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STRESSING ISSUE OF A PIEZOCERAMIC CANTILEVER WITH LONGITUDINAL POLARISATION AND ELECTRODE COATINGS

UDC (621.315:MATLAB)

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Abstract. *The main subject of this study is the investigation of the free vibration of a rectangular prismatic piezoceramic cantilever with longitudinal polarization and electrode coatings. Based on the general solution of coupled equations for piezoceramic material, applying the equations of electro-elasticity and satisfying electrical and mechanical conditions for the stress of a cantilever made from PZT4 piezoceramic material, componential displacements, electric potential, specific strain, electric field, and piezoelectric displacement, are determined and numerically obtained with Matlab software package. Based on the obtained equations and simulation results, it is possible to optimize the dimensions of the cantilever and determine the type of piezoceramic.*

Key words: *longitudinal polarization, piezoceramic cantilever, PZT4 piezoceramic material, Matlab*

1. INTRODUCTION

Piezoceramic beam-like elements are the essential excitation elements in various ultrasonic devices (actuators and sensors). A beam with a fixed support at one end and without support at the other end is called a cantilever (or clamped-free) beam [1]. The ability to detect vibrations using the piezoelectric material has become a viable energy harvesting source [2]. Piezoelectric energy harvesting using cantilever-type structures has been extensively investigated due to its potential application in supplying power for wireless sensor networks [3]. The most commonly used piezoelectric materials are the following: Quartz, Polycrystalline ceramic, Lead Zirconate Titanate (PZT) [4].

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Piezoelectric devices are complex multi-physics systems requiring advanced methodologies to maximise their performance [5]. Problems related to the design of piezoelectric devices are power limitations due to the physical properties of the material from which the device is made, accurate determination of the resonant frequency, derivation of equations that describe the relationships between electromechanical quantities, etc. [6]. Vibrations modelling of a cantilever is studied theoretically in this paper so that the power conversion capability could be enhanced.

Knowing the natural resonant frequencies of piezoceramic elements is the initial condition when designing different devices [7]. A three-dimensional model is ideal for accurate analysis of these devices [8]. In the most general case, a complete three-dimensional analysis leads to a very complex system of nonlinear equations that are difficult to solve. For this reason, an effort has been made here to solve this problem by presenting an analysis of a two-dimensional (2D) model of thin cantilevers in a Cartesian coordinate system.

In this paper, the general stress case of a rectangular prismatic piezoceramic cantilever with longitudinal polarization will be observed. The observed cantilever is loaded at the free end with a concentrated force. Besides, the cantilever also contains electrode coatings on the sides of $z=\pm h/2$. It is assumed that an electric potential of $2U_0$ is brought to the electrodes. Also, the assumption is that the influence of electromechanical characteristics of electrode coatings can be neglected. First, solutions for component displacements and electric potential in the polynomial form are assumed [9]. The mutually independent coefficients are determined from the given boundary conditions on the surfaces of a rectangular piezoceramic body. Then, the exact solutions for the component displacements and electric potential are determined. Afterwards, using Cauchy's kinematic equations, one may obtain solutions for component mechanical deformations: dilatation and slide.

2. FUNDAMENTAL EQUATIONS

In the observed case, if a constant concentrated force $\vec{F}$ is applied, as shown in Fig. 1, there is only the occurrence of normal mechanical stress σ_x in the axial direction (along the x-axis) and tangential mechanical stress in the cross-sectional plane τ_{xz} aimed in the direction of the force $\vec{F}$.

The vector of the mechanical displacement components is represented by the projections of the displacement vector (through the Cartesian components of the displacement vectors) u, v and w: $\vec{s}=u(x, y, z)\vec{i} + v(x, y, z)\vec{j} + w(x, y, z)\vec{k}$. Meanwhile, volumetric forces are neglected [10]. Also, the assumption was introduced that the mechanical displacement components u and w are independent of the y coordinate, i.e.:

$$u = u(x, z), \quad w = w(x, z), \tag{1}$$

while the component of the piezoelectric displacement vector in the y-axis direction is equal to zero:

$$D_y = 0. \tag{2}$$

The boundary conditions on the sides where the electrode coatings are ($z=\pm h/2$) can be expressed as follows [11]:

$$\sigma_z\Big|_{z=\pm h/2}=0,\ \tau_{xz}\Big|_{z=\pm h/2}=0,\ \ \psi\Big|_{z=\pm h/2}=\pm U_0, \tag{3}$$

where ψ is an electric potential.

If the cross-sectional dimensions of the rectangular cantilever are small compared to the length l, the boundary conditions (3) are fulfilled using expressions from the reference [11], where the mechanical stress components for the observed case are:

$$\sigma_x=-\frac{F}{I_y}xz,\ \ \tau_{xz}=-\frac{F}{16I_y}(h^2-4z^2), \tag{4}$$

where $I_y=bh^3/12$ is the axial moment of inertia for surfaces parallel to the front x=0.

The following integral conditions are applied on the surface x=0:

$$\sigma_x\Big|_{x=0}=0,\ \ -2b\int_{-h/2}^{h/2}\tau_{xz}\Big|_{x=0}dz=F\,. \tag{5}$$

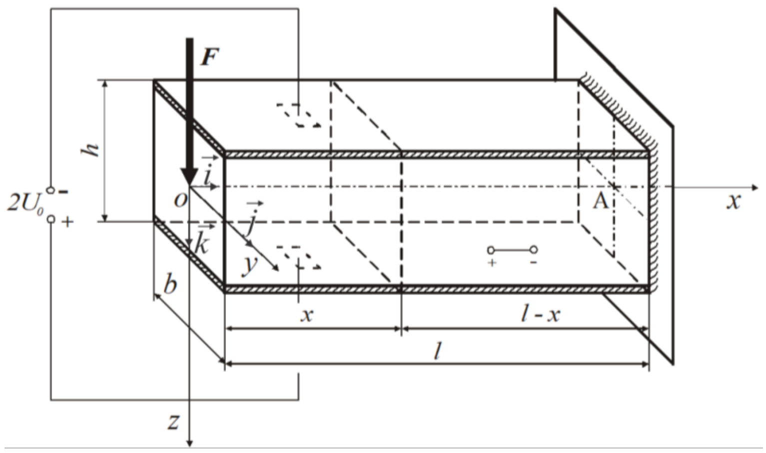

Fig. 1 Stressing of piezoceramic cantilever with longitudinal polarisation and electrodes

The conditions for rigid clamping of the cantilever for the front surface $x=l$ are:

$$w(l,\ 0)=0,\ \ \frac{\partial w}{\partial z}\Big|_{\substack{x=l\\z=0}}=0. \tag{6}$$

If the displacement vector $\vec{s}$ components u and w, as well as the electric potential ψ , are represented in the polynomial form [12], the following expressions are obtained:

$$\begin{gathered} u=a_2z+a_4x^2z,\\ w=b_0+b_2x+b_3x^3,\\ \psi=c_1z+c_4z^3. \end{gathered} \tag{7}$$

If the equations of electrostatics:

$$div\vec{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = 0,\ \ \vec{E} = -grad\psi = -\left(\frac{\partial \psi}{\partial x}\vec{i} + \frac{\partial \psi}{\partial y}\vec{j} + \frac{\partial \psi}{\partial z}\vec{k}\right), \tag{8}$$

as well as Cauchy's kinematic equations:

$$\begin{gathered} \varepsilon_x = \frac{\partial u}{\partial x},\ \ \varepsilon_y = \frac{\partial v}{\partial y}, \\ \varepsilon_z = \frac{\partial w}{\partial z},\ \ \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \\ \gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x},\ \ \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}, \end{gathered} \tag{9}$$

are applied in expression (7), the following system of equations is obtained:

$$\begin{gathered} E_x = 0,\ \ E_y = 0, \\ E_z = -c_1 - 3c_4 z^2, \\ \varepsilon_x = 2a_4 xz, \\ \gamma_{xz} = a_2 + b_2 + (a_4 + 3b_3)x^2, \\ \varepsilon_y = 0,\ \ \varepsilon_z = 0,\ \ \gamma_{xy} = 0,\ \ \gamma_{yz} = 0, \end{gathered} \tag{10}$$

where a_2, a_4, b_0, b_2, b_3, c_1 and c_4 are newly introduced coefficients, ε_x, ε_y and ε_z are dilatation components, γ_{xy}, γ_{xz} and γ_{yz} are slide components, E_x, E_y and E_z are the electric field vector ($\vec{E}$) components. Given the initial assumption that for the observed case of a cantilever loaded at the free end with concentrated force, there is only the occurrence of normal mechanical stress σ_x, and tangential mechanical stress τ_{xz}, and mechanical displacement component v satisfy the following relations $\partial v/\partial x=\partial v/\partial y=\partial v/\partial z=0$.

For the adopted longitudinal polarization, equation which describes the mutual coupling between electrical and mechanical variables has the following form:

$$\begin{gathered} \varepsilon_x = \varepsilon_{33}^E \sigma_x + \varepsilon_{13}^E(\sigma_y + \sigma_z) + b_{33}E_x, \\ \varepsilon_y = \varepsilon_{13}^E \sigma_x + \varepsilon_{11}^E \sigma_y + \varepsilon_{12}^E \sigma_z + b_{31}E_x, \\ \varepsilon_z = \varepsilon_{13}^E \sigma_x + \varepsilon_{12}^E \sigma_y + \varepsilon_{11}^E \sigma_z + b_{31}E_x, \\ \gamma_{xy} = \varepsilon_{44}^E \tau_{xy} + b_{15}E_y, \\ \gamma_{xz} = \varepsilon_{44}^E \tau_{xz} + b_{15}E_z, \\ \gamma_{yz} = 2(\varepsilon_{11}^E - \varepsilon_{12}^E)\tau_{yz}, \\ D_x = d_{33}^\sigma E_x + b_{33}\sigma_x + b_{31}(\sigma_y + \sigma_z), \\ D_y = d_{11}^\sigma E_y + b_{15}\tau_{xy}, \\ D_z = d_{11}^\sigma E_z + b_{15}\tau_{xz}. \end{gathered} \tag{11}$$

where D_x, D_y and D_z [C/m^2] are piezoelectric displacement vector components, $\varepsilon_{11}{}^E$, $\varepsilon_{12}{}^E$, $\varepsilon_{13}{}^E$, $\varepsilon_{33}{}^E$, and $\varepsilon_{44}{}^E$ [m^2/N] are coefficients of elastic power at a given (zero) electric field, b_{31}, b_{15}, and b_{33} [C/N] are piezoelectric constants (piezomodules), $d_{11}{}^\sigma$ and $d_{33}{}^\sigma$ [F/m] are dielectric permeabilities at a given zero mechanical stress.

Seven unknown coefficients: a_2, a_4, b_0, b_2, b_3, c_1, and c_4, which enter into expressions (7) and (10), have to be determined to fulfil the system of equations of electro-elasticity (11) and boundary conditions (3), (5), and (6):

$$
\begin{gathered}
2a_4 xz = -\frac{F\varepsilon_{33}^E}{I_y} xz, \\
a_2 + b_2 + (a_4 + 3b_3)x^2 = -\frac{F\varepsilon_{44}^E}{8I_y} h^2 + \frac{F\varepsilon_{44}^E}{2I_y} z^2 - b_{15}c_1 - 3c_4 b_{15} z^2, \\
D_x = -\frac{Fb_{33}}{I_y} xz, \quad D_z = -d_{11}^\sigma c_1 - \frac{Fb_{15}}{2I_y}\frac{h^2}{4} + \left(\frac{Fb_{15}}{2I_y} - 3d_{11}^\sigma c_4\right) z^2, \\
-b_{33}\frac{F}{I_y} z + 2\left(\frac{Fb_{15}}{2I_y} - 3d_{11}^\sigma c_4\right) z \Bigg|_{div\vec{D}=0} = 0\,, \\
c_1 \frac{h}{2} + c_4 \frac{h^3}{8} \Bigg|_{z=\frac{h}{2}} = U_0, \\
-c_1 \frac{h}{2} + c_4 \frac{h^3}{8} \Bigg|_{z=-\frac{h}{2}} = -U_0, \\
w\Big|_{\substack{z=0\\x=l}} = b_0 + b_2 l + b_3 l^3 = 0, \quad \frac{\partial w}{\partial x}\Big|_{\substack{z=0\\x=l}} = b_2 + 3b_3 l^2 = 0.
\end{gathered} \tag{12}
$$

From the system of equations (12) unknown coefficients are determined as:

$$
\begin{gathered}
a_2 = \frac{F\varepsilon_{33}^E}{2I_y} l^2 - \frac{2U_0}{h} b_{15} - \frac{F\varepsilon_{44}^E}{3I_y}\frac{h^2}{4}, \quad a_4 = -\frac{F\varepsilon_{33}^E}{2I_y}, \\
b_0 = \frac{F\varepsilon_{33}^E}{3I_y} l^3, \quad b_2 = -\frac{F\varepsilon_{33}^E}{2I_y} l^2, \quad b_3 = \frac{F\varepsilon_{33}^E}{6I_y}, \\
c_1 = \frac{2U_0}{h} - \frac{F}{6I_y}\frac{b_{15} - b_{33}}{d_{11}^\sigma}\frac{h^2}{4}, \quad c_4 = \frac{F}{6I_y}\frac{b_{15} - b_{33}}{d_{11}^\sigma}.
\end{gathered} \tag{13}
$$

By introducing the obtained values for coefficients (13) into expressions (7) and (10), one gets solutions for componential displacements of the displacement vector $\vec{S}$, electric potential ψ, specific strains (dilatation ε_x and slide γ_{xz}), electric field E_z, and piezoelectric displacements D_x and D_z, for the rectangular prismatic cantilever with longitudinal polarisation and electrode coatings on the sides $z=\pm h/2$, in the form of:

$$
u = \left(\frac{F\varepsilon_{33}^E}{2I_y} l^2 - \frac{2U_0}{h} b_{15} - \frac{F\varepsilon_{44}^E}{3I_y}\frac{h^2}{4}\right) z - \frac{F\varepsilon_{33}^E}{2I_y} x^2 z, \tag{14}
$$

$$w=\frac{F\varepsilon_{33}^{E}}{3I_y}l^3-\frac{F\varepsilon_{33}^{E}}{2I_y}l^2x+\frac{F\varepsilon_{33}^{E}}{6I_y}x^3, \tag{15}$$

$$\psi=\left(\frac{2U_0}{h}-\frac{F}{6I_y}\frac{b_{15}-b_{33}}{d_{11}^{\sigma}}\frac{h^2}{4}\right)z+\frac{F}{6I_y}\frac{b_{15}-b_{33}}{d_{11}^{\sigma}}z^3, \tag{16}$$

$$\varepsilon_x=-\frac{F\varepsilon_{33}^{E}}{I_y}xz, \tag{17}$$

$$\gamma_{xz}=-\frac{2U_0}{h}b_{15}-\frac{F\varepsilon_{44}^{E}}{3I_y}\frac{h^2}{4}, \tag{18}$$

$$E_z=-\frac{2U_0}{h}+\frac{F}{2I_y}\frac{b_{15}-b_{33}}{d_{11}^{\sigma}}\left(\frac{h^2}{12}-z^2\right). \tag{19}$$

$$D_x=-b_{33}\frac{F}{I_y}xz. \tag{20}$$

$$D_z=-\frac{2U_0}{h}d_{11}^{\sigma}-\frac{F}{6I_y}(2b_{15}+b_{33})\frac{h^2}{4}+\frac{Fb_{33}}{2I_y}z^2. \tag{21}$$

3. Numerical Analysis

The subject of observation in this paper is stressing of a piezoceramic PZT4 [13] cantilever with a longitudinal type of polarization and electrode coatings, with the following dimensions: b=4.1 mm, h=20.1 mm, l=30.1 mm and density ρ=7500 kg/m^3, loaded by the concentrated force (see Fig. 1). Material coefficients used in the analysis are listed in Table 1.

Table 1 Design parameters of the piezoceramic cantilever

Symbol	Value
ε_{13}^E	$-5.31\cdot10^{-12}$
ε_{33}^E	$15.5\cdot10^{-12}$
ε_{44}^E	$39\cdot10^{-12}$
b_{31}	$-123\cdot10^{-12}$ m/V
b_{33}	$289\cdot10^{-12}$ m/V
b_{15}	$496\cdot10^{-12}$ m/V
ε_0	$8.854\cdot10^{-12}$ F/m
d_{11}	$1475\cdot\varepsilon_0$ F/m
d_{33}	$1300\cdot\varepsilon_0$ F/m

Based on equations (14)-(21) a numerical analysis was performed using Matlab software, and bi-parametric surfaces of state were obtained for: componential displacement $u=u(x, z)$ (see Fig. 2); componential displacement $w=w(x, F)$ (see Fig. 3); electric potential $\psi=\psi(z, F)$ (see Fig. 4); specific strain – dilatation $\varepsilon_x=\varepsilon_x(x, z)$ (see Fig. 5); specific strain – slide $\gamma_{xy}=\gamma_{xy}(U_0, F)$ (see Fig. 6); electric field $E_z=E_z(z, F, U_0)$ (see Figs. 7 and 8); piezoelectric displacement $D_x=D_x(x, z)$ (see Fig. 9); and piezoelectric displacement $D_z=D_z(z, F)$ (see Fig. 10).

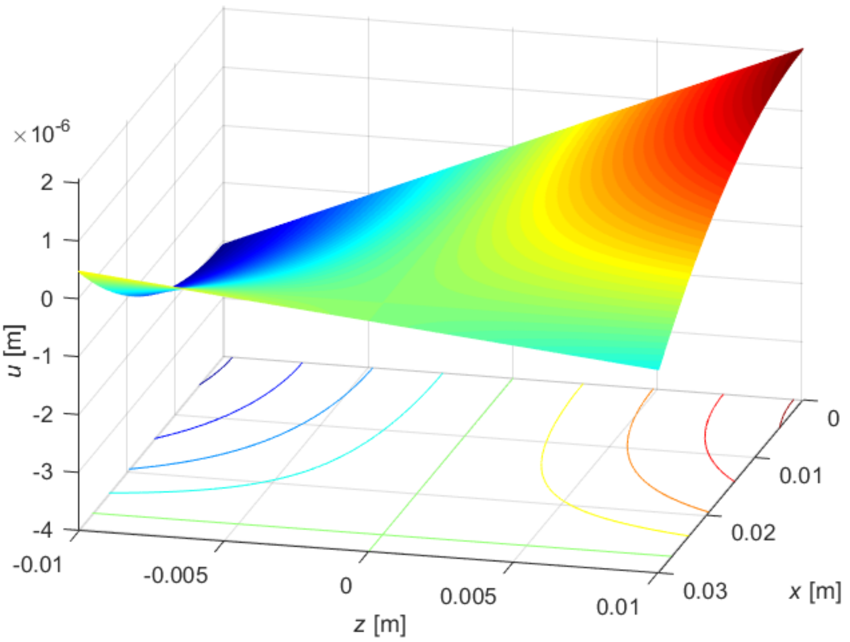

Fig. 2 Componential displacement $u=u(x, z)$

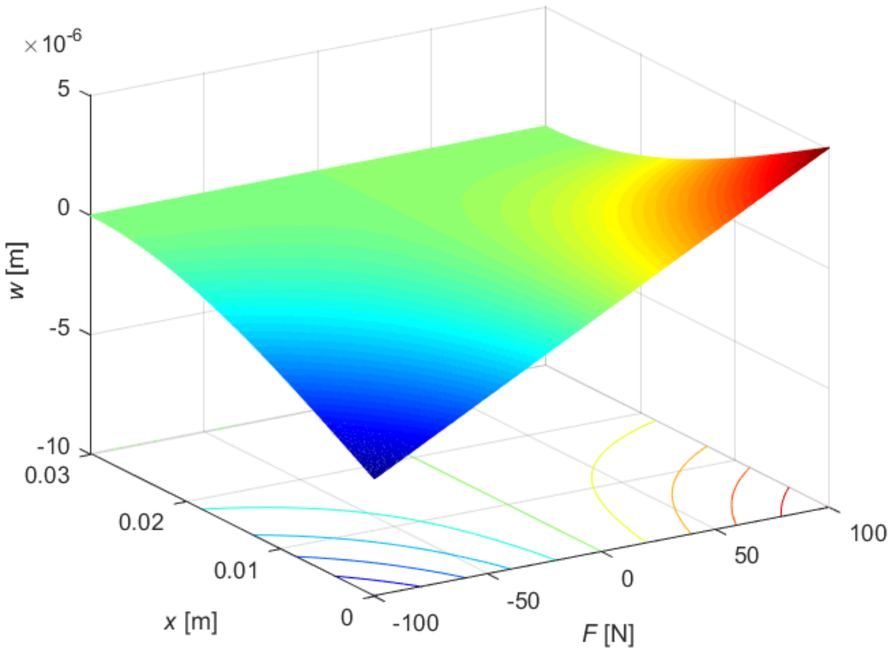

Fig. 3 Componential displacement $w=w(x, F)$

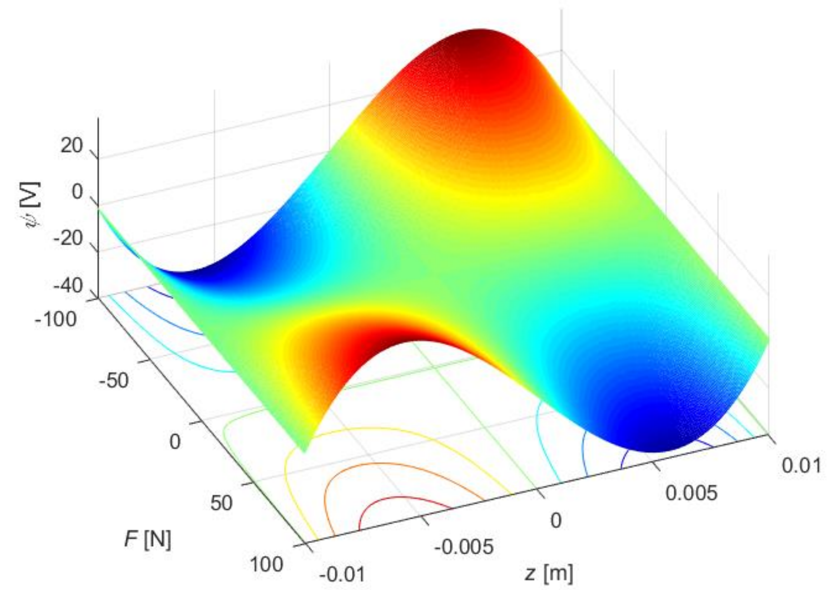

Fig. 4 Electric potential ψ=ψ(z, F)

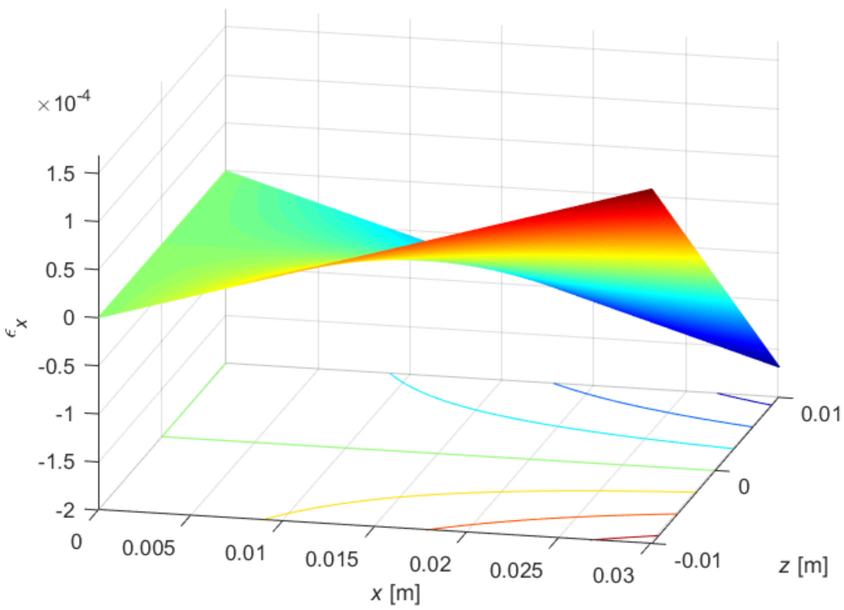

Fig. 5 Specific strain – dilatation ε_x= ε_x(x, z)

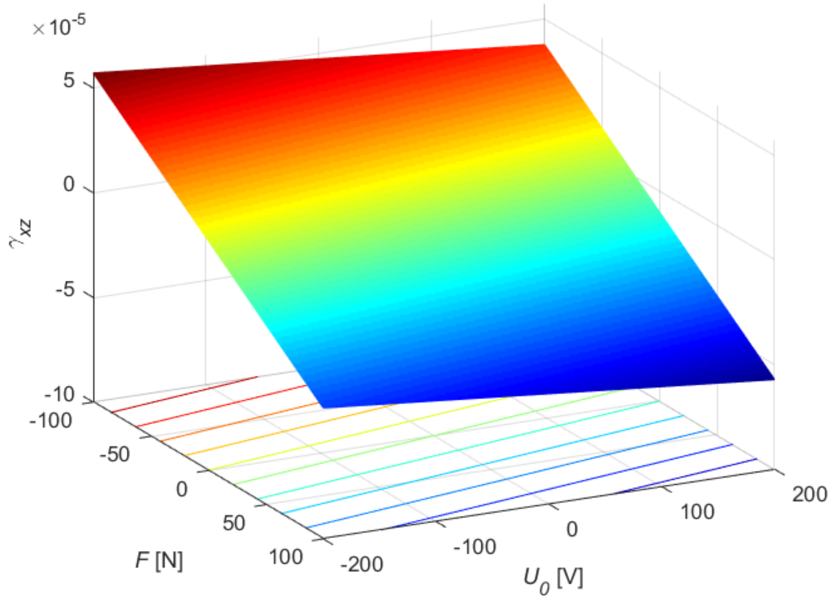

Fig. 6 Specific strain – slide $\gamma_{xy}= \gamma_{xy}(U_0, F)$

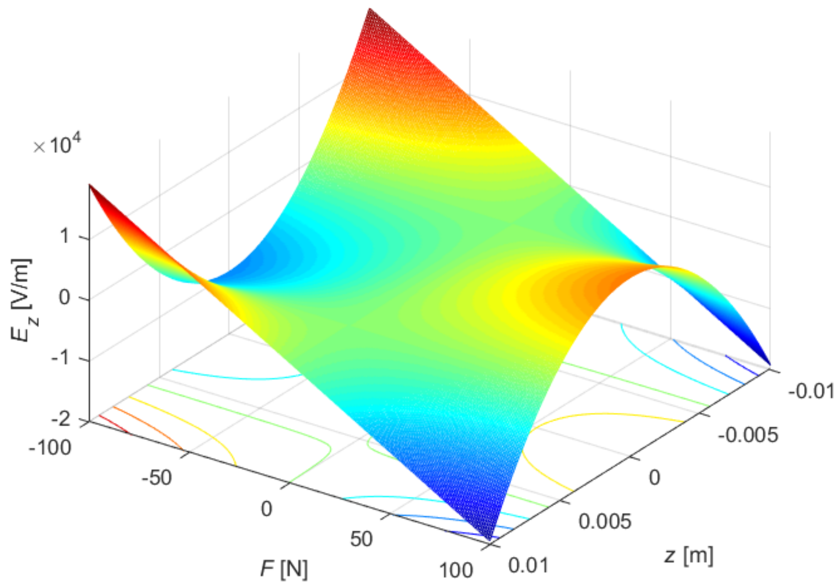

Fig. 7 Electric field E_z=$E_z(z, F)$

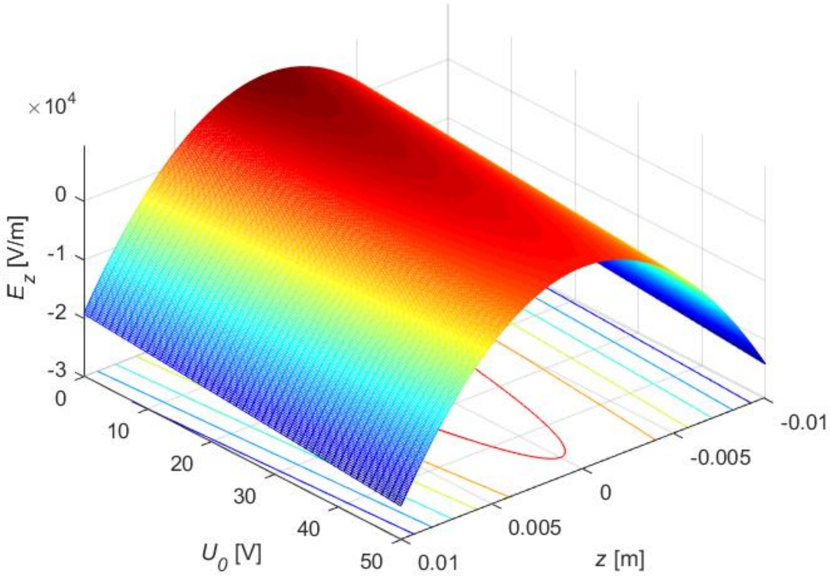

Fig. 8 Electric field E_z=E_z(z, U_0)

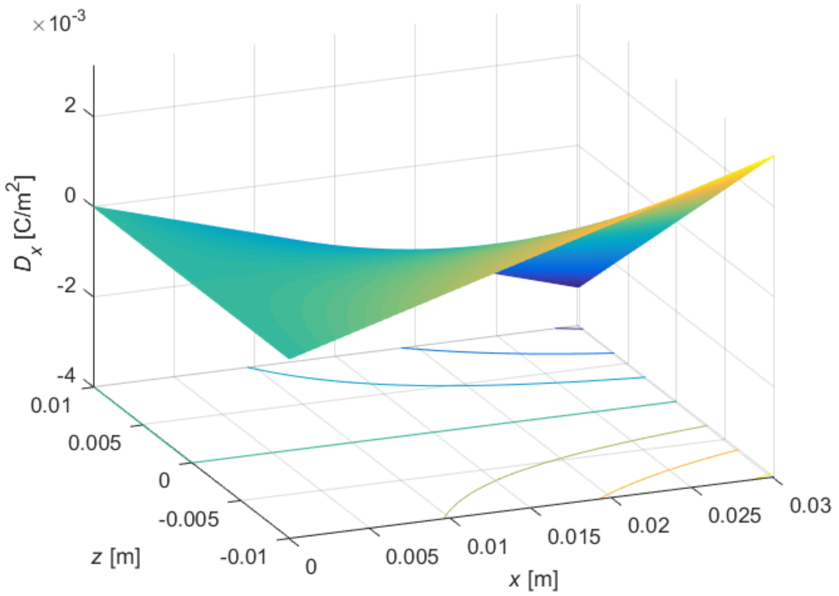

Fig. 9 Piezoelectric displacement D_x=D_x(x, z)

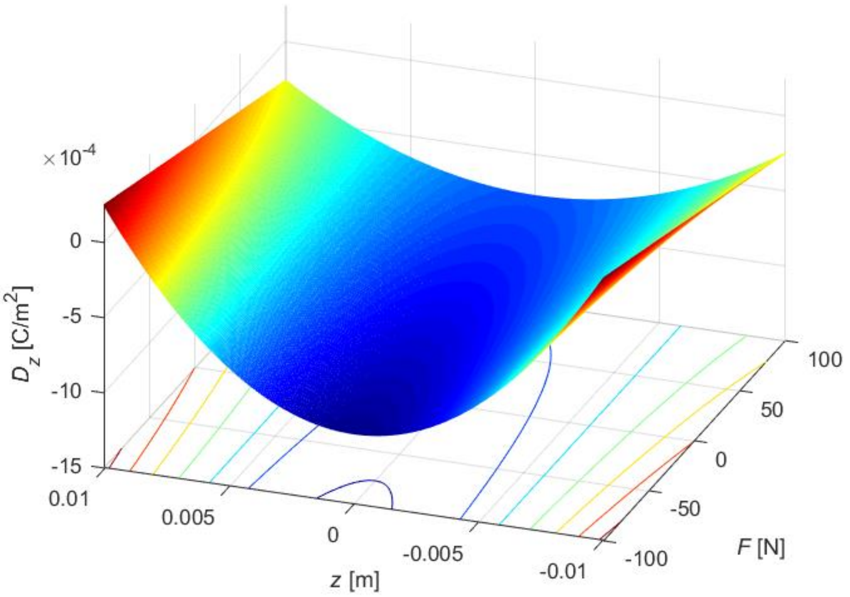

Fig. 10 Piezoelectric displacement D_z=D_z(z, F)

4. CONCLUSION

In this paper, the overall qualitative image of the stressing issue of the rectangular prismatic piezoceramic cantilever with longitudinal polarisation and with electrode coatings is observed.

Conclusions derived from numerically obtained bi-parametric surfaces of state using the Matlab software package are:

- Fig. 2 shows that the value of componential displacement is equal to zero for z=0, which is to be expected based on equation (14). On the other hand, the oscillation node occurs at x=0.0281 m for the selected dimensions of the observed piezoceramic PZT4 cantilever. At that spot, in the observed regime, it is possible to, for example, fix the cantilever without affecting its oscillations.
- Fig. 3 shows the fulfilled conditions of rigid clamping for the front surface x=l (w=0).
- For the electric potential ψ (see Fig. 4) and the electric field E_z (see Fig. 7) an almost sine law of change along the z-axis of the cantilever is justified, where the maximal difference of electric potential occurs at spots where E_z=0 (for z = ±0.0058 m), and vice versa for z=±h/2. The electric field E_z=E_z(z, U_0) (see Fig. 8) has a characteristic half-cylinder spatial surface as well as the piezoelectric displacement D_z (see Fig. 10).
- Bi-parametric surfaces of dilatation ε_x (see Fig. 5) have extreme values on the frontal surface x=l. For bi-parametric surfaces of slide γ_{xy} (see Fig. 6), a linear dependence is justified.

- Piezoelectric displacement D_x (see Fig. 9) has a characteristic saddle-shaped spatial surface.
- If the velocity of the longitudinal wave propagation in the piezoceramic electro-elastic rod is taken into account, which is $c = 1\Big/\sqrt{\rho\varepsilon_{33}^{E}(1-k_{33}^{2})}$, as well as the coefficient of electromechanical static relation in the longitudinal direction $k_{33}^{2} = b_{33}^{2}\Big/\varepsilon_{33}^{E} d_{33}^{\sigma}$, one may get a resonant oscillation frequency of 174.28 kHz.

This kind of analysis enables the prediction of the characteristics of piezoceramic cantilevers (as actuators or sensors) with the presented configuration when designing piezoceramic devices. The measurement of mechanical displacements of the cantilever will be the subject of further research.

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