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ONE NUMERICAL METHOD FOR STRIKE FORCE DETERMINATION AT THE STEP MOTORS*

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Abstract. *A system which contains two or more multilayer solenoid coils is a kernel of step motors. There is a "strong" permanent magnet inside the solenoid coils. The magnet is being moved by the action of electromagnetic field force, which is caused by the coils. Numerical calculations combined with analytical expressions for electromagnetic field strength that exists in the core of step motors, magnetic flux and force on moving magnet, which transforms translational motion into rotational motion, are given in this paper. By comparing with an experimental result, it will be shown that the proposed technique is accurate and effective in calculating both electromagnetic field distribution and strike force of permanent magnet.*

Key words: *Multilayer solenoid coils, Permanent magnet, Strike force, Electromagnetic field, Maxwell's equations, Bessel's functions*

1. INTRODUCTION

For a long time, most of current numerical methods, including finite element method (FEM), have not been efficient in analyzing the electromagnetic fields of very thin coils [1-4], [7-8], magnets [6] or cable terminations [9-12]. Some numerical methods are applied for those problems solving, but CPU time was long. Because of that one hybrid method is suggested in this paper.

The hereby considered system is made of two multilayer cylindrical, solenoid coils and strong permanent magnet (made of NdFeB), obtained by gluing two little parts. The first coil on the left is main (master) coil. The second coil is following (slave) coil, and its function is to return the magnet to the previous state.

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The coils are supplied with alternating current with different modulus and frequencies. It is possible to change the strength of force on the magnet by varying modulus and frequency of coils current. Due to complicity of calculation, there is not any exact analytical solution of those systems.

As a consequence of coils supplying with alternating current, electromagnetic field is formed inside and outside of them. This field induces electromagnetic field in the magnet. The magnet can be maximally pulled out or pulled in the master coil region under the action of both fields (Fig. 1).

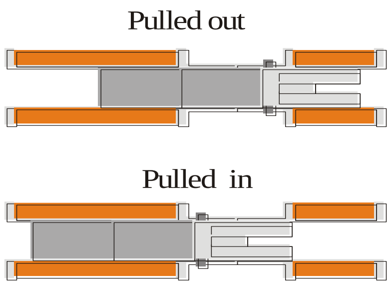

Fig. 1 Magnet positions, inside the coils

The main task is to adjust frequency and maximum value of coils current. The process of adjusting is done in several loops. When strike force on magnet is maximal, the process is stopped. Choosing parameters in this way is long, hard and never gives optimal values for the system parameters. Genetic algorithm is used for eliminating this problem.

2. Theoretical Background

Cylindrical solenoid with several thin layers uniformly placed windings of a circular cross section, having radius a, is considered. The outer solenoid's radius is b. Solenoid length is d_1. The coils current is time harmonic, $i = I_m \cos(\omega t)$, which is treated as uniform along the length of the winding, in every layer.

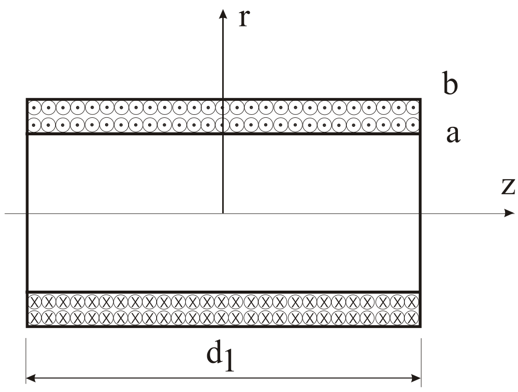

Fig. 2 Multilayer solenoid

Cylindrical coordinate system, r, θ, z, is positioned as in Fig. 2, so solenoid axis coincides with z-axis. Magnetic field vector has radial component, H_r, and axial component, H_z, but electric field vector, $\vec{E}$, has only angular component, E_θ:

$$\vec{H}(r,z) = H_r(r,z)\,\hat{r} + H_z(r,z)\,\hat{z}\,, \tag{1}$$

$$\vec{E}(r,z) = E_\theta(r,z)\,\hat{\theta}\,, \tag{2}$$

whose modulus depends on radial and axial coordinate.

Further on, by solving Maxwell's equations [5, 13], electromagnetic field inside and outside satisfies differential equation:

$$\frac{\partial^2 E_\theta}{\partial r^2} + \frac{1}{r}\frac{\partial E_\theta}{\partial r} + \left(k_0^2 - \frac{1}{r^2}\right)E_\theta + \frac{\partial^2 E_\theta}{\partial z^2} = 0\,, \tag{3}$$

where $k_0 = \sqrt{\varepsilon_0 \mu_0}$.

It is possible to apply separating variable method ($E_\theta(r,z) = R(r)\cdot Z(z)$). Two differential equations are obtained:

$$\frac{\dfrac{d^2R(r)}{dr^2} + \dfrac{1}{r}\dfrac{dR(r)}{dr}}{R(r)} = -m^2\,, \tag{4}$$

$$\frac{d^2Z(r)}{dz^2} = m^2\,. \tag{5}$$

For the first equation, solutions are Bessel's functions of the first kind, $J_1(p\,r)$, and the second kind, $Y_1(p\,r)$, but of the first index.

Hyperbolic functions are solutions for the axial component of electrical field:

$$Z(z) = D_1\cosh(m\,z) + D_2\sinh(m\,z)\,. \tag{6}$$

If $p = \sqrt{k_0^2 + m^2}$, electric and magnetic field components are:

$$E_\theta(r,z) = \begin{cases} A\,J_1(p\,r)\cosh(mz), & 0 < r < a;\ |z| < d_1/2 \\ \left(BJ_1(p\,r) + CY_1(p\,r)\right)\cosh(mz), & r > b;\ |z| < d_1/2 \\ D\,J_1(p\,r)e^{-mz}, & z > d_1/2 \end{cases}\,, \tag{7}$$

$$H_r(r,z) = \frac{m}{j\omega\mu_0}\begin{cases} A\,J_1(p\,r)\sinh(mz), & 0 < r < a;\ |z| < d_1/2 \\ \left(BJ_1(p\,r) + CY_1(p\,r)\right)\sinh(mz), & r > b;\ |z| < d_1/2 \\ -D\,J_1(p\,r)e^{-mz}, & z > d_1/2 \end{cases}\,, \tag{8}$$

$$H_z(r,z) = -\frac{p}{j\omega\mu_0}\begin{cases} A J_0(p r)\cosh(mz), & 0<r<a;\ |z|<d_1/2 \\ \left(BJ_0(p r)+CY_0(p r) \right)\cosh(mz), & r>b;\ |z|<d_1/2 \\ D J_0(p r) e^{-mz}, & z>d_1/2 \end{cases} . \quad (9)$$

Bessel's functions of the first kind, $J_0(p\, r)$, and the second kind, $Y_0(p\, r)$, of the zero index, appeared in previous expressions.

Unknown integration constants can be determined by using boundary conditions:

a) $H_r\left(r=a-0,\ |z|<\frac{d_1}{2} \right)=0$,

b) $H_r\left(r=b+0, |z|<\frac{d_1}{2} \right)=0$,

c) $\int\limits_{z=0}^{\frac{d_1}{2}} H_z(r=a-0,z)\,dz - \int\limits_{z=0}^{\frac{d_1}{2}} H_z(r=b+0,z)\,dz = \frac{NI}{2}$, and

d) Point matching method [14] for well-known electromagnetic field [3-4] at points along $r=0$.

3. STRIKE FORCE CALCULATION

A system of two coils, having circular cross sections, and one less permanent magnet, with the same cross section, placed inside solenoids, is considered (Fig.3).

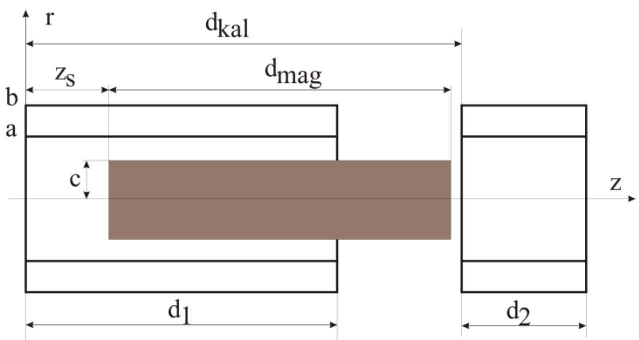

Fig. 3 System of two solenoids (main, length d_1, and slave, length d_2), and moving magnet (length d_{mag})

The first coil on the left is the main or master. Inner radius is a , outer solenoid's radius is b. Solenoid length is d_1. The second coil is "following", placed on distance d_{kal} from the beginning of first coils. Its length is d_2.

Magnet is cylindrically shaped, where z_s is a distance from the system's beginning. Its radius is $c < a$, and its length is d_{mag}. In calculations, it is treated as solenoid, where magnetization, M, is replaced with $NI / d_{mag} = N'i$.

Induced eddy currents cannot be neglected [2], but Ampere's micro currents are neglected because of uniform magnetization ($\dot{J}_m = \nabla \times \dot{M} = 0$).

If x_{0n} are Bessel's function zeros, and:

$$p_n = \frac{x_{0n}}{a}, \tag{10}$$

$$m_n = \sqrt{x_{0n}{}^2 - (k_0 a)^2}\,\frac{1}{a}, \tag{11}$$

$$\sqrt{k_0^2 + m_n^2}\,a = x_{0n}, \tag{12}$$

$$g_{i,j} = \sqrt{x_{0n}^2 - (k_0 a)^2}\,\frac{z - \frac{d_i}{2} - j\,d_{kal}}{a}, \tag{13}$$

electric and magnetic field components, for this case, are:

$$E_\theta(r,z) = \begin{cases} \sum_{n=1}^{\infty} J_1\left(x_{0n}\frac{r}{a}\right)(A'_n \cosh(g_{1,0}) + D''_n e^{g_{2,1}}), & 0 < r < a;\ z < d_1 \\ \sum_{n=1}^{\infty} J_1\left(x_{0n}\frac{r}{a}\right)(D'_n e^{-g_{1,0}} + D''_n e^{g_{2,1}}), & 0 < r < a;\ d_1 < z < d_{kal} \\ \sum_{n=1}^{\infty} J_1\left(x_{0n}\frac{r}{a}\right)(D'_n e^{-g_{1,0}} + A''_n \cosh(g_{2,1})), & 0 < r < a;\ d_{kal} < z < d_{kal} + d_2 \end{cases}, \tag{14}$$

$$H_r(r,z) = \frac{1}{j\omega\mu_0 a}\begin{cases} \sum_{n=1}^{\infty} \sqrt{x_{0n}^2 - (k_0 a)^2}\,J_1\left(x_{0n}\frac{r}{a}\right)(A'_n \sinh(g_{1,0}) + D''_n e^{g_{2,1}}), \\ \sum_{n=1}^{\infty} \sqrt{x_{0n}^2 - (k_0 a)^2}\,J_1\left(x_{0n}\frac{r}{a}\right)(-D'_n e^{-g_{1,0}} + D''_n e^{g_{2,1}}), \\ \sum_{n=1}^{\infty} \sqrt{x_{0n}^2 - (k_0 a)^2}\,J_1\left(x_{0n}\frac{r}{a}\right)(-D'_n e^{-g_{1,0}} + A''_n \sinh(g_{2,1})), \end{cases}, \tag{15}$$

$$H_z(r,z) = -\frac{1}{j\omega\mu_0 a}\begin{cases} \sum_{n=1}^{\infty} x_{0n} J_0\left(x_{0n}\frac{r}{a}\right)(A'_n \cosh(g_{1,0}) + D''_n e^{g_{2,1}}), \\ \sum_{n=1}^{\infty} x_{0n} J_0\left(x_{0n}\frac{r}{a}\right)(D'_n e^{-g_{1,0}} + D''_n e^{g_{2,1}}), \\ \sum_{n=1}^{\infty} x_{0n} J_0\left(x_{0n}\frac{r}{a}\right)(D'_n e^{-g_{1,0}} + A''_n \cosh(g_{2,1})), \end{cases}. \tag{16}$$

Strike force on magnet can be calculated by using the following procedure:

$$dA = F_z\,dz = I_{mag} d\phi,\ F_z = I_{mag}\frac{d\phi}{dz}, \tag{17}$$

$$F_z = I_{mag} \frac{d\phi}{dz}, \tag{18}$$

$$dI_{mag} = M\, dz'. \tag{19}$$

Magnetic flux is:

$$\phi(z) = \int_s \vec{B}\, d\vec{s} = \int_{\theta=0}^{2\pi} \int_{r=0}^{c} \mu_0 H_z(r,z) r\, d\theta\, dr = 2\pi \int_{r=0}^{c} \mu_0 H_z(r,z) r\, dr\,. \tag{20}$$

For time harmonic current, $i = I_m \cos(\omega t)$,

$$\phi(z) = \frac{2\pi}{-j\omega a} \int_{r=0}^{c} \sum_{n=1}^{\infty} x_{0n}\, J_0\left(x_{0n} \frac{r}{a} \right) r\, dr \times f(z,n)\,, \tag{21}$$

or

$$\phi(z) = \frac{2\pi\, c}{-j\omega} \sum_{n=1}^{\infty} \left(J_1\left(x_{0n} \frac{c}{a} \right) f(z,n) \right), \tag{22}$$

$$f(z,n) = \begin{cases} A'_n \cosh(g_{1,0}) + D''_n e^{g_{2,1}}, & 0 < r < a;\;\; z < d_1 \\ D'_n e^{-g_{1,0}} + D''_n e^{g_{2,1}}, & 0 < r < a;\;\; d_1 < z < d_{kal} \\ D'_n e^{-g_{1,0}} + A''_n \cosh(g_{2,1}), & 0 < r < a;\;\; d_{kal} < z < d_{kal} + d_2 \end{cases}\,. \tag{23}$$

Respecting previous expressions, the force on the magnet is calculated as:

$$dF_z = dI_{mag} \frac{d\phi}{dz} = N'_2\, I_{mag}\, dz \frac{d\phi}{dz} = M\, d\phi, \tag{24}$$

$$F_z(z_s) = M \left(\int_{z_s}^{d_1} dF_z + \int_{d_1}^{z_s + d_{mag}} dF_z \right), \tag{25}$$

$$F(z_s) = j \frac{cM}{f} \sum_{n=1}^{\infty} J_1\left(x_{0n} \frac{c}{a} \right) \left(\begin{array}{l} A'_n \left(\cosh\left(\sqrt{x_{0n}^2 - (k_0 a)^2}\, \dfrac{d_1}{2a} \right) - \cosh\left(\sqrt{x_{0n}^2 - (k_0 a)^2}\, \dfrac{z_s - \frac{d_1}{2}}{a} \right) \right) + \\ + D''_n\, e^{\sqrt{x_{0n}^2 - (k_0 a)^2}\, \frac{z_s - d_{kal} - \frac{d_2}{2}}{a}} \end{array} \right). \tag{26}$$

It is possible to obtain more precise results if the helicoidal character of coils winding up is included. The electric field has axial component, except for the angular component, and magnetic field has all three components:

$$\vec{H}(r,z) = H_r(r,z)\hat{r} + H_?(r,z)\hat{?} + H_z(r,z)\hat{z}, \tag{27}$$

$$\vec{E}(r,z) = E_?(r,z)\hat{?} + E_z(r,z)\hat{z}\,. \tag{28}$$

Variable separating method $(H_\theta(r,z) = R(r)\,Z(z))$ is applied again. Now, a system of differential equations is:

$$\frac{\frac{d^2R(r)}{dr^2}+\frac{1}{r}\frac{dR(r)}{dr}}{R(r)}+\left(k_0^2-\frac{1}{r^2}\right)+\frac{d^2Z(z)}{dz^2}=0\,, \tag{29}$$

$$\frac{\frac{d^2R(r)}{dr^2}+\frac{1}{r}\frac{dR(r)}{dr}}{R(r)}+\left(k_0^2-\frac{1}{r^2}\right)=-m^2\,, \tag{30}$$

$$\frac{d^2Z(z)}{dz^2}=m^2\,, \tag{31}$$

$$\frac{d^2R(r)}{dr^2}+\frac{1}{r}\frac{d\,R(r)}{dr}+\left(k_0^2+m^2-\frac{1}{r^2}\right)R(r)=0\,, \tag{32}$$

where are

$$x = p\,r;\ p=\sqrt{k_0^2+m^2}\,. \tag{33}$$

$H_0(r,z)$ is determined as

$$H_\theta(r,z)=\begin{cases} A\,J_1(pr)\cosh(mz), & 0<r<a;\ \ |z|<d_1/2 \\ (BJ_1(pr)+CY_1(pr))\cosh(mz), & r>b;\ \ |z|<d_1/2 \\ D\,J_1(pr)\,e^{-mz}, & z>d_1/2 \end{cases}\,. \tag{34}$$

Axial component of magnetic field can be calculated by using both radial and angular magnetic field components. For other values it is obtained:

$$\frac{\partial H_z}{\partial z}=-\left(\frac{1}{r}H_r+\frac{\partial H_r}{\partial r}\right), \tag{35}$$

$$H_r(r,z)=\frac{1}{j\omega\mu_0}\frac{\partial E_\theta(r,z)}{\partial z}\,, \tag{36}$$

$$\frac{\partial^2 H_r}{\partial r^2}+\frac{1}{r}\frac{\partial H_r}{\partial r}+\left(k_0^2-\frac{1}{r^2}\right)H_r+\frac{\partial^2 H_r}{\partial z^2}=0\,. \tag{37}$$

When magnet is moving, magnet flux is changing, and it causes induction.

For dynamic regime, it is necessary to solve differential equation:

$$m_{mag}\frac{dv(z_s,t)}{dt}=F(z_s)\,, \tag{38}$$

where m_{mag} is weight of magnet and $v(z_s,t)$ is magnet velocity. Induced current is:

$$i_{ind}=-\frac{1}{R_{zice}}\frac{d\Phi}{dt}\,, \tag{39}$$

and R_{zice} is its resistance.

4. NUMERICAL RESULTS

In order to verify the reliability of the numerical model and find out the main factors of influencing the electromagnetic field distribution in this system, a series of experiments are carried out in the laboratory. The maximal values of current for main and slave solenoids are limited on $I_m = 2\,A$ because of very small considered system dimensions. Applied conductor, having diameter of $0.25\,mm$ and coils with ten layers, is analyzed. The number of curls per solenoid's length is 75 (in one layer). A compensating method is applied for strike force measurements and two examples have been considered.

a) Results are shown for system where inner radii of cross section of main and slave solenoids are $a = 5.6\,mm$, outer radii are $b = 10\,mm$. Master solenoid length is $d_1 = 20\,mm$, but slave solenoid length is $d_2 = 10\,mm$. Magnet has circular cross section, whose diameter is $2c = 5\,mm$ and whose length is $d_{mag} = 20\,mm$. Magnet's weight is $m_{mag} = 4\,g$.

Static and dynamic strike force on magnet at the end of its moving, when it's moving is finished, is shown in Table 1.

Table 1 Static and dynamic strike force on magnet at the end of its moving, versus position of slave solenoid

d_{kal}(mm)	F_{st} (mN)	F_{din} (mN)	F_{st} (mN) M	F_{din} (mN) M
34.4–6	43.519179	47.272590	41.43	44.57
34.4–4	80.483703	87.425203	82.01	85.83
34.4–2	168.81788	183.37797	167.79	179.97
34.4	238.68280	259.26854	238.03	258.50
34.4+2	269.63820	292.89376	270.56	293.38
34.4+4	284.00473	308.49936	289.95	311.07
34.4+6	291.27918	316.40122	300.43	316.88
34.4+8	295.27348	320.74001	302.59	318.00
34.4+10	297.62130	323.29032	300.04	322.15
With out slave coil	302.56582	328.66130	302.71	329.04

The measured values are represented by last two columns (M). The intensity of force should be maximal in this moment, because then translational motion transforms in angular. Position of the second coil influences force. It is possible to achieve a maximal strength of force without the second solenoid, but magnet has to be returned to the start position.

A force on the permanent magnet strongly depends on its Position. This is shown in Table 2.

Table 2 Static and dynamic strike force on magnet at the end of its moving, versus position of magnet

z_s (mm)	F_{st} (mN)	F_{din} (mN)	F_{st} (mN) M	F_{din} (mN) M
2.0	154.61411	0.0000000	155.06	0.0000000
2.5	180.60391	12.963427	182.92	10.42
3.0	201.61775	25.926854	202.13	21.15
3.5	218.59392	38.890282	217.09	35.38
4.0	232.29741	51.853709	231.52	49.65
4.5	243.33689	64.817136	241.42	63.19
5.0	252.18944	77.780563	250.66	77.17
5.5	259.22450	90.743990	256.49	90.05
6.0	264.72429	103.70742	261.93	105.00
6.5	268.90007	116.67084	268.18	118.28
7.0	271.90442	129.63427	272.85	130.35
7.5	273.84016	142.59770	274.59	143.92
8.0	274.76617	155.56113	276.32	158.55
8.5	274.70066	168.52455	274.98	170.05
9.0	273.62190	181.48798	274.45	185.17
9.5	271.46657	194.45141	272.56	200.45
10.0	268.12564	207.41484	269.11	211.94
10.5	263.43770	220.37826	264.15	222.25
11.0	257.17966	233.34169	260.15	234.55
11.5	249.05530	246.30512	253.30	246.20
12.0	238.68280	259.26854	241.45	258.35

Resultant current distribution in solenoid's coil, static force distribution and dynamic force distribution are shown in Fig. 4, Fig. 5 and Fig. 6.

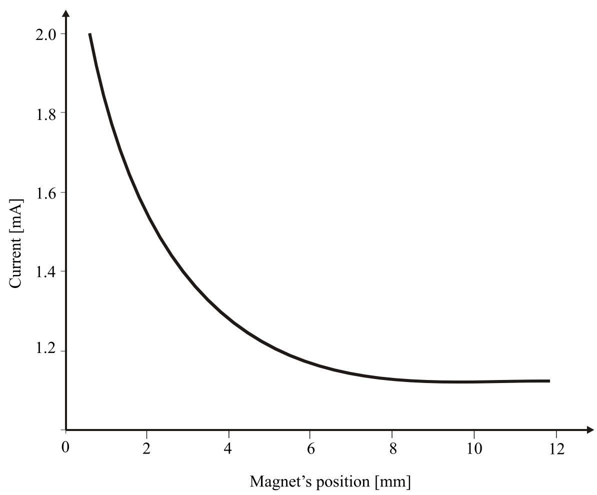

Fig. 4 Current distribution (in mA) versus position of magnet (z_s(m))

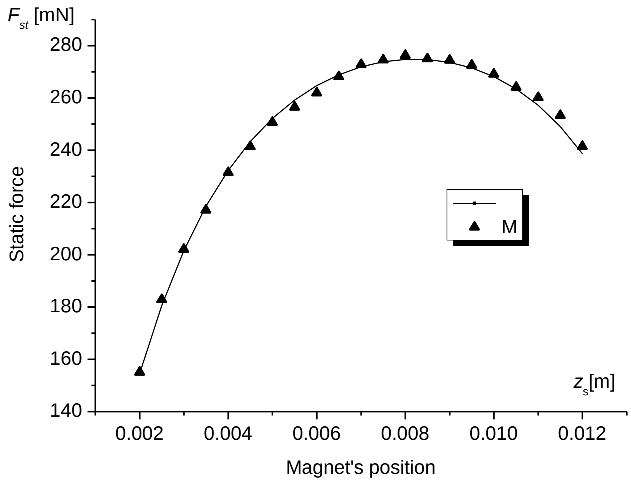

Fig. 5 Static force distribution on the magnet versus position of magnet (z_s(m))

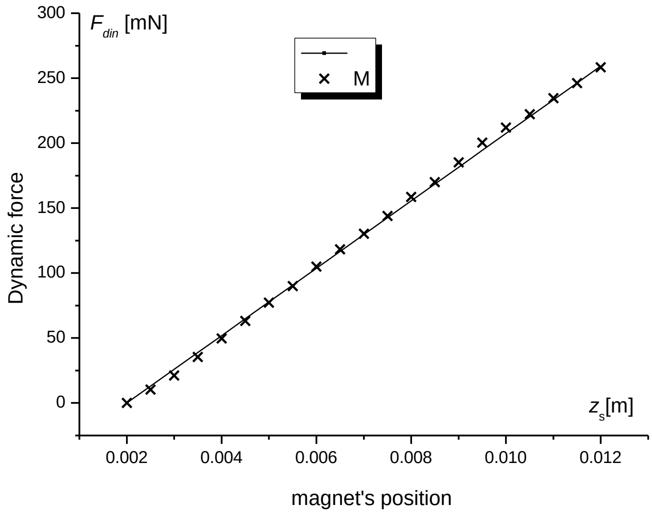

Fig. 6 Dynamic force distribution on the magnet versus position of magnet (z_s(m))

b) The next results are presented for the system where $a = 2.2$ *mm*, $b = 4$ *mm*, $d_1 = 7$ *mm*, $d_1 = 5$ *mm*, $c = 2$ *mm* and $d_{mag} = 10$ *mm*. A distance between master and ‘following' solenoid is 4 *mm*. Magnet's weight is $m_{mag} = 2$ *g*.

Resultant current values for a little system (smaller than previous), versus permanent magnet's position is given in Fig. 7.

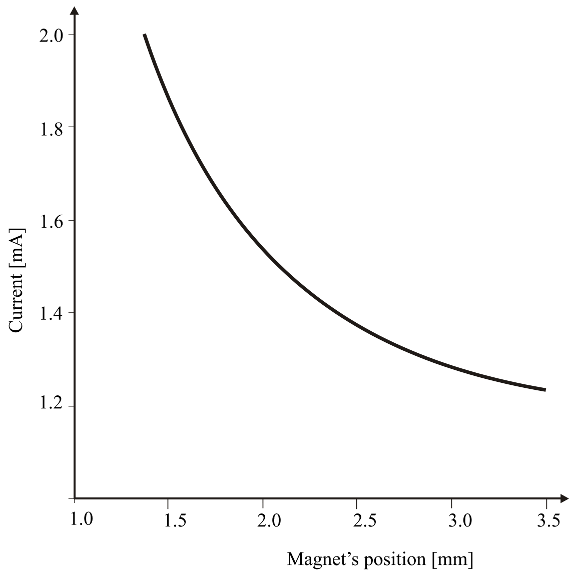

Fig. 7 Current distribution (in mA) versus position of magnet (z_s(m))

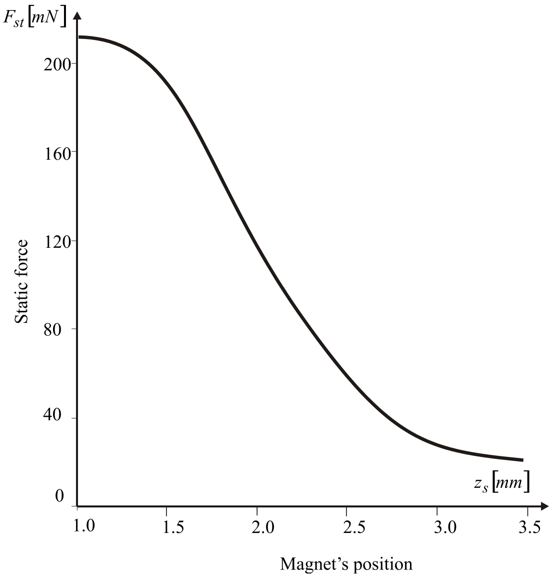

Fig. 8 Static force distribution on the magnet versus position of magnet (z_s(m))

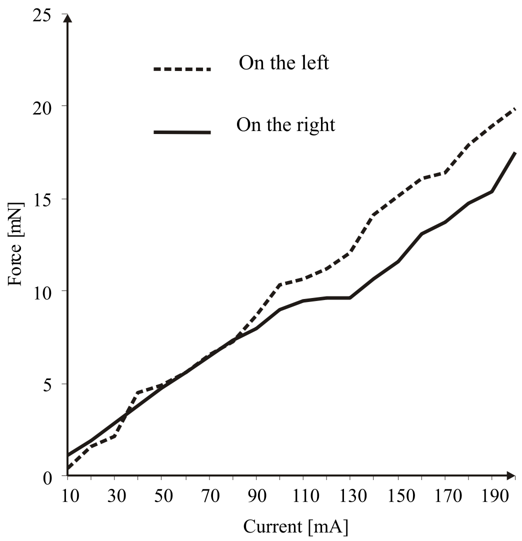

Fig. 9 Dynamic force distribution on the magnet versus current

Strike forces are different (about ten per cent) and unbalance cannot be neglected, similarly as at a moving-coil type permanent magnet linear synchronous motor [3].

5. CONCLUSION

It can be noticed that a very complex mathematical calculation is needed for this problem solution. This is probably one of the most important reasons why there is no exact determination of this problem in the literature known to the authors. In the future authors' work, similar calculations will be done for ellipsoidal form permanent magnets [6] and for bianisotropic coils [8].

Determination of Maxwell's equations in cylindrical coordinates, many differential equations and separating variable method are applied in this paper. Different regions connections and unknown integration constants can be determined using boundary conditions for electric and magnetic field component, as well as point matching method. The proposed hybrid method was validated by experimental data, obtained in our laboratory. Results are given for static and dynamic conditions of working. Air resistance and fiction, during magnet "working", are not so important, because magnet is "flying" inside solenoid coils.

Based on the results, presented in tables and figures, it is possible to conclude that strike force depends on the position of the second solenoid, magnet position and current modulus. Maximal force intensity is for the case when slave coil does not exist, but it has to return the magnet at the start position. The best results are given when magnet has a long path for moving, but system dimensions of micro step motors limit this.

Further on, during magnet moving, the added field is induced and it reduces the previous solenoid current.

Through the comparison between calculation results and measured ones, the reliability of the calculation is validated.

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