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FINITE-TIME STABILITY ANALYSIS OF DISCRETE TIME-DELAY SYSTEMS USING DISCRETE CONVOLUTION OF DELAYED STATES

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Abstract. *Finite-time stability for the linear discrete-time system with state delay was investigated in this article. Stability of the system was analyzed using both the Lyapunov-like approach and the discrete Jensen's inequality. A novel Lyapunov-like functional with a discrete convolution of delayed states was proposed and used for the derivation of the sufficient stability conditions of the investigated system. As a result, the novel stability conditions guarantee that the states of the systems do not exceed the predefined boundaries on a finite time interval. The proposed methodology was illustrated with a numerical example. A computer simulation was performed for the analysis of the dynamical behavior of this system.*

Key words: *discrete systems, finite-time stability, Jensen's discrete inequality, time delay systems*

1. INTRODUCTION

To investigate the stability of the control system, the Lyapunov method was widely used in the control system community. In some cases, the Lyapunov stability is insufficient to describe the dynamical behavior of some special classes of the system or to give satisfactory conclusions about the different types of stability. This is the case for the practical stability, where the requirements are set on the states of the system. In these situations, there are constraints on the system states trajectories, i.e. they have to stay

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within the predefined values and should not exceed them. Consequently, it is of particular interest to investigate the trajectories of the system only over a finite time interval. The described stability concept, based on the stability investigation in a limited time frame, is named as finite-time stability (FTS). In that sense, the system is stable if the states of the system do not exceed the predefined boundaries on some fixed time interval. This stability concept was introduced in the era of modern control systems [1-3], and it is still widely used nowadays as well. Initially, the concept had an academic value, and its practical applications were applied later on. With the development of the linear inequalities, the stability conditions that could be used for practical purposes were developed for both the continuous [4-8] and discrete-time systems [9-16].

Time delay is often present in electrical, mechanical, chemical, and other systems. The described latency in such systems can potentially bring the systems into instability, or its appearance can result in low performances during the transient process. A significant effort was made to investigate the dynamical behavior and stability of such systems [17-22].

The FTS concept was applicable to both the regular and time-delay systems. However, the number of the reported results of the FTS for time-delay systems is limited. Some FTS conditions of time-delay systems were reported in [23-25]. These results were obtained based on the estimation of state vectors. They were found to be either conservative or inconvenient for practical calculations. Using the linear matrix inequalities (LMI) and the Lyapunov-like functional, less restrictive FTS results for time-delay systems have been reported, [26-30].

In this article, the finite-time stability of discrete time-delay systems was investigated. A discrete Lyapunov-like functional with a discrete convolution of delayed states [31] was used for the stability investigations. The methodology used throughout the article was to combine the Lyapunov-like approach and the Jensen's discrete inequality. The novel sufficient stability conditions were presented in a form of algebraic inequalities.

Notations: The matrix transposition was denoted by a superscript 'T'. $\Re^n$ and $\Re^{n\times m}$ are the n-dimensional Euclidean spaces and the set of all real matrices having dimension n $\times$ m, respectively. $X > 0$ denotes a real positive definite matrix, while $X > Y$ implies that the matrix $X - Y$ is a positive definite matrix. $\lambda_{\max}(X)$ ($\lambda_{\min}(X)$) denotes the maximum (minimum) of eigenvalues of a real symmetric matrix X.

2. Problem Formulation

A linear discrete system with state delay was analyzed. The system was described as:

$$x(k+1) = A_0\, x(k) + A_1\, x(k-h) \tag{1}$$

with a known vector function of the initial conditions:

$$x(j) = \psi(j), \quad j \in \{-h, -h+1, \dots, 0\} \tag{2}$$

where $x(k) \in \Re^n$ is a state vector, $A_0 \in \Re^{n\times n}$ and $A_1 \in \Re^{n\times n}$ are known constant matrices, h is a constant time delay. The initial condition $\psi(k)$ is the a priori known vector function for each $k \in \{-h, h+1,...,0\}$

Definition 1 The linear discrete time-delay system (1), which satisfies the initial condition (2), is said to be finite-time stable with respect to (α, β, N), $\alpha < \beta$ if

$$\sup_{k \in \{-h, -h+1, \cdots, 0\}} \psi^T(k)\psi(k) \le \alpha \quad \Rightarrow \quad x^T(k)x(k) < \beta, \forall k \in \{1, 2, \cdots, N\} \tag{3}$$

Lemma 1 (Jensen's discrete inequality) For any positive symmetric constant matrix $M \in \mathbb{R}^{n\times n}$, scalar n and a vector function $f(k): \{-h, -h+1, \cdots, 0\} \to \mathbb{R}^n$ the following inequality is valid:

$$\left(\sum_{k=1}^{n} f(k)\right)^T M \sum_{j=1}^{n} f(k) \le n \sum_{k=1}^{n} f(k)^T \, M \, f(k) \tag{4}$$

Lemma 2 For any symmetric positive definite matrix $\Gamma = \Gamma^T > 0$, the following expressions hold:

$$2u(k)v(k-h) \le u^T(k)\Gamma u(k) + v^T(k-h)\Gamma^{-1}v(k-h) \tag{5}$$

$$-2u(k)v(k-h) \le u^T(k)\Gamma u(k) + v^T(k-h)\Gamma^{-1}v(k-h) \tag{6}$$

3. Main Result

In this section, a novel discrete Lyapunov-like functional with a discrete convolution of delayed states is defined. The functional was used to find a sufficient delay-dependent FTS condition. The definition for this class of the functional was initially introduced in [31]. The following lemma defines this functional and determines its characteristics.

Lemma 3. Consider the time delay system (1). Let a scalar, aggregation function is defined as:

$$V(x_k) = y^T(x_k)Py(x_k) \tag{7}$$

where vector $y(t)$ is defined by discrete convolution:

$$\begin{aligned} y(x_k) &= x(k) + \sum_{j=1}^{h} Q(j)x(k-j), \\ x_k &= x(k+\vartheta), \quad \vartheta \in \{-h, \; -h+1, \; \dots, 0\} \end{aligned} \tag{8}$$

and $Q(j)$ is the $n \times n$ discrete matrix function:

$$Q(j+1) = A^j(A - A_0), \quad j \in \{0, 1, \cdots, h-1\}\ , \tag{9}$$

where matrix A is a solution of the matrix equation (10):

$$A^{h+1} - A^h A_0 - A_1 = 0 \tag{10}$$

The forward difference $\Delta V(x_k) = V(x_{k+1}) - V(x_k)$ of expression (7) along the trajectory of system (1) is calculated as:

$$\Delta V(x_k) = y^T(x_k)(A^T PA - P)y(x_k) \tag{11}$$

Proof. The forward difference of (7) along the solutions of system (1) is:

$$\Delta V(x_k) = \Delta y^T(x_k) P y(x_k) + y^T(x_k) P \Delta y(x_k) + \Delta y^T(x_k) P \Delta y(x_k) \tag{12}$$

Term $\Delta y(x_k)$ was derived in the following manner:

$$\begin{aligned} \Delta y(x_k) &= \Delta x(k) + \sum_{j=1}^{h} Q(j)\,\Delta x(k-j) \\ &= \underbrace{[A_0 + Q(1) - I]}_{A-I} x(k) + \underbrace{(Q(2) - Q(1))}_{\Delta Q(1)} x(k-1) + \cdots \\ &\quad + \underbrace{(Q(h) - Q(h-1))}_{\Delta Q(h-1)} x(k-h+1) \quad + \underbrace{(A_1 - Q(h))}_{\Delta Q(h)} x(k-h) \\ &= [A_0 + Q(1) - I] x(k) + \sum_{j=1}^{h} \Delta Q(j) x(k-j) \end{aligned} \tag{13}$$

The following matrices are defined as:

$$A = A_0 + Q(1) \tag{14}$$

$$\Delta Q(j) = Q(j+1) - Q(j), \quad j = 1, 2, \cdots, h-1 \tag{15}$$

$$\Delta Q(h) = A_1 - Q(h) \tag{16}$$

Using the expression:

$$\Delta Q(j) = (A - I) Q(j), \quad j = 1, 2, \ldots, h \tag{17}$$

and calculating $\Delta y(x_k)$ and $\Delta V(x_k)$, it can be obtained:

$$\Delta y(x_k) = (A - I) y(x_k) \tag{18}$$

$$\begin{aligned} \Delta V(x_k) &= y^T(x_k)(A-I)^T P y(x_k) + y^T(x_k) P (A-I) y(x_k) \\ &\quad + y^T(x_k)(A-I)^T P (A-I) y(x_k) \\ &= y^T(x_k)(A^T P A - P) y(x_k) \end{aligned} \tag{19}$$

This completes the proof.

Matrix $Q(j)$ can be determined in the following way: from (16) and (17), it can be calculated:

$$\begin{aligned} Q(j+1) &= Q(j) + (A-I)Q(j) = AQ(j) = A^2 Q(j-1) = \cdots = A^j Q(1) \\ &= A^j (A - A_0), \qquad j \in \{0, 1, \cdots, h-1\} \end{aligned} \tag{20}$$

with the final condition

$$Q(h+1) = Q(h) + A_1 - Q(h) = A_1 \tag{21}$$

From (20) and (21), it can be obtained:

$$Q(h+1) = A^h (A - A_0) = A_1 \tag{22}$$

i.e.

$$A^{h+1} - A^h A_0 - A_1 = 0 \tag{23}$$

which had to be demonstrated.

Using the result obtained in Lemma 3, it is possible to present the main results of this study. The following theorem gives the sufficient condition of FTS of system (1).

Theorem 1 Linear discrete time delay system (1) with

$$\begin{gathered} x^T(k-j)x(k-j) < q\, x^T(k)x(k), \quad q>0, \\ j \in \{-h, -h+1, \ldots, 0\}, \ \ \forall k \in \{1, 2, \cdots, N\} \end{gathered} \tag{24}$$

is finite-time stable with respect to $\{\alpha, \beta, T\}, \alpha < \beta$, if there exist two positive scalars, μ and ε, such that:

$$\frac{1+\mu h+\mu^{-1}\delta+h\delta}{1-\varepsilon h-\varepsilon^{-1}q\delta}\gamma^N < \frac{\beta}{\alpha} \tag{25}$$

$$\varepsilon \in (\max\{\varepsilon_1, 0\}, \varepsilon_2), \quad \varepsilon_{1,2} = \frac{1 \pm \sqrt{1-4q\delta h}}{2h}, \tag{26}$$

$$q\delta h < 1/4 \tag{27}$$

$$\delta_i > 0, \quad i = 1, 2, \cdots, h \tag{28}$$

where:

$$\gamma = \lambda_{\max}(A^T A - I) + 1, \tag{29}$$

$$\delta_i = \lambda_{\max}\{(A-A_0)^T (A^{j-1})^T A^{j-1}(A-A_0)\}, \quad \delta = \sum_{j=1}^{h} \delta_i \tag{30}$$

with matrix A as a solution of:

$$A^{h+1} - A^h A_0 - A_1 = 0 \tag{31}$$

Proof. For $P = I$, using (11), it follows:

$$\begin{aligned} \Delta V(x_k) &= y^T(x_k)(A^T A - I)\, y(x_k) \\ &\le \lambda_{\max}(A^T A - I) y^T(x_k)\ y(x_k) \\ &= (\gamma - 1)V(x_k) \end{aligned} \tag{32}$$

After the recalculations it can be obtained:

$$\Delta V(x_k) = V(x_{k+1}) - V(x_k) \le (\gamma - 1)V(x_k) \tag{33}$$

so that:

$$V(x_{k+1}) < \gamma V(x_k) \tag{34}$$

Applying iteratively condition (34), it follows:

$$V(x_k) < \gamma V(x_{k-1}) < \gamma^2 V(x_{k-2}) < \cdots < \gamma^k V(x_0) \tag{35}$$

In order to find the expression for $V(x_0)$, equations (7), (8) and (9) were used:

$$\begin{aligned} V(x_0) &= x^T(0)x(0) + 2x^T(0)\sum_{j=1}^{h} A^{j-1}Q(1)x(-j) \\ &+ \left[\sum_{j=1}^{h} A^{j-1}Q(1)x(-j)\right]^T \left[\sum_{j=1}^{h} A^{j-1}Q(1)x(-j)\right] \end{aligned} \tag{36}$$

Based on inequality (5) (Lemma 2), for $\Gamma = \mu I$, $\mu > 0$ it can be found that:

$$2\sum_{j=1}^{h} x^T(0)A^{j-1}Q(1)x(-j) \le \mu\sum_{j=1}^{h} x^T(0)x(0) + \mu^{-1}\sum_{j=1}^{h} x^T(-j)x(-j)\delta_j \tag{37}$$

Using Definition 1 and inequality (28), it follows:

$$2\sum_{j=1}^{h} x^T(0)A^{j-1}Q(1)x(-j) \le \mu\alpha h + \mu^{-1}\alpha\delta \tag{38}$$

Using a discrete version of the Jensen's inequality (Lemma 1), the following inequality can be obtained:

$$\begin{aligned}\left[\sum_{j=1}^{h} A^{j-1}Q(1)x(-j)\right]^T &\left[\sum_{j=1}^{h} A^{j-1}Q(1)x(-j)\right] \\ &\le h\sum_{j=1}^{h} x^T(-j)Q^T(1)(A^{j-1})^T A^{j-1}Q(1)x(-j) \\ &\le h\sum_{j=1}^{h} x^T(-j)x(-j)\delta_j \le \alpha h\delta\end{aligned} \tag{39}$$

In that case, (36) becomes:

$$\begin{aligned}V(x_0) &\le \alpha + \mu\alpha h + \mu^{-1}\alpha\delta + \alpha h\delta \\ &= \alpha(1 + \mu h + \mu^{-1}\delta + h\delta)\end{aligned} \tag{40}$$

Combining (40) with (34), it can be calculated:

$$V(x_k) < \gamma^k \alpha(1 + \mu h + \mu^{-1}\delta + h\delta) \tag{41}$$

and:

$$x^T(k)x(k) + 2x^T(k)\sum_{j=1}^{h} A^{j-1}Q(1)x(k-j) \le V(x_k) \tag{42}$$

or:

$$x^T(k)x(k) \le V(x_k) - 2x^T(k)\sum_{j=1}^{h} A^{j-1}Q(1)x(k-j) \tag{43}$$

The second term after the inequality symbol of expression (43) is to be found. Using inequality (6), for $\Gamma = \varepsilon I$, $\varepsilon > 0$ it can be derived:

$$\begin{aligned}-2\sum_{j=1}^{h} x^T(k)A^{j-1}Q(1)x(k-j) &\le \varepsilon h x^T(k)x(k) + \varepsilon^{-1}\sum_{j=1}^{h} x^T(k-j)x(k-j)\delta_j \\ &= x^T(k)x(k)\left[\varepsilon h + \varepsilon^{-1}q\sum_{j=1}^{h}\delta_j\right]\end{aligned} \tag{44}$$

Combinig (41), (43) and (44), it can be calculated:

$$x^T(k)x(k)[1 - \varepsilon h - \varepsilon^{-1}q\delta] \le \gamma^k \alpha(1 + \mu h + \mu^{-1}\delta + h\delta), \quad k = 1, 2, \cdots, N \tag{45}$$

If expressions (46) and (47) hold

$$\gamma^k \alpha(1+\mu h+\mu^{-1}\delta+h\delta)<\beta(1-\varepsilon h-\varepsilon^{-1}q\delta), \quad k=1,2,\cdots,N \tag{46}$$

$$1-\varepsilon h-\varepsilon^{-1}q\delta>0 \tag{47}$$

that implies

$$x^T(k)x(k)<\beta, \ \ k=1,2,\cdots,N \tag{48}$$

Inequality (47) is satisfied if conditions (26) and (27) are fulfilled. This completes the proof.

The estimation of parameter q from (47) is complicated in some cases. The following theorem presents a sufficient FTS condition, excluding the necessity for the estimation of q.

Theorem 2 Linear discrete time delay system (1) with a given initial condition (2) is finite-time stable with respect to $\{\alpha, \beta, T\}, \alpha<\beta$ if two positive scalars, μ and ε, exist, such that the following conditions are satisfied:

$$\frac{\alpha(1+\mu h+\mu^{-1}\delta+h\delta)}{1-\varepsilon h-\varepsilon^{-1}\delta}\gamma^N<\frac{\beta}{\alpha}\ , \tag{49}$$

$$\varepsilon\in(\max\{\varepsilon_1, 0\}, \varepsilon_2), \quad \varepsilon_{1,2}=\frac{1\pm\sqrt{1-4\delta h}}{2h}\ , \tag{50}$$

$$\delta h<1/4\ , \tag{51}$$

$$\delta_i>0, \quad i=1,2,\cdots,h\ , \tag{52}$$

where scalars γ, δ_i, δ and matrix A are defined in (29)-(31).

Proof. The proof of this theorem coincides with the proof of Theorem 1 up to expression (43). Expression (44) is reformulated as:

$$-2\sum_{j=1}^{h}x^T(k)A^{j-1}Q(1)x(k-j)\le\varepsilon h x^T(k)x(k)+\varepsilon^{-1}\sum_{j=1}^{h}x^T(k-j)x(k-j)\delta_j \tag{53}$$

Using (41), (43) and (53), it can be calculated

$$x^T(k)x(k)\le\gamma^k\alpha(1+\mu h+\mu^{-1}\delta+h\delta)+\varepsilon h x^T(k)x(k)+\varepsilon^{-1}\sum_{j=1}^{h}x^T(k-j)x(k-j)\delta_j, \\ k=1,2,\cdots,N, \quad \forall x^T(k)x(k)\in[0,\beta] \tag{54}$$

i.e.

$$x^T(k)x(k)\le\gamma^k\alpha(1+\mu h+\mu^{-1}\delta+h\delta)+\varepsilon h\beta+\varepsilon^{-1}\beta\sum_{j=1}^{h}\delta_j, \\ \forall x^T(k)x(k)\in[0,\beta] \tag{55}$$

If the following condition is satisfied

$$\gamma^k \alpha(1+\mu h+\mu^{-1}\delta+h\delta)+\varepsilon h\beta+\varepsilon^{-1}\beta\delta<\beta, \quad k=1,2,\cdots,N \tag{56}$$

then

$$x^T(k)x(k)<\beta, \ \forall k\in\{1,2,\cdots,N\} \tag{57}$$

From (56), it can be obtained:

$$0<\gamma\alpha(1+\mu h+\mu^{-1}\delta+h\delta)<\beta(1-\varepsilon h-\varepsilon^{-1}\delta) \tag{58}$$

and

$$1-\varepsilon h-\varepsilon^{-1}\delta>0 \tag{59}$$

Inequality (59) is satisfied if conditions (50) and (51) are fulfilled. This completes the proof.

Remark 1 Theorem 2 has a more pronounced practical importance since it does not require the calculations of parameter q. This parameter can be estimated from the simulation, as in Figure 2. Using the estimation of parameter q, the conditions from Theorem 1 give less conservative stability results.

Remark 2 The smaller values of parameter q satisfying (24) significantly reduce the conservativeness of criterion (25)-(28). With the analysis of condition (24), it follows that the value of parameter q can be a function of integers with time index *k*. In this case, there exists a sequence *q*(*k*) for the different values of index *k*. Sequence *q*(*k*) cannot be determined analytically.

Remark 3 Further improvement of the stability conditions proposed in Theorem 1 and Theorem 2 can be obtained by introducing a generalized matrix instead of the identity matrix in the Lyapunov-like function.

4. NUMERICAL EXAMPLE

Example 1 The following system was analyzed:

$$x(k+1)=A_0x(k)+A_1x(k-h)$$
$$A_0=\begin{bmatrix}0.6 & 0\\ 0.4 & 0.9\end{bmatrix},\quad A_1=\begin{bmatrix}0.1 & 0.2\\ 0.1 & 0.1\end{bmatrix},\quad h=1 \tag{60}$$

The dynamical behavior of the system is simulated using the conditions $\psi(k)=[1\ 1]^T$, $k = \in \{-1,0\}$. In that case, the following can be calculated:

$$\psi^T(t)\psi(t)=2=\alpha, \quad k\in\{-1,0\}$$

Figures 1 and Figure 2 show initial response $x(k)$ and the norm of state vector $x^T(k)x(k)$ of system (60), respectively. It can be concluded that the system is not asymptotically stable.

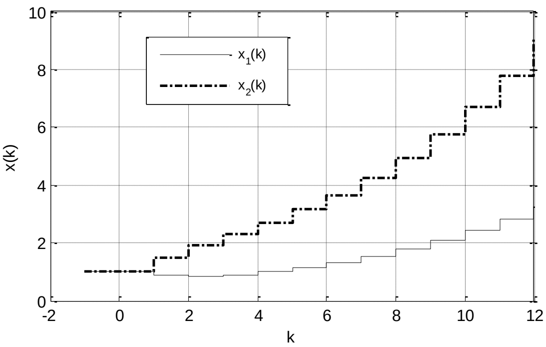

Fig. 1 The state response, $x(k)$, of the analyzed system

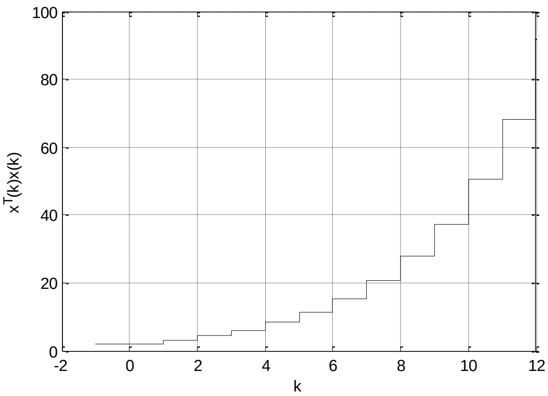

Fig. 2 The norm of the state vector, $x^T(k)x(k)$

Based on the initial response of system (60), for $N = 10$, it can be chosen

$$q = 0.75 > \frac{x^T(k-j)x(k-j)}{x^T(k)x(k)}, \quad j \in \{-1, 0\}, \ \forall k \in \{1, 2, \cdots, N\}$$

so that expression (24) is valid.

From (10) for $h = 1$, it can be found:

$$A = \begin{bmatrix} 0.7171 & 0.2953 \\ 0.4544 & 0.8603 \end{bmatrix},$$

and:

$$\gamma = 1.3790, \quad \delta = 0.1012.$$

Based on Theorem 1 for $\alpha = 2$ and $N = 10$, one can find the minimal values of parameter β, ($\beta_{m2} = 385$) such that system (60) is FTS. The values of other parameters are:

$$\varepsilon_1 = 0.0827,\ \varepsilon_2 = 0.9173,\ \varepsilon = 0.2777,$$
$$\mu = 0.3333,$$
$$\frac{1+\mu h+\mu^{-1}\delta+h\delta}{1-\varepsilon h-\varepsilon^{-1}q\delta}\gamma^N = 119.6497$$
$$\frac{\beta_{m2}}{\alpha} = 192.5000$$

Similarly, using Theorem 2 for $\alpha = 2$ and $N = 10$, the minimum value of parameter β is $\beta_{m3} = 476$ and:

$$\varepsilon_1 = 0.1142,\ \varepsilon_2 = 0.8858,\ \varepsilon = 0.3292,\quad \mu = 0.3333,$$
$$\frac{1+\mu h+\mu^{-1}\delta+h\delta}{1-\varepsilon h-\varepsilon^{-1}q\delta}\gamma^N = 128.8368,\quad \frac{\beta_{m3}}{\alpha} = 238.0000$$

It is observed that Theorem 1 gives a more favorable result than Theorems 2 does, as it uses additional information about parameter q. However, Theorem 2 is less complicated for the practical calculations since it does not require an estimation of parameter q.

It can be noticed in Figure 2 that the actual value of parameter β is estimated to be $\beta_a = 50.5$.

5. Conclusion

This article investigated the sufficient conditions for the FTS of linear discrete time-delay systems. Combining the Lyapunov-like functional, discrete convolution of delayed states and the discrete Jensen's inequality, novel sufficient delay-dependent criteria have been derived. The stability conditions were expressed in the form of algebraic inequalities. Further improvements of the results presented in this study can be obtained by replacing the identity matrix with a generalized matrix in the Lyapunov-like function.

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