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APPLICATION OF DIGITAL SLIDING MODES TO SYNCHRONIZATION OF THE WORK OF SEVERAL HYDRAULIC CYLINDERS

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Abstract. *The paper considers the problem of ensuring synchronized movement of several hydraulic cylinders. The control system for synchronization was designed by applying the variable structure control system theory. The control algorithm is based on the digital sliding mode. Measuring coordinates of the state (positions and velocities) directly on the cylinders is supposed to be possible. It is shown that such a system ensures quick synchronization of cylinders under different initial conditions. The applied algorithm was compared with conventional control algorithms. The quality of work of the considered system was illustrated by computer simulation*.

Key words: *hydraulic cylinder, variable structure system, digital sliding mode, synchronization*

1. INTRODUCTION

Hydraulic cylinders are widely applied in flexible automation of machines and manufacturing processes. As with other types of actuators (e.g. electric motors), it is frequently demanded that two or more hydraulic cylinders move synchronously. Since these cylinders can have different initial conditions and different loads, it is necessary, as soon as possible, to achieve synchronization of further movement to a given position value. This problem was examined in detail by Unbehauen and Vakilzadeh [1-4] for various types of actuators, with the application of various control algorithms of conventional type (P, PI, PID).

The basic idea is represented in Fig. 1. Fig. 1a, shows a synchronization system with two hydraulic cylinders. The referent signal is supplied to the system's input. Control signals of cylinders C1 and C2 respectively, are obtained as the sum and subtract of the referent input

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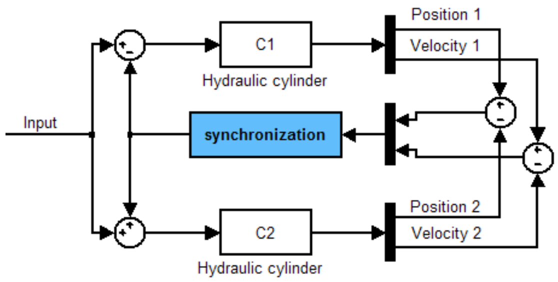
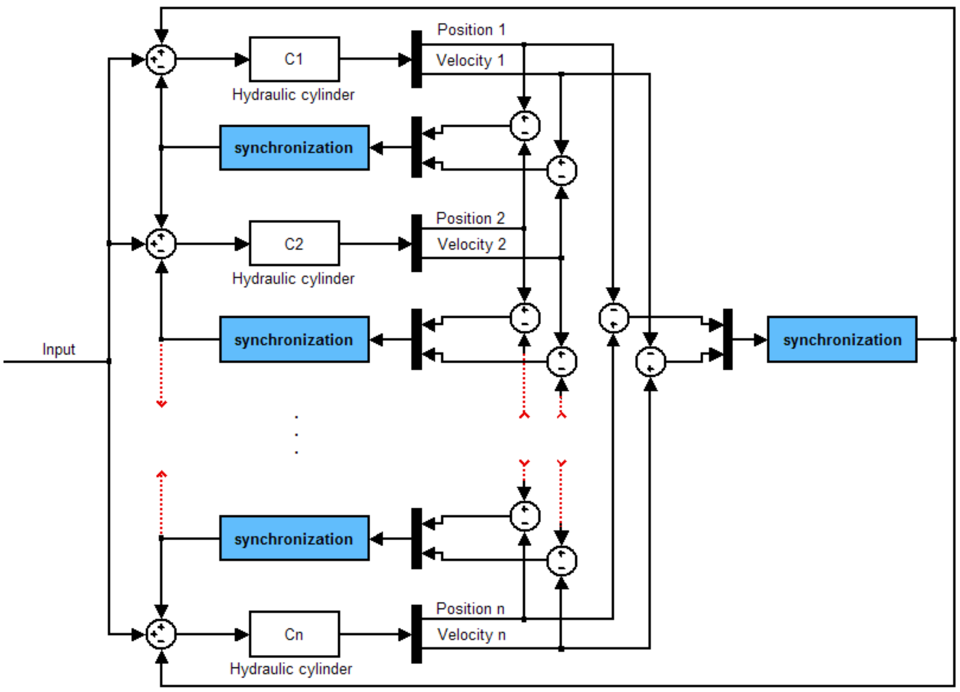
and the signal of the feedback, generated by the control mechanism (synchronizer). The feedback's signal is generated as the function of the difference of vectors of state (of position and velocity) for cylinders C1 and C2. It is also possible to view the system as a control system with an observer, where apart from the effect of the signal of the observation error on the observer's input, the effect on the object's input is also achieved. In the given case, C2 is the 'observer', while C1 is the 'object'. The basic difference from the system with the observer lies in the fact that here both the 'observer' and the 'object' are dynamic elements with the same dynamic properties and approximately the same parameters.



a)



b)

Fig. 1 Schematic representation of synchronization system:
a) two cylinders, b) several cylinders.

Fig. 1b, shows synchronization system with more than two hydraulic cylinders. Control signals of cylinders C1, C2, …, Cn respectively, are obtained as the sum and subtract of the referent input and the signal of the feedback, generated by the control synchronizer. The feedback's signal is generated as the function of the difference of vectors of state (of position and velocity) of adjacent hydraulic cylinders (C1 and C2, C2 and C3, …, Cn-1 and Cn, Cn and C1). The system can also be seen as a control system with an observer, i.e. for each hydraulic cylinder, there are two adjacent observers, the previous and the next one, which together with referent signal, synchronize the control signal at the cylinder itself. Such a synchronization algorithm for several hydraulic cylinders turned out to have better features in the sense of a shorter period of synchronization than is the case with an algorithm where any cylinder can be the referent one, while others are synchronized to it.

The main goal of this paper is exploration of the possibility of applying the algorithm of control of the variable structure with sliding working regime to the problem of synchronization. The basic features of sliding regimes, known to a small circle of experts in the field of automatic control, are:

- theoretical invariance to the external load and internal perturbations (parameters uncertainity) if maching conditions are satisfied [5] and practical robustnes;
- the character of the system's movement is known in advance;
- the movement does not depend on the object's parameters and control, but only on control parameters;
- what is necessary is not the exact knowledge of the object's parameters, but only of the range of their possible change;
- it is easier to ensure the system's stability by decomposing the problem of stability into two simpler sub-problems;
- lowering the order of the differential equation which describes movement.

Apart from the above listed features, systems with a variable structure with sliding working regimes have shortcomings. There are two basic of these:

- the necessity of measurability of the full state of the controlled object;
- the occurrence of vibration in the control signal, which may cause excitation of non-modeled dynamics of the object and undesired movement in the area of the predicted trajectory.

The first shortcoming can successfully be solved by the application of the observer, which partly alleviates the second, too. However, algorithms which solve one or both these problems have also been developed.

The paper is organized in the following way: the second part explains the method of obtaining mathematical models of control objects. The third part presents an outline of the control algorithm on the basis of [6]. The fourth part presents the effects of the application of the above-mentioned algorithm of control in comparison with conventional algorithms [1-4] by means of computer simulation.

2. MATHEMATICAL MODEL OF THE SYSTEM

A common positional hydraulic servo system is presented in Fig. 2. It is composed of double-acting hydraulic cylinder, designated as 1.0 and proportional valve 5/3 designated as 1.1, where M_L is the external mass, M_p is the piston mass, y is the position, F_L is the external load, P_1 and P_2 are the absolute pressures in the cylinder's chambers, A_1 and A_2 are the piston surfaces, V_1 and V_2 are the volumes of cylinder's chambers and i_c and u_c are the input DC current and voltage of proportional valve [6,7].

The cylinder ports are connected to a proportional valve, and piston motion is obtained by modulating the oil flow into and out of the cylinder chambers. A proportional valve provides this modulation [6,7].

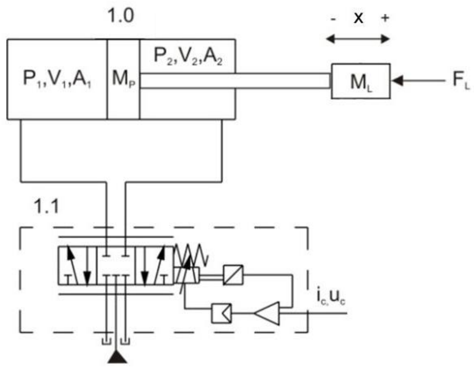


Fig. 2 Hydraulic scheme of the servo system for positioning.

The second order linearized transfer function of the presented system [6,7], can be represented as:

$$G(s) = \frac{K_p}{s^2 + K_1 s + K_2} \tag{1}$$

where K_p, K_1 and K_2 are coefficients of transfer function and they depend of cylinder dimension, tubing, proportional control valve, oil characteristics and other parameters [6,7], that is explained in detail in this paper.

3. DIGITAL SLIDING MODE CONTROL ALGORITHM

The control algorithm, whose application to the problem of synchronizing hydraulic actuators is examined in this paper, belongs to the group of digital variable structure control algorithm. The goal of synthesis of control is to achieve movement of the system in the space of the state on a pre-given hyper-surface, in systems of the higher order, i.e. on a line (most frequently a straight line), in systems of the second order. To do so, what has to be ensured is the transfer of the system's state from any initial state to the given hyper-surface and its subsequent movement on it in the sliding mode. This means that the system's phase trajectories all go into the given hyper-surface. Since it is selected so that it passes through the outcome of the space of state, which represents the state of equilibrium, the asymptotic stability of the system is also ensured. In this way, the system is brought into equilibrium according to a pre-given trajectory, which may also have attributes of optimality. To summarize, the movement of this system has three phases: (I) the phase of reaching the hyper-surface; (II) the phase of the sliding mode; (III) the phase of steady state.

If the sliding hyper-surface is marked as s(**x**), the conditions are met by satisfying the inequality

$$s(\mathbf{x})\dot{s}(\mathbf{x}) < 0 \tag{2}$$

This condition can be satisfied by applying various control algorithms. However, they must contain a relay component of type, which may give rise to parasitic movements (chattering) in the area of the hyper-surface s(**x**)=0 even in the steady state. Such movements are especially intrusive in electromechanical systems

$$U_0 \operatorname{sgn}\{s(\mathbf{x})\}, U_0 > 0 \tag{3}$$

The algorithm applied in this paper eliminates or minimizes the problem of vibration to a tolerable level. The control is formed so that it has two components: a relay component, which ensures safe and quick transfer of the system's state near the sliding hyper-surface without intersecting it, and a linear component, which brings the system's state into s(x)=0 in the following step (during one discretization period).

Since the control will be digitally implemented, it is necessary to perform time-discretization of the model (1). If T_d denotes the sampling period and

$$\mathbf{A}_{\boldsymbol{\delta}}(T_d) = \frac{e^{\mathbf{A}T_d} - \mathbf{I}_{\mathbf{n}}}{T_d}, \quad \mathbf{b}_{\boldsymbol{\delta}}(T_d) = \frac{1}{T_d}\int_0^{T_d} e^{\mathbf{A}\tau}\mathbf{b}\,d\tau, \tag{4}$$

the discrete-time state-space model of the nominal system, by using δ-transform, can be expressed in the form:

$$\boldsymbol{\delta}\mathbf{x}(kT_d) = \mathbf{A}_{\boldsymbol{\delta}}(T_d)\mathbf{x}(kT_d) + \mathbf{b}_{\boldsymbol{\delta}}(T_d)\mathbf{u}(kT_d) \tag{5}$$

The commonly used linear scalar switching function s(x) in sliding mode control (SMC) systems that defines a sliding hyperplane in the state space (s=0), along which the sliding mode is organized, is chosen as

$$s = \mathbf{c}_{\boldsymbol{\delta}}(T_d)\mathbf{x} \tag{6}$$

where $\mathbf{c}_\delta(T_d)$ is the switching function vector of appropriate dimension.

The detailed control design procedure of the selected discrete-time SMC (DSMC) algorithm is given in [8,9]. Here, only the important relations will be briefly recounted. The positioning control law is obtained as:

$$u_c = -\mathbf{c}_{\boldsymbol{\delta}}(T_d)\mathbf{A}_{\boldsymbol{\delta}}(T_d)\mathbf{x}(k) - \mathbf{\Phi}(s(k), \mathbf{X}(k)) \tag{7}$$

The switching function vector $\mathbf{c}_\delta(T_d)$, which exclusively defines system dynamics in the sliding mode, is selected according to the relation:

$$\mathbf{c}_{\boldsymbol{\delta}}(T_d) = [\mathbf{c}_{\mathbf{1}}(T_d) \mid 1]\mathbf{P}_1^{-1}(T_d) \tag{8}$$

The values required for calculating matrix $\mathbf{c}_\delta(T_d)$, are obtained through the following equations [9]:

$$c_i(T_d) = \frac{1}{(i-1)!} \frac{d^{i-1} \prod_{j=1}^{n-1} (\delta - \delta_j(T_d))}{d\delta^{i-1}} \Big|_{\delta=0} \tag{9}$$

$$\delta_i(T_d) = \frac{e^{-\alpha_i T_d} - 1}{T_d}, \quad \alpha_i > o, \quad \begin{matrix} i \neq j \Rightarrow \alpha_i \neq \alpha_j, \\ i, j = 1, \ldots, n-1 \end{matrix} \tag{10}$$

$$\mathbf{P}_1^{-1}(T_d) = [b_\delta(T_d) \quad \ldots \quad A_\delta^{n-1}(T_d) b_\delta(T_d)] \begin{bmatrix} a_1(T_d) & \cdots & a_{n-1}(T_d) & 1 \\ a_2(T_d) & \cdots & 1 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 1 & \cdots & 0 & 0 \end{bmatrix} \tag{11}$$

Function $\Phi(s,X)$ is defined as:

$$\mathbf{\Phi}(s, X) = \Phi(s) = \min\left(\frac{|s|}{T_d}, \sigma + \rho|s| \right) \mathrm{sgn}(s); \quad \begin{matrix} 0 \leq \rho T_d < 1, \\ \sigma > 0 \end{matrix} \tag{12}$$

Parameters σ and ρ define sliding mode reaching dynamics and should be chosen to provide as short as possible reaching phase.

4. Synchronization of the System with Several Hydraulic Cylinders

As was said in the introduction, synchronization of the work of several hydraulic cylinders is a sizeable problem, which can be solved by using adequate control.

There are two hydraulic systems to be considered in this section. The first system consists of two hydraulic double-acting cylinders and the second system consists of four hydraulic double-acting cylinders, whose work should be synchronized, since they have different initial positions. The system's model, given in state space form by equation (13),

$$G(s) = \frac{2.055}{s^2 + 200s} \tag{13}$$

The sliding hyper-surface coefficient is calculated by means of equation (8), for α = 15s-1. Parameters q and σ are taken to be 0 and 24 respectively.

Vector $\mathbf{c}\delta$(T)= [-15.36987 -1.024658] and $\mathbf{c}\delta(T)\mathbf{A}\delta(T)$= [0 81.95372].

In the first system, different initial positions are reflected in the fact that the piston rod of the first cylinder is already somewhat drawn out, whereas the piston rod of the other cylinder is completely pulled in.

The problem of synchronization is to bring, within a short period of time, the positions of both cylinders (x_1 and x_2) into equal positions, with no differences in further work. In order to successfully solve this problem of synchronization in the manner described in section 3, the algorithm of digital control, given in equation (7) is used, which ensures that errors in positions and velocities of the piston rods are eradicated within a short period of time.

To confirm that the algorithm of control, which enables quick synchronization, was adequately selected, Fig. 3 shows simulated results of the positions of standard hydraulic cylinders. As has been mentioned, initial positions of the cylinders differ; in this case, the piston rod of the first cylinder C1 is at initial position x_1=0.1m, whereas the piston rod of the other cylinder C2 is completely pulled in, i.e. x_2=0m.

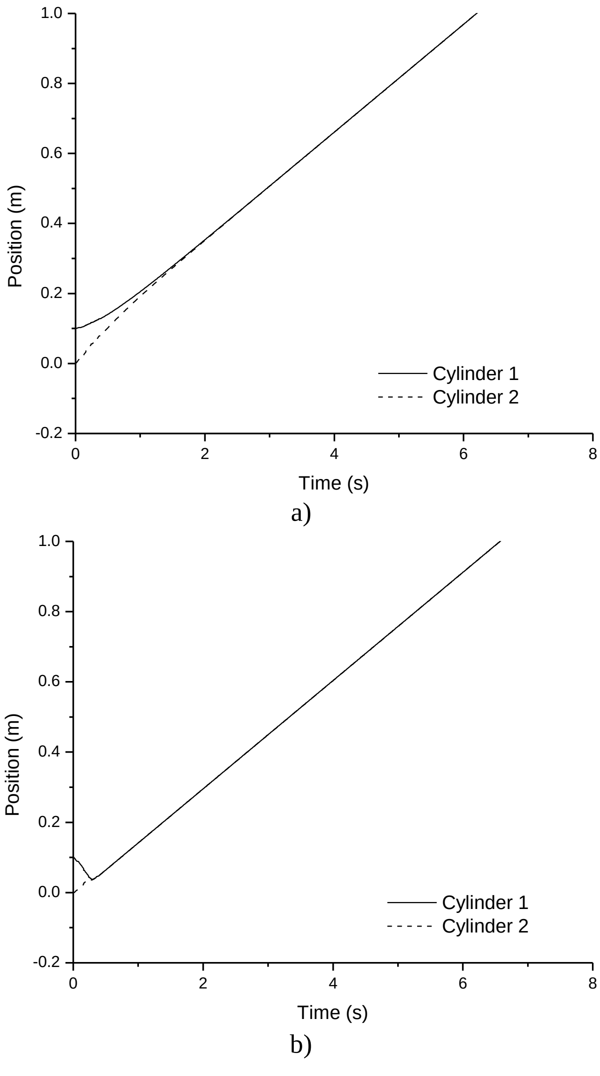
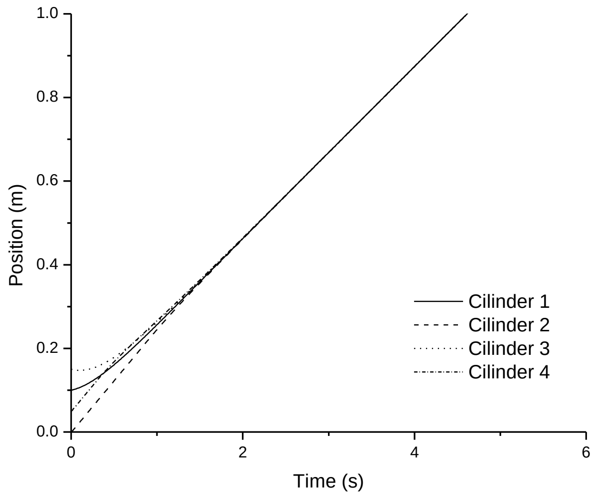


Fig. 3 Simulated results of the standard hydraulic cylinders positions: a) P controller, b) DSMC

Fig. 3a shows the work of the system when synchronization is performed with P controller, while Fig. 3b shows the result of the synchronization when DSMC is applied.

In the second system with four hydraulic cylinders, the piston rod of the first cylinder C1 is at initial position x_1=0.1m, the piston rod of the second cylinder C2 is completely pulled in, i.e. x_2=0m, the third cylinder is at initial position x_3=0.15m and the fourth cylinder is at initial position x_4=0.05m. The problem of synchronization is to bring, within a short period of time, the positions of all cylinders (x_1, x_2, x_3 and x_4) into equal positions, with no differences in further work.

Fig. 4 shows the work of the system when synchronization is performed with P controller, while Fig. 5 shows the result of the synchronization when DSMC is applied.



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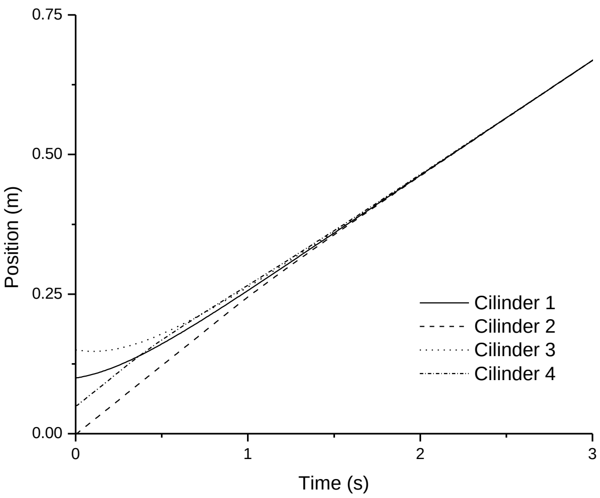

b)

Fig. 4 Simulated results of the standard hydraulic cylinders positions by P controller: a) whole diagram, b) magnified part.

For a better explanation some parts of diagrams in Fig. 4 and 5 are magnified. The Fig. 4b shows the magnified part of the diagram in the Fig. 4a, between 0 and 3s. Also, Fig 5b shows the magnified part of the diagram in the Fig. 5a, between 0 and 1s.

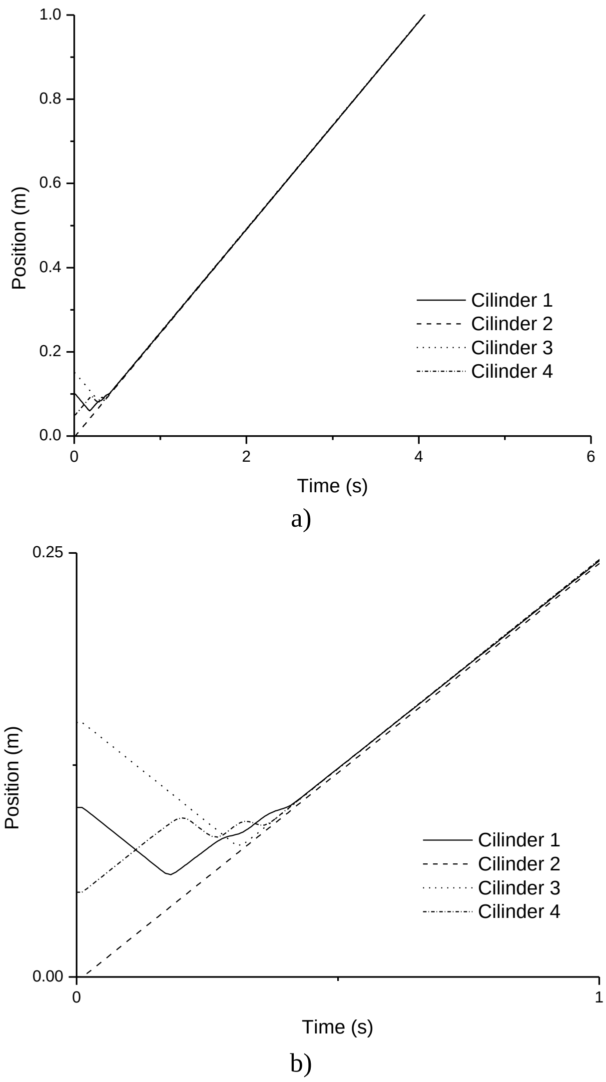

Fig. 5 Simulated results of the standard hydraulic cylinders positions by DSMC: a) whole diagram, b) magnified part

It is shown that DSMC is much better control for synchronization purposes than P controller.

CONCLUSION

The paper outlines the problem of synchronizing the work of several hydraulic cylinders, and the algorithm of digital control which solves this problem is given. The algorithm of control serves its purpose, since it ensures synchronization of the work of the cylinders within a short period of time, which was demonstrated and explained in section 4. This way of synchronizing hydraulic double-acting cylinders simplifies and reduces the price of realization to a significant extent. Such a synchronization algorithm for several hydraulic cylinders turned out to have better features, in the sense of a shorter period of synchronization than is the case with an algorithm where any cylinder can be the referent one, while others are synchronized to it.

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