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PROBABILITY OF STABILITY ESTIMATION OF DPCM SYSTEM WITH THE FIRST ORDER PREDICTOR *

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Abstract. *The probability of stability of DPCM system with the first order predictor is considered in this paper. The basis of the analysis is predictor stability (probability of stability) which is the most important and sufficient condition for stability of the whole system. Influence of predictor stability on system performances (such as signal-to-noise ratio) is considered for concrete signal.*

Key words: *Differential pulse code modulation, Predictor coefficient, Correlation coefficient, Stability condition, Probability of stability*

1. INTRODUCTION

One of the most effective techniques for signal compression is the prediction, where the prediction of the current sample is formed based on the previous samples, and then the difference between the actual sample and its prediction is quantized and transmitted. The prediction is based on the fact that samples of the most real signals are correlated. Using prediction, decorrelation of the signal is performed, i.e. the redundancy of the signal is removed. The quality of the prediction depends on the degree of correlation between samples (higher rate of correlation provides better prediction). DPCM (differential pulse code modulation) technique is based on the linear prediction, where the prediction of the actual sample is calculated as the linear combination of previous samples [1]-[4].

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DPCM system is a nonlinear feedback system. Due to the negative closed loop, although a telecommunication system, DPCM is suitable for analysis in the sense of control systems [5]. One of the most important system properties is stability. Stability of the predictor, linear part of DPCM system, is sufficient (but not necessary) for stability of the whole system.

In this paper DPCM system with the first order is considered. In the easy way we showed that the linear part of the system (predictor) is always in the stability region and it causes stability of the whole system. But this conclusion is valid in the ideal case, i.e. when system parameters are identical to the desired (projected) values. In practice, every realized system is imperfect with some stochastic parameters (with certain distribution) [6,7]. In this case, the stability of the system can only be estimated, i.e. we only know probability that the system is stable. That is the reason why we introduced the term: probability of stability [8].

The paper is organized as follows. First of all, DPCM system is briefly described with a special focus on the predictor. Stability condition for predictor is given and probability of stability of the linear part of the real DPCM system is considered. Analysis is done for different variances and for two bit rate of quantizer.

2. Theoretical Background of DPCM System

DPCM is a technique of converting an analog into a digital signal in which an analog signal is sampled and then the difference between the current sample value and its predicted value is quantized. Predicted value of the current sample is based on previous sample or samples. Basic concept of DPCM - coding a difference, is based on the fact that most source signals show significant correlation between successive samples so that quantizer uses redundancy in sample values which implies lower bit rate [9-11].

The block diagram of the DPCM encoder is shown in Fig. 1a), which consists of the quantizer, *Q*, inverse quantizer and predictor, *P*. Let R denote linear recursive filter in the feedback loop. Firstly, we will consider the DPCM encoder. The main idea of the DPCM is to form the difference d_n between the current sample x_n and its predicted value $\hat{x}_n$:

$$d_n = x_n - \hat{x}_n \,. \tag{1}$$

This difference is quantized and transmitted. In Fig. 1, e_n is the quantization error which occurrs by quantization of the difference d_n. The predicted value $\hat{x}_n$ is calculated as a linear combination of the previously quantized samples y_n. Linear predictor of N th order is described with the following equation:

$$\hat{x}_n = \sum_{i=1}^{N} a_i y_{n-i} \,, \tag{2}$$

where a_i are predictor coefficients.

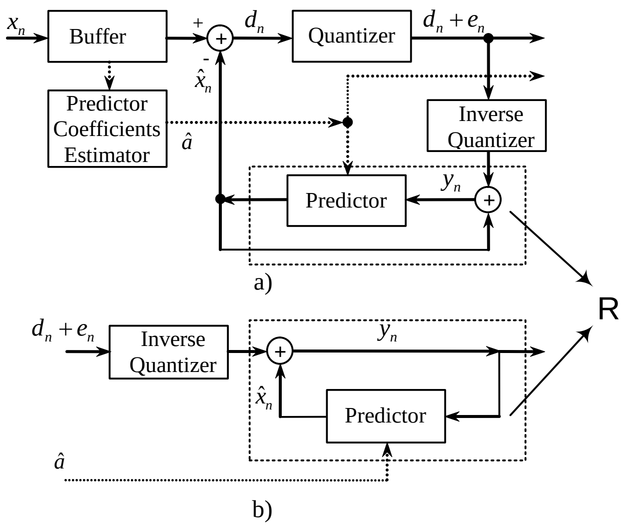

Fig. 1 DPCM system, a) Encoder, b) Decoder

In this paper, we will consider the first order predictor, where the predicted value $\hat{x}_n$ is calculated based on the quantized value y_{n-1} of the previous sample x_{n-1}, i.e.:

$$\hat{x}_n = a_1 y_{n-1} \,. \tag{3}$$

For the first order predictor the coefficient of the predictor is equal to the correlation coefficient ρ, which represents the degree of the correlation between the two consecutive samples:

$$\rho = \frac{\sum_{j=1}^{S-1} x_j x_{j+1}}{\sum_{j=1}^{S} x_j^2} \,, \tag{4}$$

where S is the total number of signal samples. In DPCM system, value of ρ is defined in advance, according to the class of signals which are considered and it is known both in the encoder and in the decoder.

DPCM system is a nonlinear feedback system. Beside predictor which is linear element, the system concludes quantizer as nonlinear element.

Linear part of DPCM system is dynamical and nonlinear part is static (doesn't depend on time).

Dynamical part of the system can be either stable or unstable. If it is stable, the whole system is also stable. Namely, quantizer is described by nonlinear static characteristic with saturation. That is the reason why the stability of the linear part is sufficient for stability of the whole system (nonlinear part stabilizes the system).

Dynamical study of the DPCM system given above shows that stability consideration of recursive filter R is very important during the design of the system. Basic requirement is that predictor coefficients are located inside stability region in parametric space or very close to this region.

3. The Probability Estimation of the Linear Recursive Filter

It is known that if recursive filter is stable, the whole DPCM is stable. But it should be noted that DPCM system can be stable even if recursive filter is unstable. However, in this case, unwanted oscillations may appear in the system. In order to avoid these oscillations it is necessary to make a detailed stability analysis of recursive filter for designing DPCM system. This means that it is need to determine predictor parameters values for which recursive filter is stable or very close to stability domain.

We will consider stability of recursive filter R with the first order predictor. Relation (3) in z-domain has the following form:

$$\hat{X}(z) = a_i z^{-1} Y(z) \; . \tag{5}$$

Transfer function of the first order predictor is:

$$W_P(z) = a_i z^{-1} . \tag{6}$$

Transfer function of the recursive filter R is:

$$W_R(z) = \frac{W_P(z)}{1 - W_P(z)} = \frac{1}{1 - a_i z^{-1}} \; . \tag{7}$$

Recursive filter is stable if all poles of transfer function (7) lie inside unit circle. This means that stability condition for the first order predictor is:

$$-1 < a_1 < 1 . \tag{8}$$

As we said, for the first order, correlation coefficient and predictor coefficient are equal: $a_1 = \rho_1$. Maximal absolute value of correlation coefficient is 1 which means that the linear filter is always in the stability region D_1 described by (8). This is important because we can easily notice how system performances change when the system is in stability region, instability region or as the most important for analysis, close to stability boundaries.

One of the most important properties in telecommunications systems is signal-to-noise ratio (SQNR). This measure compared the level of desired signal to the level of undesirable noise. Functional dependence of SQNR on correlation coefficient ρ_1 is shown in Fig. 2. Obtained results are for speech signal of 12000 samples, sampling frequency of 8 KHz (frame length $M \leq 200$) [11], and done for two values of bit rate of quantizer R = 2*bit/sample* and R = 3*bit/sample*.

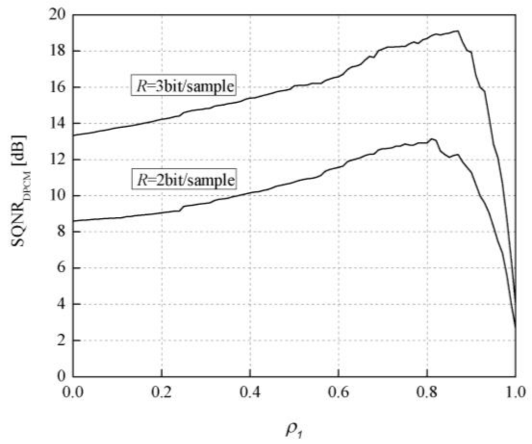

Fig. 2 Dependence of SQNR on ρ_1 in DPCM system

The maximum values of SQNR are obtained for the values of $\rho_1 = 0.81$ and $\rho_1 = 0.87$, for smaller and greater R, respectively. Optimal values for correlation coefficient ρ_1 always correspond to values for predictor coefficient which lies inside stability region of predictor $-1 < \{0.81, 0.87\} < 1$. We can see that SQNR decreases sharply when values for correlation coefficient (predictor coefficient) approach the critical value 1.

Because the predictor coefficient a_1 is always positive for the first order predictor, and its maximal value is 1, classic stability analysis is finished here because the recursive filter is always stable as well as the whole DPCM system. That is valid if we suppose that parameter a_1 is deterministic, i.e. if we could realize it with absolutely accuracy. Such system is perfect.

However, all real systems are imperfect. In this case, predictor coefficient a_1 cannot be perfectly adjusted to the value of correlation coefficient ρ_1, in practice.

Deviation between realized value and projected (desired) value of a_1 appears. These deviations have normal distribution [8]:

$$f\ (a_1) = \frac{1}{\sigma_1\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{a_1 - \overline{a}_1}{\sigma_1}\right)^2\right], \tag{9}$$

where $f\,(a_1)$ is probability density function of parameter a_1, $\overline{a}_1$ is the mean value of a_1, σ_1 is a standard deviation of parameter a_1. In this case, recursive filter can be either stable or unstable.

Note 1: Probability analysis given above is only valid for the specific case ($\sigma = 0$) and then probability of stability is 1 (as we said, R is always stable).

In general case, probability of some system stability is defined as ratio between set of stable cases and set of all possible cases [8]:

$$P_{\alpha,\beta} = \frac{\int_{\alpha}^{\beta} f(x)dx}{\int_{-\infty}^{\infty} f(x)dx}, \tag{10}$$

where α and β are stability limits.

Probability of stability in according to (8)- (10) is:

$$P_{D_1} = \frac{\int_{-1}^{1} f(a_1)da_1}{\int_{-\infty}^{\infty} f(a_1)da_1} = \frac{\frac{1}{\sigma_1\sqrt{2\pi}}\int_{-1}^{1}\exp\left[-\frac{1}{2}\left(\frac{a_1-\bar{a}_1}{\sigma_1}\right)^2\right]da_1}{\frac{1}{\sigma_1\sqrt{2\pi}}\int_{-\infty}^{\infty}\exp\left[-\frac{1}{2}\left(\frac{a_1-\bar{a}_1}{\sigma_1}\right)^2\right]da_1} \tag{11}$$

i.e.:

$$P_{D_1} = \frac{1}{\sigma_1\sqrt{2\pi}}\int_{-1}^{1}\exp\left[-\frac{1}{2}\left(\frac{a_1-\bar{a}_1}{\sigma_1}\right)^2\right]da_1 . \tag{12}$$

If we introduce substitution: $\frac{a_1-\bar{a}_1}{\sigma_1} = t$, probability of stability for linear filter R has the following form:

$$P_{D_1} = \int_{-\frac{1+\bar{a}_1}{\sigma}}^{\frac{1-\bar{a}_1}{\sigma}} \exp\left[-\frac{t^2}{2}\right]dt = \Phi\left(\frac{1-\bar{a}_1}{\sigma}\right) - \Phi\left(-\frac{1+\bar{a}_1}{\sigma}\right) = \Phi\left(\frac{1-\bar{a}_1}{\sigma}\right) + \Phi\left(\frac{1+\bar{a}_1}{\sigma}\right) - 1, \tag{13}$$

where $\Phi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x}\exp\left(\frac{-t^2}{2}\right)dt$ is Laplace function [12].

Probability of stability for different two optimal values of predictor coefficient and for three different variances is given in Table 1. Bolded values are optimal for two bit rates of quantizer in Fig. 2.

Table 1 Probabilities of stability of the recursive filter for different values of σ

	σ		
ā₁	0.2	0.4	0.6
0.30	0.9997	0.9593	0.8632
0,60	0.9772	0.8413	0.7426
0.81	**0.8289**	**0.6826**	**0.6228**
0.87	**0.7422**	**0.6274**	**0.5857**
0.90	0.6915	0.5987	0.5661

We can see that probabilities of stability of recursive filter are bigger in the case of precisely adjustment of predictor coefficient a_1 (σ_1 = 0.2). Also, we estimated probability in wide range of values of predictor coefficient. In the case of low correlated speech signal (e. g. $\bar{a}_1 = 0.3$, $\sigma_1 = 0.2$) probability of stability is high (99.97%). But for high correlated signal (e. g. $\bar{a}_1 = 0.9$, $\sigma_1 = 0.2$) linear part of DPCM system can enter into instability region (30.85%). It could not happen in the deterministic study considered at the beginning (for perfect adjustment of a_1), but it is possible in imperfect systems.

As we said, probability of stability of recursive filter (predictor) is base of probability of stability of the whole system. As quantizer is described with nonlinear characteristic (with saturation), one possible method for probability of stability estimation of the whole DPCM system is mapping stability region of recursive filter multiplying with $1/(1-q)$ where q is slope of linearized characteristic of quantizer around origin. Then stability region of the system is $[-(1-q)^{-1}, (1-q)^{-1}]$. These values are now limits in integral (12) in the case of probability estimation of the whole system.

4. Conclusion

The probability of stability of DPCM system with the first order predictor is considered in this paper. It showed that stability of predictor (linear filter R) is sufficient for the stability of the whole system and values of predictor coefficient should be in the desired boundary if we want better system performances. Real value of predictor coefficient is not always in the stability region unlike the deterministic case ($\sigma = 0$). Probability of stability estimation is done for concrete signal with specific performances.

This paper is only part of detailed stability study of DPCM system which will also include commonly used the second order predictor and predictors of higher order where stability regions are not simple and stability bounds cannot be easy determined as here. In the most general case, probability of stability of DPCM systems with high order predictor can be estimated.

References

[1] C. Fira, L. Goras, "An ECG signals compression method and its validation using NNs," IEEE Transactions on Biomedical Engineering, vol. 55, no. 4, pp. 1319–1326, 2008. [Online]. Available: http://dx.doi.org/10.1109/TBME.2008.918465

[2] H. Bahar, Y. Khiabani, "Optimal design of DPCM scheme for ECG signal handling," in *Proceedings of the 6th WSEAS International Conference on Signal, Speech and Image Processing*, Lisbon, Portugal, pp. 156–161, 2006.

[3] V. Kumar, S. Saxena, V. Giri, D. Singh, "Improved modified AZTEC technique for ECG data compression: Effect of length of parabolic filter on reconstructed signal," Computers and Electrical Engineering, vol. 31, no. 4, pp. 334–344, 2005. [Online]. Available: http://dx.doi.org/10.1016/j.compeleceng.2005.02.002

[4] N. Jayant, P. Noll, *Digital Coding of Waveforms, Principles and Applications to Speech and Video*, Prentice Hall, 1984.

[5] O. Macchi, C. Uhl, "The stability of DPCM transmission system," IEEE Transaction on Circuits and Systems II: Analog and Digital Signal Processing, vol. 39, no. 10, pp. 705–722, 1992. [Online]. Available: http://dx.doi.org/10.1109/82.199897

[6] D. Antić, Z. Jovanović, N. Danković, M. Spasić, S. Stankov, "Probability estimation of certain properties of the imperfect systems," in *Proceedings of the 7th International Symposium on Applied Computational Intelligence and Informatics, SACI 2012*, Timisoara, Romania, 2012, pp. 213–216, [Online]. Available: http://dx.doi.org/10.1109/SACI.2012.6250004

[7] D. Antić, Z. Jovanović, N. Danković, M. Spasić, S. Stankov, "Probability estimation of defined properties of the real technical systems with stochastic parameters," Transactions on Automatic Control and Computer Science, vol. 57 (71), no. 2, pp. 67–74, 2012.

[8] Z. Jovanović, N. Danković, M. Spasić, S. Stankov, Z. Icić, "Practical testing of the probability of stability of the discrete system with random parameters," in *Proceedings of 10th International Conference on Applied Electromagnetics, ПЕС 2011*, Niš, Serbia, pp. P2-4, 2011.
[9] V. M. Despotović, Z. H. Perić, L. Velimirović, V. D. Delić, "DPCM with forward gain-adaptive quantizer and switched predictor for high quality speech signals", Advances in Electrical and Computer Engineering, vol. 10, no. 4, pp. 95–98, 2010, [Online]. Available: http://dx.doi.org/10.4316/aece 2010 04015
[10] Z. Perić, A. Jocić, J. Nikolić, L. Velimirović, D. Denić, "Analysis of differential pulse code modulation with forward adaptive Lloyd-Max's quantizer for low bit-rate speech coding," Revue roumanie des sciences techniques, vol. 58, no. 4, pp. 424–434, 2013, [Online]. Available: http://www.revue.elth.pub.ro/ index.php?action=main&year=2013&issue=4
[11] Z. H. Perić, A. Jocić, M. R. Dinčić, D. B. Denić, "Compression of the highly correlated measurement signals using DPCM technique," Electronics and Electrical Engineering, http://dx.doi.org/10.5755/ j01.eee, to be published.
[12] Ž. Pauše, *Probability: Information, Stochastic Processes*, School book, Zagreb, 1974.