FACTA UNIVERSITATIS
Series: **Electronics and Energetics** Vol. 36, Nº 3, September 2023, pp. 449-464
https://doi.org/10.2298/FUEE2303449P

Original scientific paper

DETERMINING THE ACTUAL REDUCTION FACTOR OF DISTRIBUTION CABLE LINES WITH APPLIED CROSS-BONDING

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Abstract. *The considered problem appears as the consequence of the fact that many metal installations surrounding power lines in urban areas are situated under the surface of the ground and cannot be visually determined or verified. This paper presents the methodological version that enables solving the problem in the case of cable lines with cross-bonding, but it can also be applied with higher accuracy to cable lines with no applied cross-bonding than the existing one. It is based on the test measurement of currents appearing in two cable line phase conductors during a simulated ground fault in the supplied HV substation. By using the here-presented methodology it is possible to put the grounding problem of high-voltage distribution substations into realistic frameworks. Its application in practice shows that the actual conditions for solving this problem are much more favorable than was considered before.*

Key words: *substation; cable sheath; ground-fault current; grounding impedance; inductive coupling; safety conditions*

1. INTRODUCTION

During a ground fault in an HV (high-voltage) distribution network, high currents and raised potentials appear at places where they normally do not exist. Under such conditions, an HV cable line represents a very complex electrical circuit containing many conductively and inductively coupled elements. At the place of a ground fault, the fault current leaves the faulted phase conductor and returns to its sources in the power system by using all available paths. Because of that, a ground-fault current in an HV substation divides into multiple mutually different flows. One of them is of special practical importance. This fraction dissipates into the surrounding earth through the grounding system of the supplied substation and produces all potentials and potential differences (touch and step voltages) that can be dangerous and harmful within and in the vicinity of HV substations. Due to this, a special coefficient named the reduction factor of a feeding line has been introduced in the professional literature and corresponding technical standards [1, 2].

Received March 28, 2023; revised April 27, 2023; accepted May 5, 2023
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The determination of the reduction factor is nowadays a relatively easy engineer's task at the design stage of an HV substation. In the case of an overhead feeding line, it can be determined only by using the existing analytical expressions [1, 2]. However, in the case of cable lines the analytical expressions that enable taking into consideration the existence of all three metal sheaths in determining their reduction factor, for the fault anywhere along a cable line, were developed not so long ago [3, 4]. On the basis of these expressions, cable lines were introduced in the corresponding international technical standard [2]. However, in the case of cable lines with applied cross-bonding, the problem becomes still more complex so it was initially solving with the help of a computer [5], and the corresponding analytical expressions were derived only recently [6].

However, the here considered problem appears because by using the mentioned analytical expressions it is not possible to take into consideration the fact that cable lines, almost without any exception, are used in urban and suburban areas, i.e. there, where many other metal installations already have been existed. In the case of a ground fault these installations will spontaneously participate in returning the ground-fault current into the power system and in this way influence ground-fault current distribution.

The determination of the reduction factor of a distribution cable line by taking into account the surrounding metal installations was enabled for the first time by the investigations performed in the HV distribution network of Beograd [7]. The achieved solution is based on the results of the experimental measurements and an analytical procedure that enables substituting all surrounding metal installations, from the standpoint of their inductive influence on ground-fault current distribution, by only one equivalent conductor. The physical appearance and spatial position of this fictitious conductor are such that it represents a cylinder surrounding all HV cable line conductors along their entire length [7-9]. The final research results showed that the fraction of the ground-fault current flowing solely through the earth, in a typical urban area, is three to five times smaller than it has been considered before [6-8]. Certainly, this fact throws completely new light on the whole grounding problem of HV/MV (middle voltage) substations located in urban areas and dramatically changes our earlier perception concerning the magnitude of this problem.

Somewhat later this method was modified in such a manner that the introduced equivalent conductor substitutes all surrounding metal installations including the metal sheaths of the considered cable line [10-11]. This time, it is imagined as a cylinder surrounding only the phase conductor carrying a simulated ground-fault current. As well as in the previous case this conductor is completely defined by only two of its parameters. One of them is the radius of this cylindrical conductor, whereas the other one is its longitudinal resistance. However, with this methodology version, the problem was not solved for all cable lines because it cannot be applied in the case of cable lines with applied cross-bonding, which is the most common case in the contemporary HV and EHV (extra high voltage) distribution networks [14]. This case is more complex and more difficult for solving because the cable metal sheaths are cross-bonded at certain places along these cable lines and in this way, they change their spatial positions toward the faulty phase conductor.

Here developed version of the methodology for compensation of deficiency of unknown but relevant data [6-13] (or, shortly of Popović methodology), enables determining the actual reduction factor in the case of cable lines with applied cross-bonding. The data about the actual reduction factor of the feeding cable lines is indispensable for the final judgment concerning the achieved safety conditions within and in the vicinity of a supplied substation in cases where the preliminary estimations are not sufficient for such judgment

[6, 8]. Because of that, this paper can be also considered as a logical continuation of the work on solving the problem of testing and assessment of safety conditions in cases where the considered cable line in addition to the tested substation supplies also one or more transit substations. Namely, by performing the preliminary testing we obtain the actual grounding impedance of the tested substation and transferred potentials including all other relevant potential differences (step and touch voltages) on its grounding system [6]. However, in some critical cases, this is not sufficient for a definitive judgment concerning the safety conditions within and in the vicinity of the supplied substation. In such cases, an accurate estimation of the safety conditions is indispensable. It can be achieved by taking into account the inductive influence of all metal installations surrounding the feeding line in any section of its length [7, 8].

Also, based on the actual reduction factor of the feeding line it becomes possible the accurate determination of the highest potential that can appear during a ground fault on the grounding system of the supplied substation, and that can be harmful to the embedded electronic system. Measurements performed for this purpose by the method "fall-of-potential" (e.g. [1]) cannot give reliable results due to the small values of the currents that can be injected by this method through the tested grounding system into the surrounding ground, especially in the case of long feeding line supplying one or more transit substations [6].

2. Problem Description

Certainly, determining the actual reduction factor of an HV or EHV cable line implies that this line and its supplied substation are already constructed and that they are at the stage immediately preceding their putting into operation. Due to this, in most cases, the value of the actual reduction factor is necessary only for the final verification of the required safety conditions within and in the vicinity of the supplied substation [1]. Exceptionally, if conducted test measurements show that the prescribed safety criteria are not achieved, the only additional measure that could be in that case applied is improving the feeding line reduction factor by laying a copper wire in the same trench with the feeding cable line [15]. Certainly, for the more cost-effective implementation of this measure, it would be desirable for this trench not to be already covered.

For the sake of an illustration of the considered problem, the main ground-fault current fractions in HV substations located in urban areas are shown in Fig. 1.

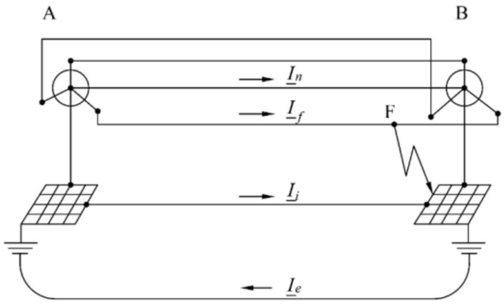

Fig. 1 Main fractions of a ground fault current

Based on Fig. 1, it is not difficult to notice that in the case of a ground fault two fractions of the ground-fault current leave the power system. One of them is dissipated into the surrounding earth through the grounding system of substation B, while the other is induced in the metal installations surrounding the feeding line. Because of that, both of these ground-fault current fractions if they are too large can cause dangerous and harmful effects in the immediate environment of the supplied substation and its feeding line.

Since the mutual separation of currents, $\underline{I}_i$ and $\underline{I}_e$ occurs along many external grounding electrodes (metal sheaths of outgoing MV cable lines) and under the surface of the ground, none of these currents can be determined only by calculations or only by measurements [16]. Also, each of the surrounding metal installations, together with the earth as the common return path, forms one electrical circuit, while all of them form a very complex and large electrical circuit with many conductively and inductively coupled elements. Due to that, such an electrical circuit is not easy to analyze even under the unrealistic assumption that all relevant data about it are known.

Besides that, the equivalent soil resistivity of the area surrounding a feeding cable line is relevant, but uncertain data because in urban conditions it cannot be exactly determined. Although there are several methods to measure soil resistivity [1], no one is applicable in urban areas. This practical impossibility stems from the fact that the surface of urban areas is already covered by buildings, streets, sidewalks, and many other permanently constructed objects; whereas many known and unknown metal installations already exist under the surface of the land belonging to an urban area.

The problem of determining the actual reduction factor was formerly solved thanks, inter alia, to the fact that the longitudinal mutual impedances between phase conductors and belonging metal cable sheaths remain unchanged along the whole cable line length [10, 11]. However, this is not the case with cable lines with applied cross-bonding and this is the reason because of which the previously developed methodology versions were not applicable in their case. Thus, the problem that should be solved can be defined in the following manner: how can we determine the actual reduction factor when the cable line is with applied cross-bonding?

3. Complete Equivalent Circuit of an HV Cable Line during a Ground Fault

Different metal installations like they are: sheaths of different types of distribution cable lines, neutral conductors of the LV (low voltage) networks, steel water pipes, foundation ground electrodes of relatively new buildings, etc. are situated in a relatively narrow and strictly defined corridor along and under each of urban streets. Some of them are insulated toward the earth, whereas some others are in direct and continuous contact with the earth. Through the mandatory terra-neutral (TN) grounding system in LV networks and consumer installations, some of them are interconnected in each of the buildings arranged along each street, whereas some others are interconnected at each MV/LV (customer) substation. As such, independently of their basic functions and spatial dispositions, during a ground fault in an HV distribution substation, these installations act as elements of a very large and spontaneously formed grounding system [16].

Now, let us assume an HV cable line consisting of three single-core cables, with all three sheaths grounded at both line ends and with no applied cross-bonding. Also let us assume the most general case, from the standpoint of their spatial formation, as is shown in Fig. 2.

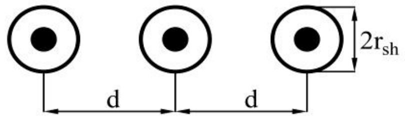

Fig. 2 Single-core cables laid in a flat formation

Also, let us assume that the total number of surrounding metal installations, including the cable line sheaths, is an arbitrarily large number, *N*. Then, the complete electrical circuit of this HV cable line, under the conditions of a simulated ground fault in supplied substation B, can be, according to e.g. [10, 11], presented as shown in Fig. 3.

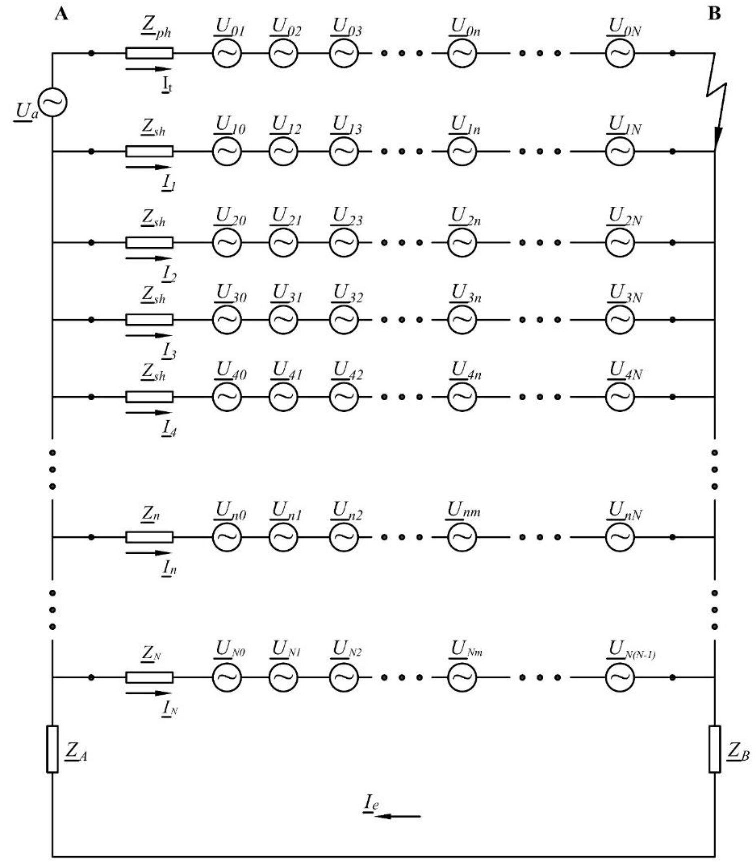

Fig. 3 Complete equivalent circuit of the assumed HV cable line

As can be seen, the presented equivalent circuit is composed of the self and mutual impedances of all surrounding metal installations and the considered cable line sheaths including the phase conductor carrying current $\underline{I}_t$. The self-impedance of an arbitrary, n^{th}, surrounding metal installation is, according to [2], determined by:

$$Z'_n \approx R'_n + \omega_t \frac{\mu_0}{8} + j\omega_t \frac{\mu_0}{2\pi} \ln \frac{\delta_t}{r_n} \;, \;\; \Omega/\text{km}, \tag{1}$$

and in the case of metal installations with a full metal cross-section, is:

$$Z'_n = R'_n + \omega_t \frac{\mu_0}{8} + j\omega_t \frac{\mu_0}{2} \left(\frac{\mu_r}{4} + \ln \frac{\delta_t}{r_n}\right), \;\; \Omega/\text{km}, \tag{2}$$

whereas mutual impedance between two arbitrary, n^{th} and m^{th}, surrounding metal installations is determined by:

$$Z'_{nm} = \omega_t \frac{\mu_0}{8} + j\omega_t \frac{\mu_0}{2\pi} \ln \frac{\delta_t}{d_{nm}} \;, \;\; \Omega/\text{km}; \quad m \neq n \tag{3}$$

The equivalent earth penetration depth is determined by:

$$\delta_t = 658 \sqrt{\frac{\rho}{f_t}} \;, \quad (\text{m}) \tag{4}$$

Current in an arbitrary surrounding installation, I_n (Fig. 3), induces in an also arbitrary (m^{th}) surrounding installation a voltage, $\underline{U}_{mn}$, which is determined by:

$$U_{mn} = -Z_{nm} I_n \,, \qquad (\text{V/km}) \quad m \neq n \tag{5}$$

Although complete, the presented equivalent circuit is not without any idealization and approximation of the actual physical model. The fact, that some of the metal installations are grounded at certain places, mainly in each of the buildings arranged along both sides of streets, has been disregarded. Also, the fact that the metal sheaths of the considered cable line are grounded at each of the cross-bonding boxes along HV or EHV cable lines has been disregarded. These facts are not important for us because we analyze only the induced currents that, as is well known, circulate only in the axial direction through the cable line sheaths and surrounding metal installations, and as such cannot produce any potential on them toward the earth. Due to this, it is not relevant whether these installations are insulated or not and whether the introduced equivalent conductor [6-13] substituting all these installations have insulation, or not. Because of the same reason, when we determine the reduction factor of HV overhead lines, we also do not take into consideration the fact that their ground wires are grounded at each tower, e.g. [2].

Also, in practice, almost each of the surrounding metal installations has more different sections that are not laid at the same distance from the considered HV cable line i.e. in parallel with it and with any other of the surrounding metal installations. However, these facts also are not of any importance for the application of this methodology [12, 13, 16].

Finally, the value of ρ in urban areas can be only approximately estimated based on the main geological characteristics of the soil surrounding the considered cable line. However, according to (1), (2), and (3), this approximation has not any greater influence on the accuracy of this methodology because impedances $\underline{Z}'_n$ and $\underline{Z}'_{nm}$ are only slightly dependent on the specific soil resistivity, ρ.

Based on the complete equivalent circuit in Fig. 3 it is obvious that the current in any of the spontaneously formed electrical circuits contains a cumulative effect of the inductive coupling with all other surrounding electrical circuits formed by the line conductors and surrounding metal installations. This fact is utilized for solving the problem by the measurements of currents in only two of the cable line conductors. In the previously developed methodology version [10, 11], one of them is the test current through the phase conductor of one freely chosen single-core cable, I_t; whereas the other one is the current in the metal sheath of this single-core cable, I_1. However, for solving the problem in the case of cable lines with cross-bonding it is necessary to introduce in the measurement circuit one more phase conductor [12, 13].

4. Equivalent Sheath of the Single-core Cable Caring Test Current

4.1. Measurement Circuit in the Casa of Lines with Cross-bonding

In the case of cable lines with applied cross-bonding the measurement circuit necessary for obtaining the necessary data can be presented, according to [12, 13], as shown in Fig. 4.

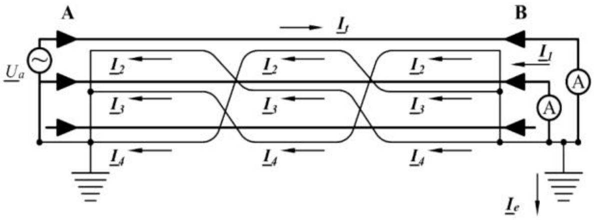

Fig. 4 The principal measurement circuit

4.2. Determination of the Relevant Parameters of the Equivalent Conductor

When we introduce an equivalent conductor (in the farther text: equivalent sheath) substituting all surrounding metal installations including all cable line sheaths, the considered cable line during the necessary measurements (Fig. 4) can be presented by a relatively simple equivalent circuit shown in Fig. 5.

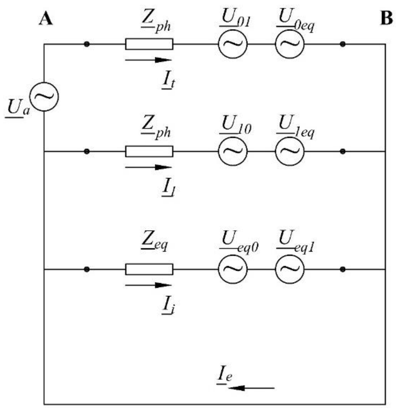

Fig. 5 Simplified equivalent circuit

As can be seen, in this circuit the grounding impedances at the ends of the line are disregarded ($\underline{Z}_A \approx \underline{Z}_B \approx 0$), which is in accordance with the definition of the feeding line reduction factor, e.g. [2]. In that way, we take into consideration only the mutual inductive influence of the faulty phase conductor and all neutral conductors on the reduction of the current dissipated through the grounding system of the supplied substation into the surrounding earth.

Based on the equivalent circuits in Figs. 3 and 5 it is obvious that the arbitrarily large number (N) of unknown induced currents is substituted by only one current, I_i. Also, numerous known and unknown surrounding metal installations including sheaths of the considered cable line are substituted by only one equivalent sheath. The relevant parameters of this sheath can be determined by using the condition that currents $\underline{I}_t$, $\underline{I}_0$, and $\underline{I}_e$ (Figs. 3 and 5) have to remain unchanged after this substitution. According to Fig. 5, this condition can be expressed by the following system of equations:

$$\begin{aligned} &1.\ Z'_{10}I_t + Z'_{ph}I_1 + Z'_{0eq}I_i = 0 \\ &2.\ Z'_{eq0}I_t + Z'_{eq1}I_1 + Z'_{eq}I_i = 0 \end{aligned} \tag{6}$$

Impedances $\underline{Z}'_{ph}$ and $\underline{Z}'_{01}$ are, according to [2], given by:

$$Z'_{ph} = R'_{ph} + \omega_t \frac{\mu_0}{8} + j\omega_t \frac{\mu_0}{2\pi}\left(\frac{\mu_r}{4} + \ln\frac{\delta_t}{r_{ph}}\right)\ , \Omega/\text{km}, \tag{7}$$

$$Z'_{01} = \omega_t \frac{\mu_0}{8} + j\omega_t \frac{\mu_0}{2\pi}\ln\frac{\delta_t}{d}\ , \Omega/\text{km}, \tag{8}$$

Based on the system of equations (6), it is interesting to note that the relative ratio between the currents $\underline{I}_t$, $\underline{I}_1$, and $\underline{I}_i$, does not depend on the voltages induced in the phase conductor carrying current $\underline{I}_t$. It means that the voltages $\underline{U}_{01}$ and $\underline{U}_{0eq}$ from the equivalent circuit in Fig. 5 are not relevant in solving the considered problem and can be omitted from this circuit. On the basis of the introduced equivalent sheath, the considered cable line and all surrounding metal installations during a simulated ground fault can be presented by a relatively simple line model whose cross-section is shown in Fig. 6.

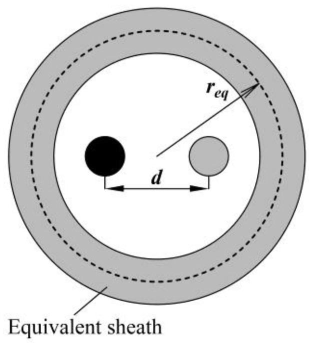

Fig. 6 Cross-section of the introduced simplified line model

The additional phase conductor in this figure is marked differently (lighter shade) to indicate that for the purpose of this measurement, it serves as one of the line neutral conductors.

On the basis, of Fig. 6, the analytical expressions for impedances Z_{eq}, Z_{0eq}, and Z_{1eq} are known in advance and are, according to [2], determined by the following expressions

$$Z'_{eq} = R'_{eq} + \omega_t \frac{\mu_0}{8} + j\omega_t \frac{\mu_0}{2\pi} \ln \frac{\delta_t}{r_{eq}} \quad , \quad \Omega/\text{km}, \tag{9}$$

$$Z'_{0eq} = Z_{1eq} = \omega_t \frac{\mu_0}{8} + j\omega_t \frac{\mu_0}{2\pi} \ln \frac{\delta_t}{r_{eq}} \quad , \quad \Omega/\text{km}, \tag{10}$$

Since currents I_t and I_1 are obtained by measurements, they can be considered as in advance known quantities, and equations (6) can be presented as:

$$\frac{Z'_{0eq} Z'_{1eq} - Z'_{eq} Z'_{01}}{Z'_{eq} Z'_{ph} - Z'^2_{1eq}} = \frac{I_1}{I_t} . \tag{11}$$

Then, since equation (11) gives the relationship between complex quantities, it can be presented in the following form:

$$\text{Re}\,\{(Z'_{0eq} Z'_{1eq} - Z'_{eq} Z'_{01}) I_t\} = \text{Re}\,\{(Z'_{eq} Z'_{ph} - Z'^2_{1eq}) I_1\},$$

$$\text{Im}\,\{(Z'_{0eq} Z'_{1eq} - Z'_{eq} Z'_{01}) I_t\} = \text{Im}\,\{(Z'_{eq} Z'_{ph} - Z'^2_{1eq}) I_1\}, \tag{12}$$

where Re and Im are the real and imaginary parts of equation (11).

In the system of equations (12), the only unknown quantities, according to (7), (8), (9), and (10), are R'_{eq} and r_{eq}. Thus, by solving this system of equations we can determine the unknown parameters R'_{eq} and r_{eq}. When we determine these parameters, for obtaining the actual reduction factor of the certain cable line we can use the analytical expression for the reduction factor of the cable line constituted of only one single-core cable. It is, according to e.g. [4, 16], determined by:

$$r_a = \frac{I_e}{I_t} = \frac{R'_{eq}}{R'_{eq} + \omega \frac{\mu_0}{8} + j \frac{\omega \mu_0}{2\pi} \ln \frac{\delta}{r_{eq}}} , \tag{13}$$

where current I_e can be determined by using the equivalent circuit in Fig. 5.

Because of the previous approximations: $\underline{Z}_A \approx \underline{Z}_B \approx 0$, it is necessary to remind that HV distribution substations are located in urban or suburban areas having many underground metal installations, that act as perfect grounding electrodes and/or as conductive connections with other grounding electrodes, like foundation earth electrodes of surrounding buildings [16]. Thus, the quantitative relationships between the impedances shown in the equivalent circuits (Figs. 3 and 5) are such that the influence of impedances $\underline{Z}_A$ and $\underline{Z}_B$ can be neglected. Based on these approximations, each current appearing in the additional phase conductor (Fig. 4) or in any other of the surrounding installations/conductors is a consequence solely of the mutual inductive coupling of each of them with all of them (Fig. 3). However, the presented methodology is developed based on the measured values of currents $\underline{I}_t$ and $\underline{I}_1$. According to this, it takes into account the fact that the real values of impedances $\underline{Z}_A$ and Z_B are somewhat larger than zero. As a result, currents $\underline{I}_t$ and $\underline{I}_1$ contain components appearing only as a consequence of potentials $\underline{Z}_A\underline{I}_e$ and $\underline{Z}_B\underline{I}_e$. However, these components of currents I_t and I_1 are negligibly small in practical conditions because of the following facts:

- current I_e is only a few percent of I_t because of strong inductive coupling between the phase conductor with current I_t on the one side and the additional phase conductor including all surrounding metal installations and cable line sheaths on the other side, and
- lengths of cable lines in HV distribution networks, L, is so large that almost always satisfies the following inequality:

$$Z_A \approx Z_B \leq Z'_{ph}L \tag{14}$$

As is shown through the measured values of test currents $\underline{I}_t$ and $\underline{I}_1$, the inductive influence of all surrounding metal installations are taken into account (Fig. 4), so that we do not need data about their individual constructive characteristics, as well as their mutual spatial positions, in solving the considered problem. Also, because of that it is not important whether the considered line is with cross-bonding or not.

Since the spatial positions of the individual single-core cables in relation to the surrounding metal installations are not identical, the presented procedure should be applied to each of the line phase conductors separately. Then, for the correct assessment of the safety conditions, we should adopt those parameters (R'_{eq} and r_{eq}) that correspond to the smallest inductive influence of the surrounding metal installations. Also, the value of equivalent soil resistivity, necessary for the application of the methodology, should be adopted as the lowest one, between several ones approximately estimated based on the main geological characteristics of the relevant urban area. Certainly, in this way, the finally determined value of the actual reduction factor is slightly higher but the error is on the side of increased safety. Also, because of the purpose of correct taking into account the favorable influence of all surrounding metal installations, it is necessary to know the following: when through the line are supplied one or more transit substations then the test circuit (Fig. 4) has to include the entire feeding line, from the supply to the supplied substation [6-8].

The here presented methodology enables the determination of the actual ground-fault current fraction dissipated into the surrounding earth through the grounding system of supplied HV substations located in urban or suburban areas. Also, it enables the determination of the actual value of the ground-fault current fractions passing through the cable line sheaths and the surrounding metal installations relevant to their thermal stress during a ground fault. Finally, it is necessary to mention that here presented methodology version can be applied also in the case of feeding lines that represent a longitudinal combination of constructively different sections (for example one overhead and one cable section).

The presented methodology version is, because of the stronger inductive influence between two phase conductors than between one of them and its sheath, more accurate than the previous ones applicable in the case of cable lines with no applied cross-bonding [10, 11]. Thus it can be said that it represents the final stage of the developing process of the methodology that enables taking into account the inductive influence of metal installations surrounding HV and EHV cable lines on the ground fault current distribution in the supplied substations [7, 8, 10, and 11]. Only on the basis of the measured values of the currents I_t and I_1 (Fig. 4) is it possible to take into account all relevant factors and parameters, including the fact that the line under consideration is with applied cross-bonding.

5. Practical Significance of the Presented Methodology

The experimental investigations in the 110 kV distribution network of Beograd show that the part of the ground-fault current passing through the grounding system of the supplied substation is 2.01, 3.65, and 5.11 times smaller in relation to its values obtained without taking into account the surrounding metal installations, e.i. only based on the corresponding analytical expressions [6-8]. So wide range of the obtained values can be explained by the fact that the experimental measurements were performed on cable lines passing through areas that are of very different degrees of urbanization. As in the ratio in which the reduction factor decreases, all potentials and all potential differences on the grounding system of the supplied HV substation also decrease one can conclude that these results throw a completely new light on the grounding problem of HV substations located in urban surroundings and dramatically change our earlier perception of its magnitude. Thus it can be said that the presented methodology enables finding more economical solutions for the grounding problem of HV substations located in urban surroundings than was possible before.

The actual reduction factor enables us to put the whole grounding problem of the supplied substation into realistic frameworks that generally lead to more economical solutions to this problem in different practical cases. Certainly, these effects are more pronounced in cases where, because of high local soil resistivity and/or a high level of short-circuit currents, special measures were considered necessary to protect personnel and the public against too-high touch voltages (e.g. bare copper wire laid in the same trench as the feeding cable line or/and in the same trench as the outgoing MV cable lines), e.g. [15]. Besides, one can expect the elimination of the strict requirement for the application of expensive MV cables with an uncovered metal sheath that acts as external grounding electrodes for supplied HV substations. For example, such a requirement was for a long time posed by the power distribution company of Beograd, intending to enable reliably solving the grounding problem of the HV substation in the condition of the high level of ground-fault currents in the supply network.

For a large number of HV distribution substations, because of practical difficulties in obtaining permission for the test circuit by using the whole feeding line, the preliminary test and assessment of safety conditions is the most rational approach [6]. These preliminary tests enable us to determine the actual impedance of the grounding system of the substations located in urban areas and the relative potential distribution on its grounding system. On the basis of them for the preliminary assessment of safety conditions, it is sufficient the feeding line reduction factor determined by calculations [2-4], as well as the actual feeding line reduction factor, but determined for the nearest transit substation that is supplied with this same cable line [6]. However, in some critical cases, it may be shown that we need an accurate estimate of the value of the maximal potential differences (transferred potentials and touch voltages) that may occur on the grounding system of the tested substation. In such cases, the determination of the actual reduction factor of the feeding cable line can enable us to avoid the additional measures for the improvement of the feeding line reduction factor and/or grounding system characteristics of the supplied substation [15, 16], if these measures are not really indispensable. Besides, the actual reduction factor of the feeding cable line allows us in some cases a preliminary assessment of the safety conditions at the design stage of the substation that will be supplied with the same cable line [6]. Finally, based on the actual reduction factor it is possible to determine the maximal potential that can be appeared during a ground fault on the grounding system of the supplied substation, and which can be harmful to the sensitive electronic equipment if the appropriate protection measures are not applied.

Since the actual reduction factor depends on numerous local factors and parameters for the designers of future HV substations and those who test the safety conditions in existing HV substations, it would be useful to have a file with data on the actual values of the reduction factor for all existing feeding cable lines in one HV distribution network. Knowing these values could be certainly useful for them because reduces the possibility of accepting erroneous measurements and/or calculation results in some particular cases.

Finally, the research results presented in [6] show that ignoring the existence of surrounding metal installations can lead to completely incorrect testing and estimation of safety conditions in some critical cases of HV distribution substations. Namely, by applying the previously known and used testing procedure in some critical cases, the prescribed safety criteria can be considered satisfied, although that is not the case, i.e. although too high potential differences and human life losses are possible [6].

6. Mathematical Interpretation of the Developed Methodology

Based on the complete equivalent circuit of an HV or EHV cable line given in Fig. 3 we can write the following system of equations:

$$\begin{aligned}
&Z_{ph}I_t + Z_{01}I_1 + Z_{02}I_2 + \cdots + Z_{0n}I_n + \cdots + Z_{0N}I_N = U_a \\
&Z_{10}I_t + Z_{11}I_1 + Z_{12}I_2 + \cdots + Z_{1n}I_n + \cdots + Z_{1N}I_N = 0 \\
&Z_{20}I_t + Z_{21}I_1 + Z_{22}I_2 + \cdots + Z_{2n}I_n + \cdots + Z_{2N}I_N = 0 \\
&\bullet \\
&\bullet \\
&\bullet \\
&Z_{n0}I_t + Z_{n1}I_1 + Z_{n2}I_2 + \cdots + Z_{nn}I_n + \cdots + Z_{nN}I_N = 0 \\
&\bullet \\
&\bullet \\
&\bullet \\
&Z_{N0}I_t + Z_{N1}I_1 + Z_{N2}I_2 + \cdots + Z_{nN}I_n + \cdots + Z_{NN}I_N = 0
\end{aligned} \tag{15}$$

Before the foreseen test measurements, in the given system of equations only the impedances: $\underline{Z}_{ph}$, $\underline{Z}_{11}$, $\underline{Z}_{22}$, $\underline{Z}_{33}$, $\underline{Z}_{01}$, $\underline{Z}_{02}$, $\underline{Z}_{03}$, $\underline{Z}_{12}$, $\underline{Z}_{13}$, and $\underline{Z}_{23}$, can be obtained by calculations and can be considered as in advance known quantities together with voltage $\underline{U}_a$. After the foreseen measurements and obtained values for currents $\underline{I}_t$ and $\underline{I}_1$, unknown quantities remain current $\underline{I}_i$ and all other impedances, from $\underline{Z}_{23}$ until $\underline{Z}_{NN}$. However, after introducing the here-defined equivalent sheath, the number of unknown quantities is significantly reduced so that we obtain the following system of only three equations:

$$\begin{aligned}
&1.\ Z_{ph}I_t + Z_{01}I_1 + Z_{0eq}I_i = U_a \\
&2.\ Z_{10}I_t + Z_{11}I_1 + Z_{1eq}I_i = 0 \\
&3.\ Z_{eq0}I_t + Z_{eq1}I_1 + Z_{eq}I_i = 0
\end{aligned} \tag{16}$$

where

$$I_i = I_2 + I_3 + I_4 + \cdots + I_N \tag{17}$$

In the system of equations (16), the unknown quantities are $\underline{I}_{\mathrm{i}}$, $\underline{Z}_{0\mathrm{eq}}$, $\underline{Z}_{1\mathrm{eq}}$, and $\underline{Z}_{\mathrm{eq}}$, and since the number of unknown quantities is greater than the number of equations this system of equations is seen generally as unsolvable. However, by introducing an equivalent sheath that represents a cylinder that encompasses the line conductors with currents $\underline{I}_{\mathrm{t}}$ and $\underline{I}_1$ (Fig. 6) along the line, the impedances $\underline{Z}_{0\mathrm{eq}}$ and $\underline{Z}_{1\mathrm{eq}}$ become equal to each other and the number of unknown quantities becomes smaller. Also, under the assumption that the introduced equivalent conductor is in the form of an equivalent cable sheath, we obtain a system of equations (12) that is solvable. It enables the determination of all unknown quantities, $\underline{I}_{\mathrm{i}}$, $\underline{Z}_{0\mathrm{eq}}$, and $\underline{Z}_{\mathrm{eq}}$. When impedances $\underline{Z}_{0\mathrm{eq}}$, and $\underline{Z}_{\mathrm{eq}}$ are determined it is not difficult by using expressions (9) and (10) to determine the relevant parameters of the introduced equivalent conductor/sheath, R'_{eq}, and r_{eq}, as well as current $\underline{I}_{\mathrm{i}}$. Here it is interesting to note that any other imagined appearance of the introduced equivalent conductor does not enable the solution to the considered problem.

By using a similar procedure it is possible to determine any (m^{th}) current in the system of equations (15), $\underline{I}_{\mathrm{m}}$ [13]. It is only necessary that during the foreseen test measurements the installation through which this current would flow is ungrounded at one of its ends, i.e. that there is no current in this installation. If the spatial position of the installation in relation to the phase conductor with current $\underline{I}_{\mathrm{t}}$ is known, when we determined the relevant parameters of the equivalent cable sheath without the participation of this installation, required current $\underline{I}_{\mathrm{m}}$ can be, according to (15), determined by using the following system of equations:

$$\begin{aligned} &1.\ Z_{ph} I_t + Z_{0eq} I_i + Z_{0eq} I_m = U_a \\ &2.\ Z_{10} I_t + Z_{1eq} I_i + Z_{1eq} I_m = 0 \\ &3.\ Z_{eq0} I_t + Z_{eq1} I_i + Z_{eq} I_m = 0 \end{aligned} \tag{18}$$

where

$$I_i = \sum_{n=2}^{n=N} I_n \quad , \quad n \neq m \tag{19}$$

When we have an HV cable line in normal operating conditions if phase conductors are located at a somewhat greater distance from each other, the problem of the determination of the inductive influence on any of the surrounding metal installations, considered separately, can be solved on the basis of the principle of superposition as shown in [13].

When we need the actual transfer capacity of HV or EHV cable lines [12], then we have the situation that instead of ground-fault current the load currents circulate in all three phase conductors and differently affect the surrounding metal installations. Thus, for determining the parameters of the equivalent neutral conductor substituting all surrounding metal installations, a test current has to circulate simultaneously through all three phase conductors. Therefore, it seems that it is not possible to use the measurement circuit presented in Fig. 4. However, the mutual distances between single-core cables of a cable line are relatively small in comparison with their distances from the surrounding metal installations. Due to this, with a negligible small error can be used the measurement circuit presented in Fig. 4 for obtaining the relevant parameters of the equivalent conductor substituting all surrounding metal installations. At this, the sheaths of the single-core cables should be ungrounded at one of the ends of the considered cable line because during normal operation the currents induced in them cancel each other when the line is with cross-bonding. After the determination of the parameters of the equivalent neutral conductor, the further calculation procedure is described in [12].

7. Conclusions

The presented methodology version enables taking into account the favorable inductive influence of metal installations existing in urban areas on the ground fault current distribution in HV substations supplied by cable lines with applied cross-bonding. In critical cases when a preliminary estimation is not sufficient, the presented measurement and calculation procedure enables the correct evaluation of the actual potentials and potential differences at periodical testing and verification of the safety conditions within and in the vicinity of these substations. Since the favorable effects of the surrounding metal installations are very pronounced, the presented methodology could be served as a basis for an amendment of the corresponding technical standards.

8. List of Symbols

The notation used has the following meaning:

A (B) – supply (supplied) substation,

F – ground fault location (supplied substation),

$\underline{I}_f$ – ground-fault current component coming from substation A,

$\underline{I}_n$ – ground-fault current fraction circulating through the line neutral conductors,

$\underline{I}_\mathrm{i}$ – ground-fault current fraction induced in the surrounding metal installations (Fig. 3) and current induced in the surrounding metal installations including cable line sheaths (Fig. 5),

$\underline{I}_e$ – ground-fault current fraction dissipated through the grounding system of substation B into the surrounding earth,

d – the distance between two adjacent single-core cables (m),

$\underline{Z}_\mathrm{sh}$ – self-impedances of the cable sheath,

r_sh – mean radius of the cable sheath (m),

$\underline{U}_\mathrm{a}$ – auxiliary voltage source,

$\underline{U}_{n0}$, $\underline{U}_{n1}$, $\underline{U}_{n2}$, …, $\underline{U}_{\mathrm{nN}}$ – voltages induced in an arbitrary (nth) metal installation by the current in each of the surrounding electrical conductors including metal installations,

$\underline{I}_\mathrm{t}$ – simulated ground-fault current through one of the phase conductors of the considered line,

$\underline{I}_1$ – current through the sheath of the single-core cable carrying test current I_t (Fig. 3) and current induced in the additional phase conductor (Fig. 4 and 5)

$\underline{I}_2$, $\underline{I}_3$ – currents induced in the sheaths of the other two single-core cables,

$\underline{I}_4$, $\underline{I}_5$, …, $\underline{I}_\mathrm{n}$, …, $\underline{I}_\mathrm{N}$ – currents induced in the individual surrounding metal installations (Fig. 3),

$\underline{Z}_\mathrm{ph}$ – self-impedance of the phase conductor,

$\underline{Z}_4$, $\underline{Z}_5$, $\underline{Z}_6$, …, $\underline{Z}_\mathrm{N}$ – self-impedances of the individual surrounding metal installations,

N – an arbitrarily large number representing the total number of surrounding metal installations enlarged by the number of the sheaths of the considered cable line,
R'_n – longitudinal resistance of an arbitrary, n^{th}, surrounding metal installation (Ω/km),
r_n – mean radius of an arbitrary, n^{th}, surrounding metal installation (m),
d_{nm} – the distance between two arbitrary, n^{th} and m^{th}, surrounding metal installations (m),
ω_t – angular test frequency: $2\pi f_t$,
μ_0 – magnetic permeability of vacuum: $4\pi \cdot 10^{-7}$ Vs/Am,
μ_r – relative magnetic permeability of the metal that is used for the installation production,
δ_t – equivalent earth penetration depth (m),
ρ – equivalent soil resistivity along and around the considered cable line (Ωm),
f_t – test circuit frequency,
$\underline{I}_2$, $\underline{I}_3$, and $\underline{I}_4$ (Fig. 4) – currents through the individual cable line sheaths,
A – ampere-meter,
$\underline{U}_{01}$ ($\underline{U}_{eq1}$) – voltages that current $\underline{I}_0$ ($\underline{I}_i$) induces in the phase conductor with current $\underline{I}_t$,
$\underline{U}_{10}$ ($\underline{U}_{1eq}$) – voltage that current $\underline{I}_t$ ($\underline{I}_i$) induces in the additional phase conductor with $\underline{I}_0$,
$\underline{U}_{eq0}$ ($\underline{U}_{eq1}$) – voltages that current $\underline{I}_t$ ($\underline{I}_0$) induces in the equivalent sheath,
$\underline{Z}'_{eq}$ – self-impedance of the introduced equivalent sheath (Ω/km),
R'_{eq} – longitudinal resistance of the equivalent sheath (Ω/km), and
r_{eq} – mean radius of the imagined cylinder representing the equivalent sheath (m).
$\underline{Z}_{01}$ - mutual impedance between the additional phase conductor and the phase conductor with current $\underline{I}_t$ (Ω/km),
$\underline{Z}'_{eq0}$ – mutual impedance between the equivalent sheath and the phase conductor with current $\underline{I}_t$ (Ω/km),
$\underline{Z}'_{1eq}$ – mutual impedance between the introduced equivalent sheath and the additional phase conductor (Ω/km),
R'_{ph} – longitudinal resistance of the phase conductor, (Ω/km),
r_{ph} – radius of the phase conductor (m),

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