last decade [59].

The author in the paper in [60] presented a rigid-flexible multibody system approach to model drill string dynamics. The research modelled the drill string as a series of uniform flexible beams interconnected by linear viscous-elastic force elements. Showing a close match between theoretical and numerical results for static, buckling, and resonant frequency problems.

In 2013, the research paper discussed the development of the rigid-flexible multibody system for modeling drillstring dynamics and the influence of model parameters on simulation accuracy and calculation time [61].

To improve the dynamic friction model of drill strings under torsional-longitudinal coupling oscillation, researchers developed a model that considers friction forces and torsional vibration. The constructed dynamic friction model exhibited good agreement with experimental findings, confirming its accuracy in forecasting friction force changes owing to torsional vibration [62].

Using a multi-body system dynamics technique, Xiu-Quan Liu and colleagues performed a mechanical analysis of deep-water drilling riser systems. According to the study, tensioners efficiently reduce the riser system's deformation, resulting in lower static displacements. The researchers do point out that compared to simple dynamic models; the application of multi-body dynamic systems is more complicated [63].

In the scientific article [64], a nonlinear dynamic model with four-degree-of-freedom (4DOF) was established to characterize the behavior of the drill-string in a deviated well. The stick-slip phenomenon, the lateral and torsional deformations of the drill string, and the fluid-damping effects were also considered.

The scientific contribution in 2021 [65]implied the development of a nonlinear dynamic model using Garlerkin's method coupled with a second-order differential equation, considering the effects of parameters such as forcing frequency, perturbation amplitude, mass ratio, and flow velocity.

In [66], a novel intelligent dynamic model was proposed to predict the drilling rate of penetration online. The proposed model showed its capacity for autonomous learning during the drilling process and it had a higher prediction accuracy than two online and five offline, well-known conventional methods.

In the same context of ROP online prediction, a multi-source information fusion-based dynamic drilling ROP model was proposed. This model takes into account the formation drillability and the time interval as updating conditions. Compared to well-known methods. This novel technique improved the effectiveness of ROP online prediction [67].

Based on two different techniques of modeling, the lumped mass method and the discrete element technique, a drill string dynamic model was proposed, taking into account rock breaking as a crucial factor. The drilling simulation results conducted with the integrated model under different ground rotational speeds (GRSs) and weight-on-bits (WOBs) demonstrated that the developed model could be effectively used to investigate rock-breaking of drill bit and drill string motion simultaneously [68].

5. Incorporate Advanced Automation and AI in Drilling Control

Incorporating automation and cutting-edge technologies within the drilling operation can amplify the control over drilling parameters. Artificial Intelligence (AI) can scrutinize real-time rig activities, telemetry data, and Bottom Hole Assembly (BHA) particulars as an illustrative instance. It can thereby formulate astute suggestions for fine-tuning drilling processes toward optimization [69].

The figure below assesses technological advancements within the oil drilling industry over the past two decades (1999-2023).

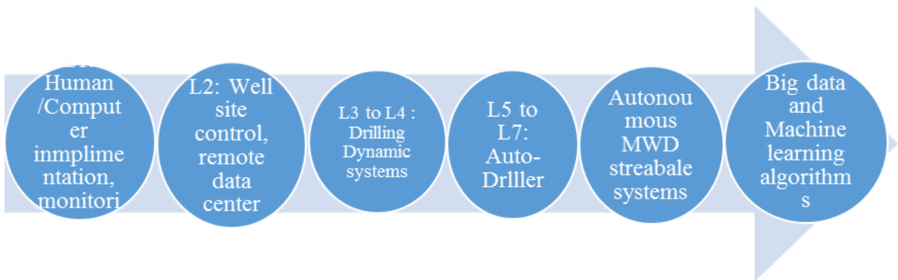

Fig. 2 Automations level in drilling operations

The objective of the investigation conducted by Kasiralvalad, 2014 in [70] is to comprehensively examine the using of nanomaterials in drilling and drilling fluids pertinent to petroleum development and production.

The review contribution conducted by Opeyemi. B et al. summarized the previous investigations that applied artificial intelligence in drilling operations optimization before 2015. They confirmed the importance of integrating these advanced tools in the control, analysis, and enhancing well drilling systems [69].

The investigation conducted in [71] showcases the efficacy of artificial neural networks (ANN) in estimating drill bit temperature and cutting force in drilling operations. The study applied artificial neural networks (ANN) to estimate the drill bit temperature and cutting force in the drilling process using coated carbide and uncoated drills. Also, the effects of the different network structures in the modeling of the drill bit temperature and cutting force were investigated.

In the work presented by Creegan and Jeffrey, an intelligent drilling optimization framework featuring an adaptive auto-driller mechanism is elucidated. This innovative approach integrates advanced artificial intelligence (AI) algorithms to enhance the efficacy of on-bottom drilling operations [72].

The study in [73] explored drilling vibration modes and penetration rate modeling in the Egyptian Western Desert. Utilizing Artificial Neural Networks and multiple linear regression, the Authors developed an advanced model to predict penetration rate and vibration levels.

The study revealed that axial vibrations could be reduced by increasing the weight of the bit and rotary speed. Conversely, lateral and torsional vibrations were enhanced by increasing rotary speed and decreasing weight on the bit.

Ramy Saadeldin et al.'s study developed an Artificial Neural Network (ANN) model to predict drill string vibrations during horizontal drilling. The model utilized surface drilling data, including mud pumping pressure, rotating speed, and top drive torque. Validation was achieved through simulation and experimental data, with a separate dataset confirming the model's accuracy. The optimized model showed a high correlation between predicted and actual vibrations, indicating its effectiveness[74].

The study in [75] introduced the Well Control Space Out technology, an internet-of-things (IoT) solution that utilized deep learning to process real-time video images of the drill string. By automatically detecting and tracking key rig components, the system provides critical data for good control and improves decision-making in time-sensitive situations.

A study by Rouzgard and Shirazi, introduced an algorithmic approach for the autonomous steering of a drill bit during directional drilling activities. In manual control scenarios, challenges may arise due to operational imprecisions and human factors, resulting in deviations from the intended drilling trajectory [76].

Equinor et al. inaugurated the world's inaugural fully automated oil and gas platform, Oseberg Vestflanken H. This platform is a remarkable technological achievement, functioning autonomously without human intervention [77].

M. N. Al-Mudhaf discussed revolutionizing drilling efficiency with Neuro-autonomous solutions applied in the Kuwait oil and gas industry. The last adopted digital technologies, particularly AI and autonomous systems have revolutionized drilling operations. The oil companies have led the way with their implementation of Neuro Autonomous Solutions, including DrillOps Automate, DD Advisor, and Auto-Curve, coupled with Schlumberger's Well Construction Rig and Blue BHA. This integrated system enhances drilling efficiency, reduces costs, and improves safety by optimizing data processing and minimizing human error [78].

6. CONCLUSION

Previous theories and scientific contributions have focused on understanding, describing, and modeling drill string behaviors. The most used methods include the finite element method, which uses multiple techniques to analyze complex structures; the lumped masse technique, which simplifies complex systems but may miss many essential parts, wave equations, and PDE approaches, which provide analytical mathematical tools; and the Cosserat theory, which requires knowledge of microscopic structure details. Mathematical models used to describe drill strings can be classified into two main categories: static models, which calculate deflections, axial forces, and twisting of drill pipes, and dynamic models, which describe BHA behaviors under dynamic phenomena like stick-slip, bit-bounce, and bit-whirl. Incorporating automation and advanced technologies such as automated rigs and AI techniques to improve control of drill string vibrations and undesirable dynamics was important to raise the level of efficiency and optimization in drilling operations in current gas and oil fields.

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