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EXPECTED UTILITY THEORY UNDER EXTREME RISKS

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Abstract. *Expected utility theory provides a framework for modeling choice of a rational individual, whose goal is to maximize expected utility to the preferences towards risk. However, extreme risks, such as, for example, a stock market crash or a natural disaster, significantly affect the function of the probability distribution of outcomes by adding the weight to the tails of the distribution. In such cases, the application of the theory of decision-making is extremely sensitive to assumptions on the probability distribution function. Therefore, this paper will provide a review of models of decision-making in terms of expected utility theory under extreme risk.*

Key words: *Expected utility, extreme risk, decision-making*

INTRODUCTION

The classical economic analysis of investment decision-making in the presence of risky and uncertain outcomes is based on the expected utility theory. This theory offers a framework for modeling a rational individual's choice whose goal is the maximum expected utility with regard to the given preferences towards risks. Assuming that a decision maker is characterized by a constant risk aversion, preferences may be described by the power utility function. On the other hand, investment outcomes modeling in the presence of risks are based on the probability theory, whereas the risk is perceived through the shape and symmetry of the expected outcomes probability distribution from the considered investment alternatives. It is most frequently assumed that the outcomes represent a random process, which can be described by a normal distribution. However, extremely risky situations, such as the stock market crash or natural disasters, have a significant effect on the function of the outcomes probability distribution, emphasizing the tails of distribution. In such cases, the application of the power utility function in estimating the expected utility may imply either no decisions or completely impossible decisions, which leads to the conclusion that the

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application of the decision theory is extremely susceptible to the assumptions regarding the probability distribution functions (Geweke, 2001).

Disregarding the size of the sample, i.e. the information set used for statistical analyses and outcome modeling, an individual cannot on certain occasions make a difference between different expected outcome distributions, which may lead to divergent rational decisions. Yet, if the information on the type of distribution is known beforehand, this fact may cause a different behavior in the conditions of extreme risks which need not be affected by a subsequently formed information set. On the other hand, research has shown that a group of efficient investment alternatives is determined by the shape and symmetry of the expected outcome distribution, which may cause the shift of efficacy boundaries. Therefore, the widely accepted Markowitz's method of optimization (Markowitz, 1952) may be modified in various ways so as to include the anomalies of financial time series – heavy-tailed and asymmetric distribution and more sophisticated measures of extreme risks.

Economic implications of the incompatibility of the expected utility theory and the statistical theory in the decision-making process have become rather evident, regarding the fact that the applied models of optimization do not only determine the decisions of individual and institutional investors, but also of regulatory bodies. Namely, the cost-benefit analysis is dominant, and in some cases obligatory analytical tool for assessing the net economic value of a new regulatory acts and measures on environment protection in the USA. Utility measurement represents an especially sensitive part of this analysis which requires a careful examination of numerous factors that define the social behavior in the conditions of ecological catastrophes (Carey, 2014; Sunstein, 2005). Therefore, some of the most important deficiencies of the expected utility theory under extreme risk will be presented in this paper. The framework of expected utility theory under risk and uncertainty will be presented in the first part of this paper. Determinants of the extreme risks will be analyzed in the second part, while its influence on the expected utility theory will be presented in the third part of the paper. In the fourth part authors will review possible adjustments of the utility function and their implications on the decision-making process.

1. Expected Utility Concept under Risk and Uncertainty

The normative theory of decision-making determines a series of principles on which the behavior of a rational individual is based – that of a decision maker. An individual's desire to lessen or avoid losses, that is to enlarge their wins (either material, emotional or any other) is implied in that individual's goals, while maximizing the personal welfare or benefit is the guiding principle in making a choice among different alternatives (Pavličić, 2014: 13). The rational choice theory is based on the model comprising two components: (1) a group of alternatives which are possible to realize, under different conditions, and (2) individuals' preferences that reflect their goals. In the situations of certainty, decision-makers make choices in a very simple and routine way even when confronted with a large number of alternatives. However, new situations characterized by risky outcomes and uncertainty may cause the change of possible alternatives so that, out of a possible subgroup of alternatives, the alternative corresponding to an individual's preferences is chosen. The fundamental study on the theory of rational choice by von Neumann and Morgenstern (1947) defined the framework and postulates of the rational choice. The expected utility theory defines the personal utility measurement in risky situations by the utility function, in which the relation of (strict) preference $\geq$ is defined in the final set of alternatives X and has the following characteristics:

- (completeness) for any two alternatives $x,\ y \in X$ it is true that $x \geq y$ or $y \geq x$ or $x \sim y$, where ~ stands for indifference;
- (transitivity) for any three alternatives $x, y, z \in X$ if $x \geq y$ and $y \geq z$ then $x \geq z$;
- (continuity) for any three alternatives $x, y, z \in X$ so that $x \geq y \geq z$ which means that there exists a certain probability p such that $\exists p \in [0,1] \ni y \sim [p : x;\ 1-p : z]$, which proves that minor changes in preferences will not change the order of preferences till the tipping point;
- (independence) for any three options $x, y, z \in X$ there is a probability $p \in [0,1]$, so that if $x \geq y$ then $px + (1-p)z \geq py + (1-p)z$, i.e. the preferences depend on the possibility of achieving a different outcome.

If $\geq$ relation of the (strict) preference is determined by the set X, the function $U: X \rightarrow R$ for which it is true that:

$$x \geq y \leftrightarrow U(x) \geq U(y) \tag{1}$$

is called the utility function of the preference relation. This function is defined for all values of $x > 0$ and is also valid for $U'(x) > 0$ and $U''(x) < 0$, so that von Neumann and Morgenstern regard the problem of decision-making as the problem of maximizing an individual's expected utility $E(U(x))$ defined as follows:

$$E(U(x)) = \int_{X \in R} U(x) d\mu(x) \tag{2}$$

where x denotes possible outcomes of the alternatives $x: R \rightarrow R^N$, and μ stands for the probability measurement of the considered outcomes which defines the distribution of the outcome probabilities in the real number set[1].

An individual's attitude towards a risk, which is expressed as an absolute risk aversion (Arrow, 1951) in the following way:

$$A(x) = -U''(x)/U'(x) \tag{3}$$

within the expected utility theory, determines the form of the utility function which is presupposed to be an individual's choice.

Assuming that an individual with some initial wealth W considers possible outcomes of a decision reflected in the change of the level of the initial wealth, shown as: $W+\varepsilon_1$ with the probability p and $W + \varepsilon_2$ with the probability $1-p$, then the expected utility $E(U(W + \varepsilon_i))$, i=1,2, may be determined in the following way:

$$E(U(W+\varepsilon_i)) = pU(W+\varepsilon_1) + (1-p)U(W+\varepsilon_2) \tag{4}$$

[1] The mathematical aspect of the decision-making issue and the axiomatic approach to the preference relation is detailed in a paper by Fishburn, P. (1999) Preference structure and their numerical representation. Theoretical Computer Science, 217, 359-383.

In case decisions do not affect the change in wealth, then the expected utility $E(U(W+\varepsilon_i))$ will be equal to the "fair" wealth utility U(W), which may be regarded as the certainty equivalent and which may be determined in the following way:

$$U(W) = U(W + p\varepsilon_1 + (1-p)\varepsilon_2) \tag{5}$$

where $E(\varepsilon_i) = p\varepsilon_1 + (1+p)\varepsilon_2 = 0$, and the individual with such an attitude towards a risk is considered to be indifferent to risks.

In common cases, when an individual is not prone to risk taking, the utility function curve is concave (Fig. 1 on the left), which means that $U(W) > E(U(W+\varepsilon_i))$ is true, whereas in the opposite case (Fig. 1 on the right), the curve may be convex – when an individual is inclined to taking risks.

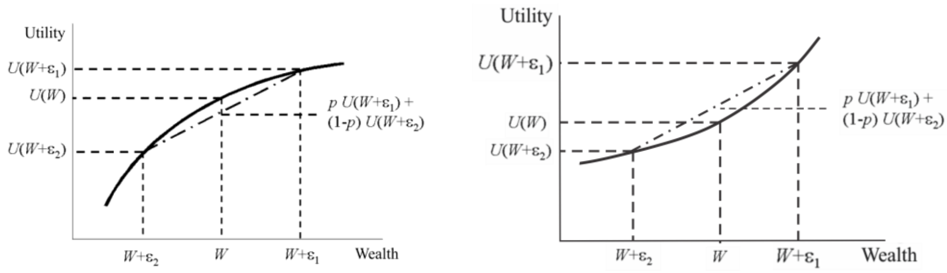

Fig. 1 Utility function: concave (left) and convex (right)

Theoretical and empirical research has shown that the most frequent forms of the investors' utility functions are quadratic function, power function and exponential function (Campbell & Viceira, 2001:19)[2], and they can be determined by the following formulas:

1. Quadratic utility function $U(W) = aW - bW^2$
2. Exponential utility function $U(W) = -\exp(-\alpha W)$
3. Power utility function $U(W) = \dfrac{W^{1-\gamma} - 1}{1-\gamma}$

However, the mathematical foundation of the expected utility depends considerably on the characteristic of independence, which implies the probability linearity. One of the most famous paradoxes which disproves the characteristic of independence in practice is Allais's paradox (Allais & Hagen, 1979). This paradox can be observed in the following experiment: The supposition is that there are three possible lottery wins: the first prize – 500,000,000$, the second prize – 100,000,000$ and the third prize – 0$, and that there are two possible scenarios. The first scenario offers the possibility of choosing one of the two lotteries: lottery A with the following probability of wins A = (0, 1, 0) and lottery B with the following possible outcomes and their probabilities respectively B = (0.1; 0.89; 0.01). The second

[2] A survey of various investors' utility functions and a relation of utility and aversion towards risks is to be found in a paper by Petrović, E., Radović, O., Stanković, J. (2013) The impact of Risk Aversion on Individual Investors Investment Decision-Making Process, Strategic Management, Volume 18 (1): 3-14 and Avdalović, V., Petrović, E., Stanković, J. (2016) Rizik i osiguranje, Ekonomski fakultet, Niš.

scenario presupposes two lotteries, as well, but the probability of their wins are as follows: A' = (0; 0.11; 0.89) and B' = (0.1; 0; 0.90). Starting from the characteristics of the preference function in the expected utility concept, it means that if decision makers prefer A instead of B in the first scenario, then they will prefer A' rather than B' in the second one. However, the largest number of respondents chose lottery A in the first scenario and lottery B' in the second one (Kahneman & Tversky, 1979), which proves the fact that the independence characteristic is "incompatible with the preference for security in the neighborhood of certainty" (Allais, 2008: 4). "Far from certainty" individuals behave rationally, after all, and estimate the expected utility of the outcome in accordance with the expected utility theory (Andreoni & Sprenger, 2010), so that this paradox may be misunderstood.

Besides this well-known criticism, there is a number of new critical papers that are based on the behavioral economy and that emphasize the fact that a strict application of the optimization method of the expected utility may create some intuitively unacceptable conclusions in certain cases (Rabin, 2000). Such anomalies become significant when the expected utility theory is applied to making decisions concerning society as a whole, since they are expressed through the ethical acceptability of decisions on the community level.

2. Determinants of Extreme Risks

Contemporary eco-social systems are exposed to a great number of correlated risks which represent a potential hazard for the survival of the whole global system. Despite the fact that risks could be categorized in different ways, the particularity of extreme risks is reflected in their frequency and intensity. Thus, extreme risks or catastrophes are all risky situations with a low probability of occurrence and enormous and unforeseen consequences (Posner, 2004). Regarding the fact that the quality and amount of available information on the causes and effects of particular risks limits the possibility of risk predictions, the prospects of generally accepted scientific methods to describe and foresee the expected effects of these risks have been challenged. With no consensus on the issue of the loss threshold, which determines whether a risk is extreme or not, all the risks whose consequences surpass some normal experience of any social system are grouped in this category. Macro catastrophes are, for example, considered to be all the events whose consequences include at least one of the following: (1) death of more than 1,000 people or disease/injury of more than 5,000 people; (2) interruption of usual daily activities on a particular territory lasting longer than one week; (3) destruction of property and infrastructure whose damage is more than 10 billion US dollars; (4) direct and indirect loss worth at least 1% GDP (Coburn et al, 2014).

The extreme risk intensity is determined by a system vulnerability and exposure to a particular risk and it affects both the eco-social system as a whole and the economic and financial subsystem. It thus represents dynamic and changeable determinants of extreme risks whose impact on the system's capacity to depreciate a particular risk may change in time and space.

The concept of vulnerability is an analytical tool which determines the level of sensitivity of physical and social systems to damage and weakness, as well as a normative framework for defining the activities aimed at a wealth increase by a risk reduction (Adger, 2006). Vulnerability may be defined as a probability that a system, subsystem or their component parts may suffer a loss due to a risk exposure (Turner et al., 2003). Depending on the field of research, vulnerability may be defined in various ways; however, it is usually understood as the function of exposure, sensitivity and adaptive capacity, which may be quantifiably presented in the following formula (Metzger, Leemans & Schroter, 2005: 255):

$$V(es,x,s,t) = f(E(es,x,s,t), S(es,x,s,t), AC(es,x,s,t)) \tag{6}$$

in which the symbols denote the following: V – vulnerability, E – exposure, S – sensitivity, AC – adaptive capacity, while es stands for the products and services of eco-systems used by sectors or a particular part of the system x in the context of the scenario s in the period of time t.

Since the potential impact (PI) of a risk is the function of exposure (E) and sensitivity (S), as presented in the following equation:

$$PI(es,x,s,t) = f(E(es,x,s,t), S(es,x,s,t)) \tag{7}$$

then, vulnerability (V) may be shown as the function of the potential impact (PI) and adaptive capacity (AC) in the following way:

$$V(es,x,s,t) = f(PI(es,x,s,t), AC(es,x,s,t)) \tag{8}$$

This simplified version of vulnerability displays and reveals the relation between various elements of the concept, but this concept operationalization is quite complex. Namely, when estimating the system vulnerability, three basic characteristics of the concept have to be considered (O'Brien, Sygna & Haugen, 2004: 3-4): (1) since the risks affecting ecosystems and their subsystems are different, it may be stated that vulnerability represents an inherently differential concept; (2) vulnerability being scale-dependent, the vulnerabilities of an individual, state, region, community and social group are all observed differently; (3) vulnerability is a dynamic concept since it may change over time depending on the system structure transformations and its functions. Considering the fact that it is a multidimensional concept, the vulnerability of an ecosystem may be observed from ecological, economic and social aspects. Moreover, current efforts to measure vulnerability tend to be *ex-ante* and are aimed at disaster risk reduction unlike the *ex-post* assessment and management of risk and vulnerability which main objection is recovery after the disaster.

The fact is that a rapid technological and economic advancement in the second half of the 20th century has changed the frequency and intensity of known risks while simultaneously creating new ones. Considering the fact that the characteristics of catastrophes are prone to change in space and time, the extreme risks classification has become a complicated task. Risks of catastrophes may be roughly divided into natural and human-induced hazards (Table 1). Natural hazards may be caused by atmospheric, geological, hydrological, seismological or any other natural dangers, as well as other external dangers out of the ecosystem of the planet Earth. The range of human-induced hazards is wider so that the number of human-induced catastrophes has been significantly greater than the number of natural disasters in the past decades.

The interdependence of the ecosystem elements has conditioned the correlation of the extreme hazards' causes and consequences so that the difference between these categories of risks is not so clear. The scientific studies usually define catastrophic risks, which may endanger the functioning of the system, as natural catastrophes caused by earthquakes, hurricanes, volcanic eruptions, etc. However, a lot of catastrophes that affect the eco-social

Table 1 Types of catastrophic risks to the ecosystem

Natural	Human-induced
▪ Natural catastrophes ▪ (earthquakes, storms, tsunami, floods, volcanic eruptions) ▪ Climatic catastrophes (drought, extremely low or high temperatures) ▪ Ecological catastrophes (rise in the sea level, fires, pollution, atmospheric changes, changes in the ocean ecosystem) ▪ External risks (meteor impact, solar storms) ▪ Epidemics (epidemic of human diseases, epidemic of animal diseases, epidemic of plant diseases)	▪ Financial shocks (price bubble of assets, financial irregularities, bank run, public debt, banking crises, stock market crash) ▪ Trade dispute (strikes, sanctions, nationalization, war rate, cartel pressures) ▪ Geopolitical conflicts (conventional wars, nuclear wars, civil wars, political influences by the external powers) ▪ Political violence (Terrorism, separatism, organized crime, civil unrests, assassinations) ▪ Technological catastrophes (nuclear catastrophes, industrial accidents, infrastructure collapse, technological accidents, Internet threats) ▪ Humanitarian catastrophes (famine, drinking water shortage, refugee crisis, collapse of social programmers system)

Source: Coburn et al. (2014)

system may be ascribed to human activity, such as: famine, resources shortage, wars, climatic changes and epidemics, financial instability and economic crises (Helbing, 2012). It is the governments that play the key role in such situations since they have to establish and develop the system resistance and protection from catastrophic risks, whereas the decisions on prospective measures imply an economic analysis of benefits and costs, as well. Besides the already mentioned particularities of manifestations of these hazards, such decisions are also determined by the risk aversion of decision-makers. The way in which social and political institutions influence the preferences of individuals and the way in which individual preferences are aggregated in a social choice represent the crucial components of the decision-making process which often exceed the framework of the expected utility theory. Therefore, what follows is a survey of the basic flaws of this concept in the presence of extreme risks, as well as the consequences of decision-making.

3. Expected Utility Concept under Extreme Risks

Regarding the intensity of risks, individuals, institutions and creators of macroeconomic politics are very frequently confronted with different options and alternatives in the presence of extreme risks in various spheres of social life (such as finances, insurance, traffic safety measures, health protection politics, measures for avoiding and overcoming the consequences of economic crises, nuclear and climatic catastrophes). The combination of the probability distribution of possible heavy-tailed outcomes and the power utility function of a decision-maker does not only imply a limitless expected utility but also a limitless expected marginal utility, which would mean that an individual should postpone any kind of consumption at present in order to avoid potential catastrophic damages in the future (Ikefuji et al, 2010). This phenomenon is called "tyranny of catastrophic risk" and occurs when the utility function is not limited from below, i.e. $\lim_{W\to 0} U(W) = -\infty$, which can be shown in a simplified model (Buchholz & Schymura, 2012: 3-6) as follows:

Supposing the investment alternative in question had only two outcomes, the optimistic scenario outcome being $W + \varepsilon_1 = 1$, and the pessimistic scenario outcome varying and, in the worst possible situation, which is the low loss limit, equaled 0, i.e. $1 > W + \varepsilon_2 \geq 0$. The outcome in which $W + \varepsilon_2 = 0$ represents the case of absolute catastrophe, that is a total wealth loss, while the set of outcomes, whose values are in the range between 0 and $W+\varepsilon_1$, are the situations in which a part of wealth is to be lost in case of risks. If the probability of the optimistic scenario realization is denoted as p, then the probability of pessimistic scenarios realization is $1-p$, i.e. in case of $W+\varepsilon_2$ it can be marked as p_2. The probability of the outcome $W+\varepsilon_2$, whose value is either 0 or inclining to 0, is also very small, e.g. $p_2 = 10^{-6}$. The economic intuition would require that these risks be considered when deciding, but with acceptable limits, since a rational investor would not want to lose the more probable earnings for the sake of the protection from the risks extremely unlikely to occur. Otherwise, "the tyranny of catastrophic risks" may completely terminate normal activities. If the same situation is observed on the level of society supposing that a decision-maker negates the possibility of the optimistic scenario realization by giving priority to pessimistic scenario avoidance, society will, due to an increased level of protection from catastrophic risks, miss the chances to enlarge the wealth and well-being of individuals. Since the decision-makers' preferences concerning risks are different and determined by the utility function, it may be assumed that the decision will depend on the utility function characteristics.

Supposing the individual's preferences towards risks might be described by the utility function $U(x_i)$, which is defined for all outcomes as x_i ($x_i = W + \varepsilon_i$), $x_i > 0$ and for which it is true that $U'(x_i) > 0$ and $U''(x_i) < 0$. Observing the set of investment alternatives with the outcomes x_i, $i = 1,2,\ldots, k + 1$, and a discreet probability distribution of the outcome x_i, it may be concluded that the expected outcome of the considered alternatives is $P = ((x_1, p_1);\ldots;(x_{k+1}, p_{k+1}))$, while the expected utility, which may be regarded as the certainty equivalent, $m_u(P_x)$ represents a sum of the expected utility of all the outcomes pondered by appropriate probabilities. The expected utility of such an outcome may be presented in the following way:

$$U(m_u(P_x)) = \sum_{i=1}^{k+1} p_i U(x_i) \tag{9}$$

The state $k + 1$ represents the state of an expected catastrophic risk whose probability of occurrence p_{k+1} may vary, but is inclined to zero. In order to focus only on the influence of various levels of probability p_{k+1} on the assessment of investment alternatives, we will suppose that the potential probabilities of the states in which catastrophic damages $\bar{p}_i$, $i = 1,2,\ldots, k$ are constant. If the probability p_{k+1} is known, then the probability of the outcome realization is x_i, $i = 1,2,\ldots, k$, $p_i(p_{k+1}) = (1 - p_{k+1})\bar{p}_i$. For any combination of potential probabilities $(\bar{p}_1,\ldots,\bar{p}_k)$

and outcomes $(\bar{x}_1,\ldots,\bar{x}_k)$ in "normal" situations, i.e. the situations in which catastrophic risks are highly unlikely to occur, the expected outcome of the alternative with a potentially catastrophic outcome may be determined in the following way: $P_x(x_{k+1}, p_{k+1}) = ((\bar{x}_1, p_1(p_{k+1}); ...; (\bar{x}_k, p_k(p_{k+1}); (x_{k+1}, p_{k+1})))$. If $p_{k+1}= 0$, such an alternative may be identified with the alternative with an expected catastrophic outcome $P_g = ((\bar{x}_1, \bar{p}_1);\ldots;(\bar{x}_k, \bar{p}_k))$. In general, every project $P = (x_{k+1}, p_{k+1})$ represents a combinations of a catastrophic outcome and an anticipated P_g. Therefore, it can be concluded that "the tyranny of the catastrophic risks" is valid for the given utility function $U(x_i)$ if for any P_g there exists a series of alternatives with potentially catastrophic outcomes $(P_x^{(n)})_{n\in N}$ in which $\lim_{n\to\infty} p_{k+1}^{(n)} = 0$ and $\lim_{n\to\infty} m_u(P_x^{(n)}) = 0$. In other words, it is possible that the expected outcome utility of the considered alternative is inclining to zero even when the probability of the catastrophic risk is inconsiderably small. In that situation, the catastrophic risk, regardless of the probability of its occurrence and expected outcome, largely influences the estimation of the alternative. This phenomenon occurs whenever the utility function $U(x_i)$ is not limited from below, which means that it cannot be claimed that the expected utility theory necessarily underestimates the low probability outcomes, but that it is basically dependent upon the utility function type. The utility function limitation depends on the level of risk aversion, while a sufficient level of risk aversion determines the value of the relative risk aversion coefficient $xA(x) \geq 1$ for every $x > 0$.

On the other hand, if the utility function $U(x_i)$ is limited from below, then each series of alternatives with potentially catastrophic outcomes $(P_x^{(n)})_{n\in N}$, for which $\lim_{n\to\infty} p_{k+1}^{(n)} = 0$ and $\lim_{n\to\infty} m_u(P_x^{(n)}) = 0$, has an expected utility convergent to the expected utility without the catastrophic risk $\sum_{i=1}^{k+1} \bar{p}_i U(\bar{x}_i)$, if the catastrophic damage $x_{k+1}^{(n)}$ is limited from above.

Both cases prove that the expected utility theory cannot be valid in the presence of extreme risks because it will either induce the extreme risks dominance or completely negate their existence. A proper treatment of these hazards should include the fact that the price individuals and society are prepared to pay in order to avoid catastrophic consequences is limited, regardless of the risk impact and consequences irreversibility (Ikefuji et al., 2010). The aforementioned assertions indicate that the expected utility model should be modified so as to avoid all the weaknesses of the model concerning the supposed probability distribution of outcomes and utility function forms.

4. Decision-making under Extreme Risks

The expected utility theory has been criticized and modified by a great number of authors. The supposition that decision-makers are familiar with the probability distribution of the realization of considered alternatives has been particularly discussed as a serious flaw. Regarding the fact that in most cases investors are not given an opportunity to choose from the options with objective probabilities, one of the most influential versions of this theory is, in fact, the theory of the subjective expected utility (Savage, 1972). The concept of decision-making is based on the utility function, but the objective probabilities are replaced by the subjective ones, i.e. the preference relation is characterized by the following: ordering of the options, sure-thing principle, weak comparative probability, non-degeneracy, continuity in low-probability events and uniform monotony (Al-Najjar & De Castro, 2010). Similarly to the previous theory, this theory was not empirically validated. A simple experiment, which proves that individuals prefer games (lottery) with known (objective) outcome probabilities, implies the ambiguity aversion on the part of a decision-maker, known as Ellsberg's paradox (Ellsberg, 1961; Keynes, 1921). This paradox was discovered while conducting the following experiment: bowl A contains randomly placed 50 red and 50 black balls, and bowl B contains 100 balls, placed at random, as well, but with no information on the exact number of red and black balls in it. The prize goes to anyone who accidentally picks up the ball of the previously specified color. The majority of the experiment participants preferred to pick up from bowl A regardless of the given color, which directly disproves the postulates of the theory of the subjective expected utility. Namely, if a respondent is required to pick up a red ball and they choose to do that from bowl A, it lowers the probability of picking up a red ball from bowl B by ½. On the other hand, following the same logic, it means that the probability of picking up a black ball from bowl B is higher by ½, since the sum of probabilities of both outcomes has to equal 1. Anyway, the experiment results indicate that the ambiguity aversion is a very powerful and robust phenomenon.

Different non-expected utility theories have explained the choice of investors by altering or completely omitting a questionable feature of independence, i.e. the principle of a rational choice certainty. The most famous ones are: generalized expected utility theory (Machina, 1982), weighted expected utility theory (Fishburn, 1983), rank-dependent utility theory (Quiggin, 1982), prospect theory (Kahneman & Tversky, 1979), cumulative prospect theory (Kahneman & Taversky, 1992), regret theory (Loomes & Sugden, 1987), dual utility theory (Yaari, 1987), and many others (Starmer, 2000).

The issue of extreme risks is discussed in the theory of rank-dependent utility, which supposes that individuals rank their options according to the cumulative distribution function, not according to the subjective probabilities. Maintaining all the aforementioned features of the preference relation of the rational investors and being based on the rank of the probable outcomes of the options x_i in the rising order, this theory offers the solution of maximizing for the following targeted investors' functions

$$E(U(x_i)) = \int_0^{\infty} \frac{\partial g(\Phi(x_i))}{\partial x_i} U(x_i)dx_i \tag{10}$$

where $\Phi(x_i)$ denotes the probability that the outcome x_i will be lower than a value p, while the $g(\cdot)$ function of ranking probability of possible outcomes is such that $g(0) = 0$ and $g(1) = 1$.

Contemporary attempts at improving the expected utility theory are basically concerned with the decision-making optimization in the cases of climatic changes. Weitzman's research on extreme climatic changes (Weitzman, 2009) presumes the presence of a lower limit of consumption determined by the parameter of the statistical value of life. He proves that the expected discount rate approaches infinity, but he also states that it is very difficult to determine the value of this parameter. Ikefuji et al. (2010) define sufficient and necessary conditions for the expected utility model in the presence of extreme risks by considering various utility functions. Not setting any limits to the probability distribution, they conclude that the generally accepted power utility function should not be considered in the process of deciding if there exists a non-negligible risk model. The exponential function and Pareto function of utility are more acceptable instead.

Despite possible improvements, the concept of expected utility predicts average reactions to the pondered average risk, where the point of pondering is the risk probability

(Chichilnisky, 2011: 5). The prospect theory explains that individuals overestimate the potential losses in reality, while simultaneously underestimating potential wins in the presence of risks, and this asymmetry cannot be explained by the theoretical wealth function nor by the generally accepted risk aversion function (Kahneman & Tversky, 1979). The decision-maker's behavior, whose choice is conditioned by both risk aversion and possible outcome ranks, is explained by the cumulative prospect theory (Kahneman & Tversky, 1992) and it may be formally shown as the problem of maximizing of the following function

$$E(U(x_i)) = \int_{-\infty}^{0} \frac{\partial g^-(\Phi(x_i))}{\partial x_i} U^-(x_i)dx_i + \int_{0}^{+\infty} \frac{\partial g^+(\Phi(x_i))}{\partial x_i} U^+(x_i)dx_i \tag{11}$$

Although this theory offers a certain level of flexibility in modeling the decision-making process in relation to the expected utility theory, the function $g^-(\cdot)$, $u^-(\cdot)$, $g^+(\cdot)$ and $u^+(\cdot)$ is extremely difficult to identify.

Being in an extremely risky situation, an individual does not think rationally since the decision-makers are prone to overestimating the probability of extreme outcomes. Research has shown that deciding under the pressure of extreme emotions results in extreme and simplified reactions, such as "fight or run", not in ranking the alternatives on the basis of their probability, as described by the theory of expected utility. Therefore, it may be proved that ranking alternatives according to von Neumann and Morgnestern in the presence of extreme risks is insensitive to the low probability outcomes (Chichilnisky, 2011), i.e. it follows:

$$E(U(x)) \geq E(U(y)) \Leftrightarrow \exists \varepsilon > 0,\, \varepsilon = \varepsilon(x, y) : E(U(x')) \geq E(U(y')) \tag{12}$$

each x' and y' are such that $x' = x$ and $y' = y$, except in case of $A \subset R : \mu(A) < \varepsilon$.

If the ranking of alternatives is focused on the outcomes with a low frequency of repetition, then this kind of ranking is insensitive to the outcomes with a high repetition frequency (Chichilnisky, 2011), i.e. it follows:

$$E(U(x)) \geq E(U(y)) \Leftrightarrow \exists M > 0,\, M = M(x, y) : E(U(x')) \geq E(U(y')) \tag{13}$$

each x' and y' are such that $x' = x$ and $y' = y$, except in case of $A \subset R : \mu(A) > M$.

With the purpose of treating "average" outcomes and the outcomes with an extreme level of probability in the same way, Chichilnisky (1996, 2009, 2011) proposes new axioms of the preference relation, such as: linearity and continuity, sensitivity to low probability outcomes and to frequent outcomes, and she formulates the decision-making problem as the maximizing of the following function:

$$E(U(x_i)) = \lambda \int_{X \in R} U(x_i) d\mu(x_i) + (1-\lambda)\Phi(U(x_i)) \tag{14}$$

for $\lambda \in (0,1)$ and the final additive function Φ, $\Phi: L \to R$, in which L represents a set of alternatives $L = L_\infty(R)$.

The first part of the formula (14) corresponds to the expected utility function, where frequent outcomes are ranked, while the second part of the function is determined by the probability measure which ranks low probability outcomes, i.e. the measure with heavy tails. Thus, the catastrophic risks are ranked more properly, while the function is sensitive to both low and high frequency outcomes. This approach offers various results in relation to the classical expected utility theory, but all the aforementioned models have not yet been applied in the investment theory and practice (Grechuk & Zabarankin, 2014).

CONCLUSION

The consequences of the financial markets instability, as well as higher vulnerability and exposure of socio-economic systems to catastrophic risks induced by either natural causes or human activity cannot be ignored in the models of decision-making optimization. Although the economics of heavy-tailed distributed risks raises difficult conceptual issues that cause the cost-benefit analysis to appear more subjective, its application should not be evaded. Economic analysis in these circumstances should consider probability distribution of such events, interaction between uncertainty and temporal dimension and model of human behavior. Preferences of decision makers, which integrate all challenges of the analysis, are usually modeled within expected utility framework.

Although the relevance of catastrophic risks cannot be neglected, it is also necessary to consider the fact that the price of their reduction that individuals and/or society are ready to pay is limited. Classical optimization models include mainly average risks, which makes them inadequate in the presence of extreme risks. The concept of the expected utility theory may be thus seriously challenged because of the phenomenon of "the tyranny of catastrophic risks". If a certain utility function is not limited from below, then even a minimum probability of the catastrophic damage may induce a complete dominance of the catastrophic risk. The price of preventing catastrophes being very high, these decisions lead to a complete decline of consumption or investment, at the moment of making a decision in order to prevent a possible absolute loss. An alternative solution within the concept of the expected utility theory is that the utility function be limited from below, which may also have extreme consequences. Namely, extreme risks which are characterized by a low level of probability may be completely dismissed in the process of decision-making and thus inadequately considered, which may be also regarded as unethical in case of decisions related to the whole society. Possible solutions within the expected utility theory are either to introduce proper threshold levels for the extreme risk, or to completely abandon this concept and accept the unexpected utility theory. However, the implementations of alternative concepts in actual situations have not been so frequent due to its complexity.

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TEORIJA OČEKIVANE KORISNOSTI U USLOVIMA EKSTREMNIH RIZIKA

Teorija očekivane korisnosti pruža okvir za modeliranje izbora racionalnog pojedinca čiji je cilj maksimiranje očekivane korisnosti uz date preferencije prema riziku. Međutim, ekstremni rizici, kao što su, na primer, krah berze ili elementarna nepogoda, značajno utiču na funkciju raspodele verovatnoće ishoda dodajući težinu repovima raspodele. U takvim slučajevima, primena teorije odlučivanja zasnovanoj na očekivanoj korisnosti je izuzetno osetljiva na pretpostavke o funkciji raspodele verovatnoće. Stoga će u ovom radu biti dat pregled modela odlučivanja u okviru teorije očekivane korisnosti u uslovima ekstremnih rizika.

Ključne reči: *očekivana korisnost, ekstremni rizik, odlučivanje*