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VISCOELASTIC DOUBLE BEAM SYSTEM STABILITY ANALYSIS USING ARTIFICIAL NEURAL NETWORKS

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Abstract. *This paper presents neural networks application and their advantages in estimation and analysis of numerical results obtained from previous research of dynamic stability of double beam system under stochastic loading processes. For determining the bounds of the almost sure stability of the double-beam system, direct Lyapunov method was applied and conventional numerical methods for stability region determination were used. Numerical results obtained by dynamic stability analysis were then used as a training data for artificial neural network (ANN). Application of ANN allows reduced computation time for accurate determination of stability regions, while the main advantage of trained ANN is that it estimates results even in cases where conventional numerical method fails.*

Key words: *Artificial neural network, viscoelastic double beam system, stability regions, direct Lyapunov method.*

1. INTRODUCTION

The past decades have witnessed a continuously growing interest in the development of diverse models based on techniques of Computational Intelligence (CI), whose predominant technologies include neural networks, fuzzy sets, and evolutionary methods [1].

Feedforward networks often have one or more hidden layers of nonlinear neurons followed by an output layer of linear neurons. Multiple layers of neurons with nonlinear transfer functions allow the network to learn nonlinear relationships between input and output vectors [2]. Artificial neural networks are interesting for classification and regression purposes due to their universal approximation property and their fast training if sequential

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training based on backpropagation is adopted [3]. Due to a complex interconnection between the input patterns of neural network and the architecture of neural network, the selection of neural network architecture must be done simultaneously. These aspects requires the formulation of search problem and the investigation of search techniques which are capable of facilitating model development work and resulting more reliable and robust neural network models [4].

The purpose of the present paper is to develop neural network tool for advanced numerical analysis. The network is trained with numerical data obtained from double beam system stability analysis. Feed forward artificial neural network is designed as an alternative to conventional numerical methods and it allows accurate analysis with reduced calculation time and gives results even in cases where conventional methods fail. The learning method of ANNs enables this system to learn from given training data sets, and due to the massive parallelism of the ANNs real-time processing of larger data sets is provided [5]. The principal contribution of this paper is to significantly reduce calculation time as well as to predict desired results when they cannot be calculated using conventional numerical method.

The present paper is organized as follows. Governing differential equations of analyzed system are given in Section 2. The neural network estimator is presented in Section 3. The neural network numerical procedure of determining the boundaries of stability is given in Section 4. Section 5 ends the paper with concluding remarks.

2. ANALYSED MODEL

Numerical ANN training set results are taken from [6] where direct Lyapunov method was used for double beam system (Fig. 1) stability analysis.

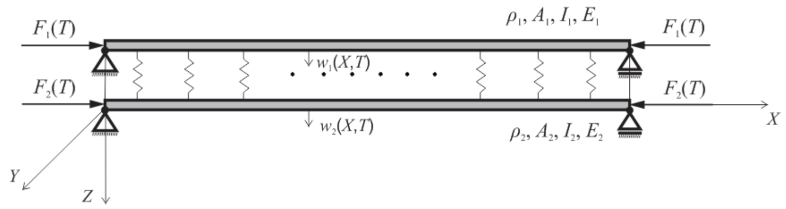

Fig. 1 Elastically connected double beam system

The coupled governing differential equations for transverse vibrations of the system can be expressed by

$$\begin{aligned}
&\rho_1 A_1 \frac{\partial^2 w_1}{\partial t^2} + \bar{\nu}_1 \frac{\partial^5 w_1}{\partial t \partial X^4} + E_1 I_1 \frac{\partial^4 w_1}{\partial X^4} + F_1(t) \frac{\partial^2 w_1}{\partial X^2} + \bar{K}(w_1 - w_2) = 0, \\
&\rho_2 A_2 \frac{\partial^2 w_2}{\partial t^2} + \bar{\nu}_2 \frac{\partial^5 w_2}{\partial t \ \partial X^4} + E_2 I_2 \frac{\partial^4 w_2}{\partial X^4} + F_2(t) \frac{\partial^2 w_2}{\partial X^2} + \bar{K}(w_2 - w_1) = 0,
\end{aligned} \tag{1}$$

where w_i (i =1, 2) denotes the transverse beam displacement, ρ_i mass density, $\bar{\nu}_i$ retardation time, X axial coordinate, t time, $E_i I_i$ bending stiffness of the beam, $\bar{K}$ the stiffness modulus of the Winkler elastic layer, and $F_1(t)$, $F_2(t)$ are the time-dependent stochastic processes.

Now, the following parameters were used to non-dimensionalize equations (1)

$$X = Lx,\ 2\nu_i = \frac{\bar{\nu}_i}{\rho A},\ e_i = \frac{E_i I_i}{\rho A L^4},\ f_{oi} + f_i(t) = \frac{F_i(t)}{\rho A L^2},\ K = \frac{\bar{K}}{\rho A},\ (i = 1, 2) \tag{2}$$

where v_i, e_i and K are the reduced retardation time, beams stiffnesses and stiffness of the Winkler layer, respectively, f_{oi} and f_i(t) are the reduced constant and stochastic component of axial forces.

Now, they have the form

$$\begin{aligned}
&\frac{\partial^2 w_1}{\partial t^2} + 2\nu_1 \frac{\partial^5 w_1}{\partial t \partial x^4} + e_1 \frac{\partial^4 w_1}{\partial x^4} + (f_{o1} + f_1(t)) \frac{\partial^2 w_1}{\partial x^2} + K(w_1 - w_2) = 0, \\
&\frac{\partial^2 w_2}{\partial t^2} + 2\nu_2 \frac{\partial^5 w_2}{\partial t \partial x^4} + e_2 \frac{\partial^4 w_2}{\partial x^4} + (f_{o2} + f_2(t)) \frac{\partial^2 w_2}{\partial x^2} + K(w_2 - w_1) = 0.
\end{aligned} \tag{3}$$

Calculations were made by using Gauss-Christoffel quadratures; for the Gaussian process the parameters of a Gauss-Hermite quadrature is taken. For the harmonic process we set $f_1(t) = H\cos(\omega t + \theta)$, where H, ω are the fixed amplitude and frequency, and θ is the uniform distributed phase on the interval $[0,2\pi)$, and a Gauss-Chebyshev quadrature is used. In order to compare both processes, the variance of the harmonic process $\sigma^2_{1H} = H^2/2$ is used. The stability regions are given as functions of the reduced retardation time v, system parameters e_1, e_2, K, reduced deterministic load f_{o1} and deterministic axial load ratio χ, where $\chi = f_{o1}/f_{o2}$.

3. Neural Network Approach for Numerical Analysis

Based on analysis explained in previous sections, obtained results are showing that not in all the cases conventional methods are usable. For some arbitrary parameters conventional numerical methods fail to provide variance for some values of retardation time v, or fail to provide any results. Therefore the advanced concept of neural network estimator is presented in this paper that successfully calculates variance in cases where conventional methods fail. Other advantages that we have been aiming for were reduced calculation time and simplicity of developed tool for numerical analysis that allows quick analysis for arbitrary parameters within the training range [2,5,7,8].

The ANN usually involves very simple dynamical schemes as nodes, and very complex networks of connections, an approach known as connectionism. Once the neural model of the process is built, off-line trained, it can be continuously updated on-line, in real-time, by minimizing the difference between its predicted output value, and its target value. Feed forward networks often have one or more hidden layers of nonlinear neurons followed by an output layer of linear neurons. Multiple layers of neurons with nonlinear transfer functions allow the network to learn nonlinear relationships between input and output vectors [2]. For the numerical analysis artificial neural network with one hidden layer is designed, trained and tested. The network has five inputs (reduced retardation time $v_1 = v_2 = v$, beams stiffnesses $e_1 = e_2$, reduced stiffness of the Winkler layer K, reduced constant component of axial force f_{o1} and deterministic axial load ratio χ) and two outputs (variances for Gaussian and harmonic stochastic process). The input layer has 5 neurons, hidden layer has 10 neurons, and output layer has 2 neurons. This multilayer feed forward network was trained for function approximation (nonlinear regression). The training process required a set of examples of proper network behavior network inputs u and target outputs *y* [8].

The process of training a neural network involves tuning the values of the weights and biases of the network to optimize network performance, as defined by the network performance function. The common performance function for feed forward networks is mean square error [3,4] between the network outputs $\hat{y}$ and the target outputs *y*. It is defined as follows

$$F = mse = \frac{1}{N}\sum_{i=1}^{N}(y-\hat{y})^2 \tag{4}$$

For training, testing and validation of developed multilayer feed forward network 756 data sets were used, with random data division and Levenberg-Marquardt back-propagation training algorithm. Used data set was created by selecting four variables K, $e_1 = e_2$, f_{01} and χ within the range of K=[50,2000], $e_1 = e_2 = [1,4]$, $f_{01} = [5,15]$, $\chi = [-1,1]$ and retardation time goes to 0.01 with descrete time step of 0.0005. Optimization methods for performance function uses the Jacobian of the network errors with respect to the weights. The gradient and the Jacobian are calculated using a *backpropagation* algorithm, which involves performing computations backward through the network. After 1000 iterations the regression value of training set of neural network estimator was $R = 1$, regression value of validation is $R = 0.99991$, regression value of test is R = 0,99999 and regression value of neural network is $R = 0.99998$ (Fig. 2).

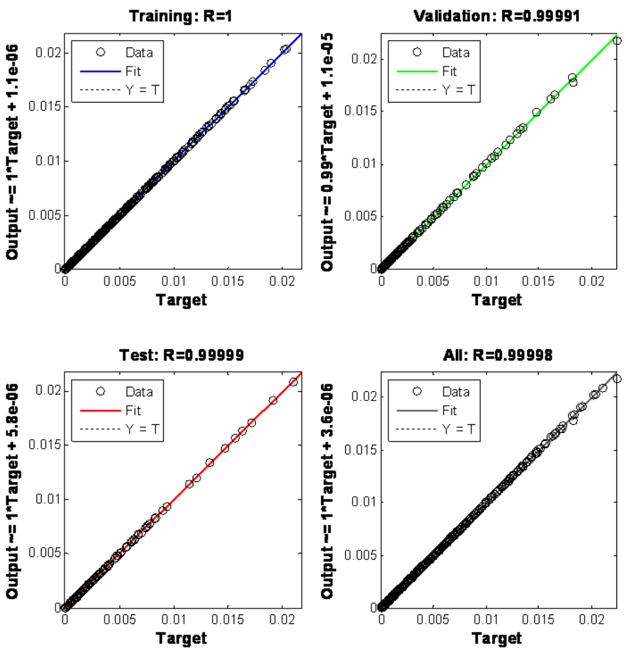

Fig. 2 Neural network training regression

4. Numerical Results

As in [6], we will consider the case when upper beam is loaded by deterministic and stochastic forces, and the lower beam is subjected only to deterministic load, i.e. $f_2(t) = 0$ and $f_{o2} \neq 0$. Stochastic component acting on upper beam is modeled as Gaussian or harmonic process with zero mean and variances σ_{1G}^2, σ_{1H}^2, respectively.

For beam parameters that were used in training, validation and testing, comparation of results achieved by conventional methods (solid line) and results achieved by neural network (dot line) are shown in Fig. 3 (results presented by upper line are for Gaussian process and results presented by lower line are for harmonic process).

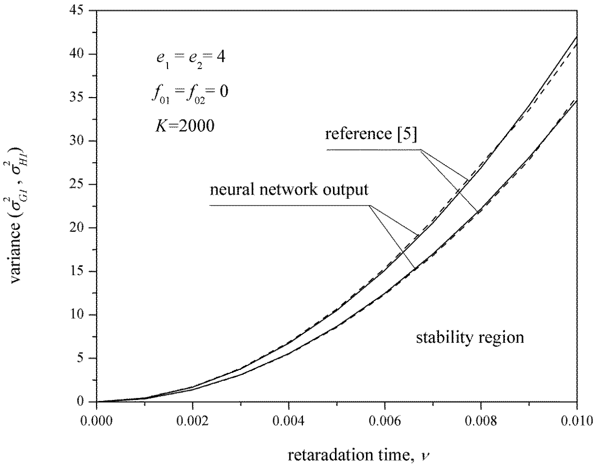

Fig. 3 Numerical results and neural network outputs comparison

It is obvious that neural network can estimate valid analysis results of the double beam system stability.

As example, 3D graphs are created for different values of deterministic load ratios (Fig. 4) and for different values of Winkler elastic layer stiffness (Fig. 5) where these data were obtained from previously trained neural network.

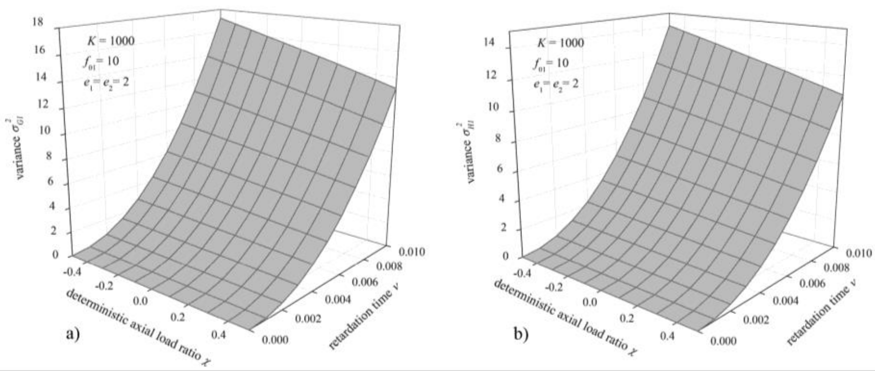

Fig. 4 Stability regions for a) Gaussian and b) harmonic process in function of deterministic axial load ratios

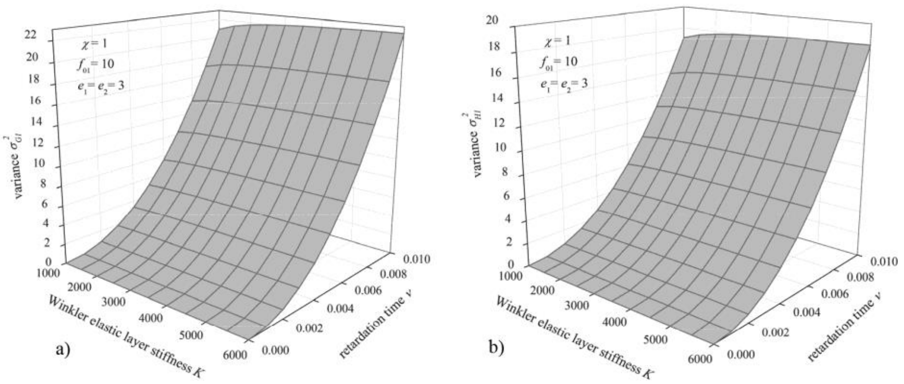

Fig. 5 Stability regions for a) Gaussian and b) harmonic process in function of a Winkler elastic layer stiffness

5. CONCLUSION

There are numerous advantages that presented method gives. After neural network is trained - weights and biases are tuned, it allows very quick and accurate analysis. With simple change of the input parameters it successfully calculates corresponding outputs. Trained neural network estimator uses significantly reduced calculation time. Another very important advantage is that it gives results in cases where conventional methods fail.

During the analysis of conventional methods, authors have noticed that for some set of the beam parameters conventional methods fail to provide results in some specific retardation time. Developed neural network allows us to determine stochastic process variance in that specific retardation time.

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