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**Original scientific paper**

# ON THE COMPARISON OF DIFFERENT SERIAL CONCATENATED SCHEMES BASED ON POLAR AND LDPC CODES*

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**Abstract**. *Nowadays concatenated codes are actively developed for different applications of error-correcting theory. In this paper we propose a new method for constructing concatenated codes consisting of some outer error-correcting code and a particular designed inner low-density parity-check (LDPC) code. We consider polarization-adjusted convolutional (PAC) code and LDPC code as outer code of suggested construction. A specialized optimization algorithm was developed to generate inner code with particular error-reducing properties. By using the woven codes decoder with this design, the complexity of the decoding procedure is reduced compared to traditional polar and LDPC codes decoders, while still providing error-correcting characteristics. Additionally, we enhance the performance of our system by puncturing the encoded symbols. The resulting concatenated constructions outperform low-rate LDPC code from the 5G standard and polar code. Furthermore, we demonstrate the performance of code with a special parity-check matrix which consists of parity-check matrices of inner and outer codes from our proposed construction.*

**Key words**: *error correcting codes, LDPC, optimization of codes, PAC codes, concatenated codes, error reducing regime*

## 1. INTRODUCTION

To date the channel coding is an integral part of all communication systems. The error-correcting construction proposed in this paper is based on three techniques: concatenation of two codes with inner error-reducing one, optimization codes to each other and

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concatenated decoder reducing the overall complexity. We applied these methods to low-density parity-check (LDPC) and polarization-adjusted convolutional (PAC) codes.

LDPC codes were introduced by R. W. Gallager in 1962 [2] along with some decoding methods of such codes. However, active analysis of LDPC codes did not begin until after the publication of MacKay's work [3] in the late 1990s.

Currently, the most common method for constructing LDPC codes is based on the expanding a small core matrix, known as a base matrix. This is achieved by replacing each non-zero element with a circulant matrix. The base matrix can be obtained through density evolution or exhaustive search process [4]. Once the base matrix has been determined, an extending algorithm such as ACE [5] can be applied to obtain the parity-check matrix of the LDPC code.

Polar codes were proposed by Arikan in [6] in 2008. Arikan proved that such codes could achieve the capacity of any symmetric binary-input discrete memoryless channels utilizing the successive cancellation (SC) decoder with the code length going to infinity. The existing irrespective channels belong to two groups: less reliable and more reliable channels. Information bits are transmitted by polar codes over reliable channels which have greater reliability. Through the polarization procedure applied recursively in the polar encoding process, the channels are divided into two extreme groups: the noisy sub-channels and almost noisy-free sub-channels. The major task during polar codes' development is choosing appropriate subsets of high reliable and low reliable sub-channels (information and frozen sets).

PAC codes were presented by Arikan in 2019 as a polar coding scheme that improves the performance of the classic polar code [7]. PAC codes differ from the polar ones in that a rate one convolutional code is applied before polar transform. According to [7], such an additional step reduces a capacity loss of sub-channels due to their non-fixed inputs in PAC coding. Several recent papers compare performance and complexity of application successive cancellation list (SCL) and sequential decoders to PAC codes [8],[9]. While offering comparable performance, SCL has constant complexity with an advantage on low signal-to-noise (SNR) ratios.

The concept of integrating several codes into a single communication system was first proposed by Forney in 1966 [10]. Serial and parallel concatenated codes (CC) are the two main types of concatenated constructions.

Nowadays, the most significant and current issue is the selection of the corresponding component codes of concatenated scheme and their coordination with each other. Several papers devoted to CC examine codes such as LDPC, polar or convolutional codes as components of construction. In such studies, the polar code component is typically decoded by belief propagation (BP) algorithm to obtain the soft output of the decoder. The authors of paper [11] use LDPC code as an outer component and polar code decoded by BP as an inner component of the construction. The same approach was utilized in papers [12], [13], where polar code is decoded by BP algorithm, which results in poor error-correcting performance of the entire scheme.

Another approach to constructing concatenated codes is to use inner code that improves the channel. The author of the work [14] considers outer high-rate polar code and inner low-density generator matrix (LDGM) code which is known as the code allowed to improve the channel. But non-optimized regular LDGM code and BP decoder of polar code are utilized in this work. Sparse regression codes (SAPRCs), which are currently under active investigation for various scenarios [15], can be considered as candidates for inner error-reducing code. Authors of the paper [16] extended the approach of [17], which combines inner LDPC code and outer staircase code, by applying a sparse regression scheme to LDPC code. Results of [16] demonstrate a good error-reducing capabilities of the presented inner code while maintaining low complexity, which is important for future network generations that require low power consumption.

In this paper, we examine a new efficient concatenated coding schemes which combine PAC and LDPC codes through an optimization process. We propose a serial concatenated construction with permutation between code components, which differs from previous approaches in that it utilizes the inner LDPC code to enhance the communication channel for the outer decoder rather than attempting to directly decode the received word. To achieve this, we introduce an optimization algorithm that enhances error-reducing properties of the inner LPDC code, thereby improving the overall performance of the concatenated code with a fixed outer code. Consequently, the inner LDPC code is used for error reducing and outer – for error correction. Our research builds upon our previous work devoted to concatenated codes consisting of two LDPC codes [18]. In this paper we further develop this approach by improving its performance using certain methods and propose concatenated code that consists of an outer error-correcting PAC code and inner error-reducing LDPC code. More precisely, we fix inner error reducing LDPC code and apply optimization algorithm to outer LDPC. Furthermore, we demonstrate the performance of our construction with puncturing the parity-check symbols of outer code. Besides this, we explore a parity-check matrix consisting of inner and outer codes' parity-check matrices and decode such code by BP decoder of LDPC. Additionally, we compare the complexities of two proposed concatenated solutions with classical decoders of LDPC and polar codes. Finally, we present simulation results to compare the performance of our schemes and LDPC codes from the 5G standard.

## 2. Code Construction

The proposed concatenated code consists of inner $C_i\,(N_i, K_i = N_o)$ error-reducing LDPC code and some outer error-correcting $C_o\,(N_o = K_i, K_o)$ code. We denote the interleaver between components of CC by $P$. The inner LDPC code enhances transmission on bit level, decreasing bit error level for the outer decoder. Let us first describe the encoding and decoding processes of the proposed coding scheme. A separate section is devoted to the description of a special soft output decoder of outer PAC code.

### 2.1. Encoding Procedure

The encoding algorithm of the proposed concatenated code is schematically illustrated in Fig. 1.

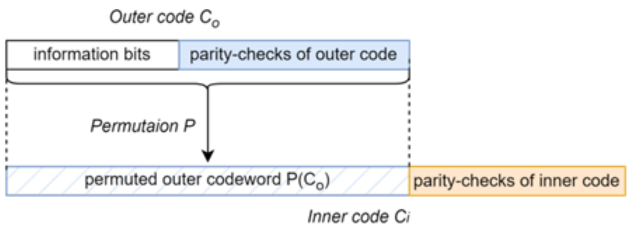


**Fig. 1** Encoder of the proposed concatenated constriction

The encoding process of our construction involves the following steps:

1. The information vector $u = (u_1, \dots, u_{Ko})$ is generated to be transmitted.
2. Information vector $u$ is encoded by outer $C_o$ ($N_o = K_i$, $K_o$) code.
3. The codeword of outer code is interleaved by $P$: $P(C_o)$. The length of $P$ is $N_o$.
4. Permuted outer $P(C_o)$ codeword is encoded using inner $C_i$ ($N_i$, $K_i = N_o$) error reducing LDPC code.

The structure of the presented concatenated code is based on the method of serial concatenation, but with added permutation between code components. The concept of permutation was derived from parallel concatenation technique. Generally, such an encoder follows the same encoding procedure as that of woven codes proposed by S. Host et al for convolutional codes [19]. A random permutation randomizes the output of the outer code, which leads to the independence of errors at the input of the outer decoder and, consequently, an improvement in the performance of the coding method.

### 2.2. Decoding algorithm

Let us demonstrate the decoding process for the proposed concatenated schemes. We adopted the idea of the decoding algorithm from the paper [20] devoted to woven codes. This decoder is an iterative one, and, therefore, the output of one component of the decoder is used at the input of the other in the next iteration. For this reason, the component decoders in the suggested design should be soft-input soft-output (SISO) decoders. Calculation of the complexity of such a concatenated decoder with different code components is presented in 4th section.

Let us designate the outer and inner components of the decoder as $Dec_{Co}$ and $Dec_{Ci}$ respectively. The vector of log-likelihood ratios (LLRs) $L_{ch} = (l_{ch}^{(1)}, l_{ch}^{(2)}, \dots, l_{ch}^{(\mathrm{Ni})})$ calculated based on the values from the channel:

$$l_{ch}^{(i)} \triangleq ln \frac{Pr(x_i = 0|y_i)}{Pr(x_i = 1\ |y_i)} \tag{1}$$

We denote the input vector of the concatenated decoding procedure as $L_1$. The outputs of the outer and inner decoders are denoted as $L_{out}$ and $L_{in}$. The decoding process utilizes the inverse permutation $P^{-1}$ at each iteration. The following steps are included in each iteration of the decoding algorithm:

1. The input to the inner decoder is formed through concatenation of two vectors:

$$L_1 = [L_{ch} + L_{out} | L_{ch}], \tag{2}$$

   where the left side of the concatenated vector represents the information part of the inner code, and the right side represents parity-check part. $L_{out}$ is the vector of extrinsics of outer code received on the previous iteration. It is worth noting that $L_{out}$ is a vector of zeros on the first iteration.

2. $L_1$ is processed by inner decoder, and the LLRs per information symbol are calculated:

$$L_{in} = Dec_{C_i}(L_1) - L_{out} \tag{3}$$

3. The outer decoder is applied to $L_{in}$ and, if this is not the last iteration, the LLRs per code symbol are calculated:

$$L_{out}=P(Dec_{C_o}\left(P^{-1}(L_{in})\right)-L_{in}) \quad (4)$$

A significant benefit of the described decoding algorithm is a noticeable reduction in complexity compared to applying classic decoders of LDPC and polar codes. Fourth section is devoted to this question.

## 3. Soft Successive Cancellation List Decoder of PAC Code

The main problem with using polar codes (and, accordingly, PAC codes) as components of concatenated constructions is lack of suitable soft-input soft-output decoding algorithms. Classic SCL algorithm is a hard decision decoder in fact. Being a representative of sequential decoders, this algorithm demonstrates high variance of output LLRs corresponding to the latest decoded symbols. Consequently, conventional sequential decoder of polar codes like SCL is not suitable for outer decoder of PAC code in a concatenated scheme.

The example of magnitudes of input/output LLRs of SCL decoder applying for PAC codeword is presented in Fig. 2. A large increase in LLRs values can be found after using SCL decoder.

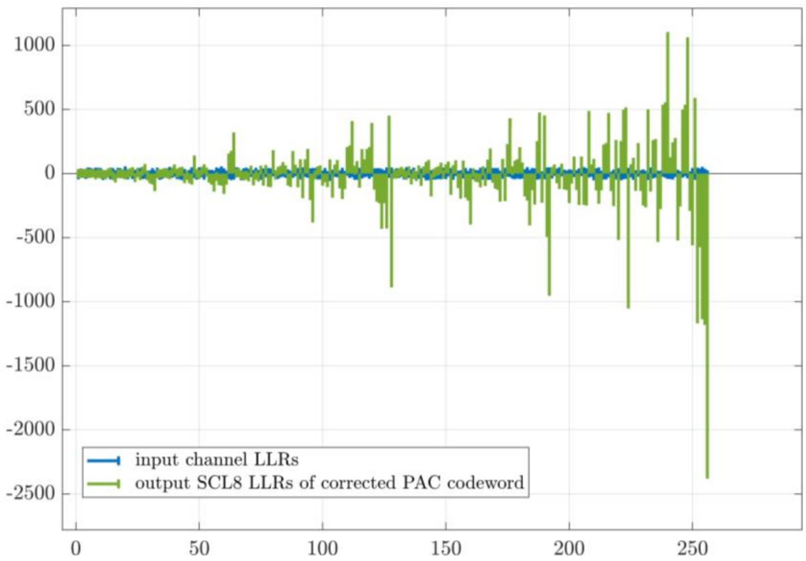


**Fig. 2** Input/Output LLR values of (256,240) PAC SCL8 decoder, Es/No = 3.6 dB

The main idea of most approaches of constructing SISO decoder of polar codes is applying BP algorithm to graph representation of polar code. Such a method results in poor performance and authors of [21] suggest a novel SISO decoder of polar code which combines both SCL and BP approaches to maintain performance of classic polar decoders with hard decisions. We utilize this solution in our concatenated decoder for PAC code component.

The soft SCL algorithm used in our research is described in [21] in details. We only recap its main steps and features in this section. The soft SCL decoder essentially comes down to three stages:

1. Applying the classic log-based SCL decoder to LLR values from the channel.
2. SC decoder with only left node update step is applied to the same channel LLRs, output of the previous step is taken as a frozen set of this SC decoder.
3. If the CRC check is failed, then the current result of the decoding process is returned, else – BP is applied to the output of SC decoder step and the resulting vector is returned as soft output of the decoder.

The main difference between presented decoding steps and proposed in [21] is verifying CRC precoding before application BP algorithm. This additional step reduces the overall complexity and makes results more predictable.

The use of the given algorithm results in observing reasonable output LLR values and maintaining performance of the basic list decoder of PAC code. Fig. 3 demonstrates the input/output LLRs of a soft SCL decoder. The code parameters are the same as in Fig. 2 except the additional CRC-8 bits in the codeword of PAC, as the described soft SCL decoder requires CRC checks, in contrast to classic SCL decoder of PAC codes.

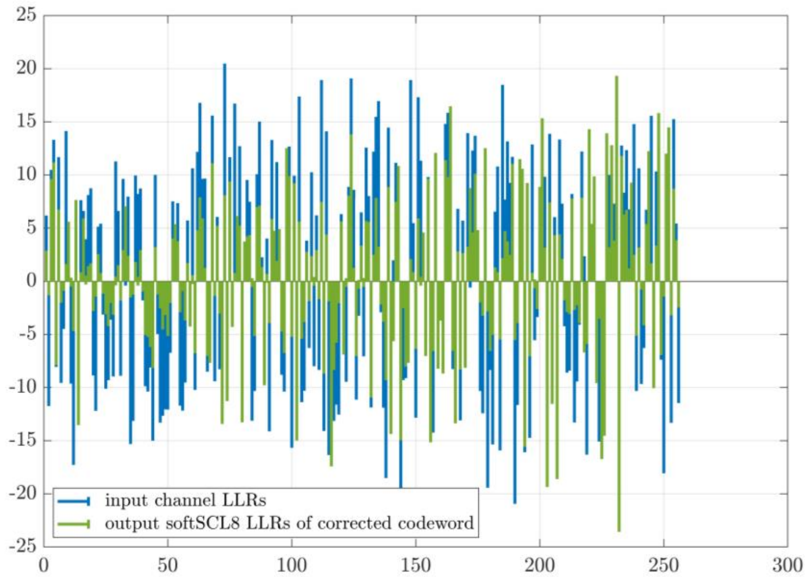


**Fig. 3** Input/Output LLRs of (256,240+8) PAC softSCL8 decoder, Es/No = 3.6 dB

## 4. Comparison of Decoders' Complexity

An important advantage of a proposed concatenated decoder is a noticeable reduction in complexity compared to applying classic decoders of LDPC and polar codes. Fourth section is devoted to this question.

Let us compare the complexity of woven decoder applied to the proposed concatenated schemes (concatenation of two LDPC and concatenated code with outer PAC and inner LDPC codes) with the complexity of BP decoder and SCL decoder.

The complexity of the classic BP decoder is equal to $I \cdot N \log N$, where $I$ – number of iterations of BP and $N$ – the length of the LDPC code. The complexity of the SCL decoder is equal to $L \cdot N \log N$, where $L$ – list size of the decoder and $N$ – the length of the polar code.

By comparison, the complexity $C_\Sigma$ of CC-decoding for the proposed combination of two LDPC is as follows:

$$C_\Sigma = S \cdot (I_o \cdot N_o \log N_o + I_i \cdot N_i \log N_i), \tag{5}$$

where $S$ – number of iterations of woven-like decoder, $I_i$, $I_o$ – number of BP iterations of inner and outer LDPC decoders respectively, $N_i$, $N_o$ – the lengths of the inner and outer LDPC components of the code respectively.

It is worth noting, that the overall reduction in complexity of CC-decoding of two serial concatenated LDPC codes is obtained until the condition that the overall number of BP iterations is held and explained by the division of complexity between component codes with a smaller code length.

Wherein, in the case of combination of PAC and LDPC codes, a soft SCL decoder is utilized as an outer decoder. It is clear from the description in [21], that its complexity is $(L+1) \cdot N \log N$ since the complexity of additional steps takes calculations as an SC decoder. Accordingly, the complexity of concatenated decoding for such a construction is as follows:

$$C_\Sigma = S \cdot ((L+1) \cdot N_o \log N_o + I_i \cdot N_i \log N_i) \tag{6}$$

The gain from using woven decoder for different lengths of concatenated code with two LDPC compared to classic BP and SCL decoders is shown in Table 1 ($R_o$ – the rate of the outer code, $R_i$ – the rate of the inner code). The gain from using this approach to concatenation of PAC and LDPC codes is presented in Table 2. Code parameters of CC were chosen to be similar to those used for the simulation results in 7th section. It can be found that the specified gain in complexity grows as the rate of the outer code increases (maintaining the overall CC rate). In the case of outer PAC and inner LDPC codes, the number of iterations of CC decoder was chosen equal to 1, so we used a non-iterative version of woven decoder since, when observing the simulations, the performance of construction is not reduced. The list size of soft SCL is equal to 4, the number of BP iterations for inner LDPC code was chosen 20.

**Table 1** Gain from using woven-like decoder (two LDPC) compared to BP50 and SCL32

| Code parameters | Gain vs BP50, % | Gain vs SCL32, % |
|---|---|---|
| (8000,2000), $R_i$=1/3, $R_o$=3/4 | 35.0 | -0.0098 |
| (8000,2000), $R_i$=1/2, $R_o$=1/2 | 25.0 | -14.2 |
| (4000,1000), $R_i$=1/3, $R_o$=3/4 | 36.0 | -0.0007 |
| (4000,1000), $R_i$=1/2, $R_o$=1/2 | 27.0 | -13.8 |
| (500,125), $R_i$=1/3, $R_o$=3/4 | 36.4 | 0.5 |
| (500,125), $R_i$=1/2, $R_o$=1/2 | 27.8 | -12.8 |

**Table 2** Gain from using woven-like decoder (CC with PAC and LDPC) compared to BP50 and SCL32

| Code parameters | Gain vs BP50, % | Gain vs SCL32, % |
|---|---|---|
| (4000,1000), $R_i$=8/30, $R_o$=30/32 | 58.3 | 34.8 |
| (4000,1000), $R_i$=1/2, $R_o$=1/2 | 56.3 | 31.8 |
| (500,125), $R_i$=8/30, $R_o$=30/32 | 58.4 | 35.0 |
| (500,125), $R_i$=1/2, $R_o$=1/2 | 56.5 | 32.0 |

## 5. Optimization Algorithm

We propose a specific error-reducing LDPC code as an inner component in the concatenated code. An optimization process that modifies the base matrix of an inner LDPC code was proposed in [18] and is briefly described in this section. After extension the base matrix, the parity-check matrix of error reducing LDPC code is obtained. We also improve the error-correcting properties of the entire construction by applying optimization not only to the inner LDPC code, but also to the outer LDPC code, while fixing the optimized inner component.

### 5.1. Optimization Algorithm for Inner LDPC Code

The goal of the suggested optimization algorithm discussed in this section is to find the base matrix of an inner LDPC code with the lowest possible signal-to-noise ratio (SNR) where the target frame error rate (FER) value of the entire concatenated code is achieved.

A detailed description of the suggested procedure can be found in [18], and here we demonstrate bird-view description.

The proposed iterative optimization algorithm randomly selects and flips a certain number of positions in the inner code's core matrix at each iteration. Thereafter, the base matrix is extended to a full parity-check matrix of the inner code. The performance of the concatenated code consisting of new inner LDPC code with this modified parity-check matrix and the fixed outer PAC/LDPC code is calculated in an AWGN channel. Finally, the process compares the SNR value on which target FER is obtained with the one from the previous best result. If this value is lower than the previous one, the current base matrix is saved as the current best core matrix of inner code for the next iteration. The outer code is selected and fixed at the beginning of the described process. The number of flipped positions at one iteration increases during optimization algorithm to prevent the convergence to local minima.

Inner error reducing LDPC code improves the superchannel of our scheme in terms of bit error rate (BER). The FER characteristic of such code is very poor on about full SNR range. This code reduces the number of bit errors, thereby improving the superchannel and, therefore, some chosen outer code can decode the received information.

### 5.2. Optimization of Outer LDPC Code

A potential advantage of a construction consisting of two LDPC codes over a concatenated scheme with inner LDPC code and outer PAC code lies in the ability to apply the same optimization process not only to the inner LDPC code but also to the outer code. Specifically, we fix previously optimized inner code in the concatenated code and implement the optimization algorithm for the outer LDPC code with the same target FER of the entire coding scheme. Fig. 4 shows the comparison of the performance of a CC of length *N*=8840 and rate *R*≈1/4 through three phases of optimization: concatenated construction before optimization (simply the raw concatenation of LDPC codes from the 5G standard with the corresponding rates), concatenated construction following optimization of the inner code, concatenated construction following optimization both the inner and outer codes.

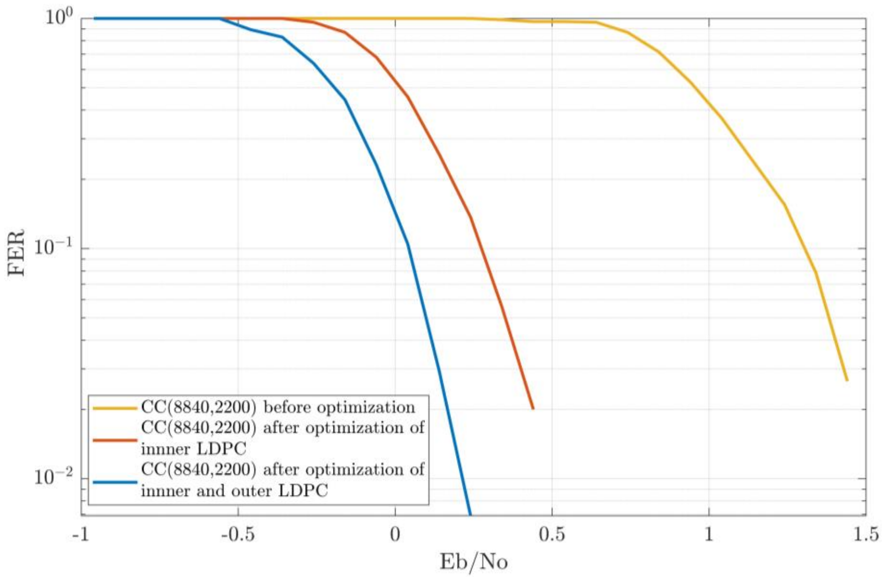


**Fig. 4** Three stages of optimization of the concatenated construction with two LDPC

## 6. Concatenated Code as LDPC

In this section we demonstrate another approach of using the optimization and concatenation techniques. We propose a new parity-check matrix that is composed of optimized inner and outer parity-check matrices of LDPC codes from a concatenated code construction. The parity-check matrix of outer LDPC code is located at the top the suggested new matrix, the parity-check matrix of inner LDPC – at the bottom, with the remaining section filled with zeros. The LDPC code with the resulted parity-check matrix can be considered as a whole with a BP decoding algorithm at the receiver side.

Let us compare the performance of this solution decoded by a classical BP decoder, proposed concatenated code decoded using a woven-like decoding algorithm and LDPC code from the 5G standard with the same parameters (length and coding rate) in Fig. 5.

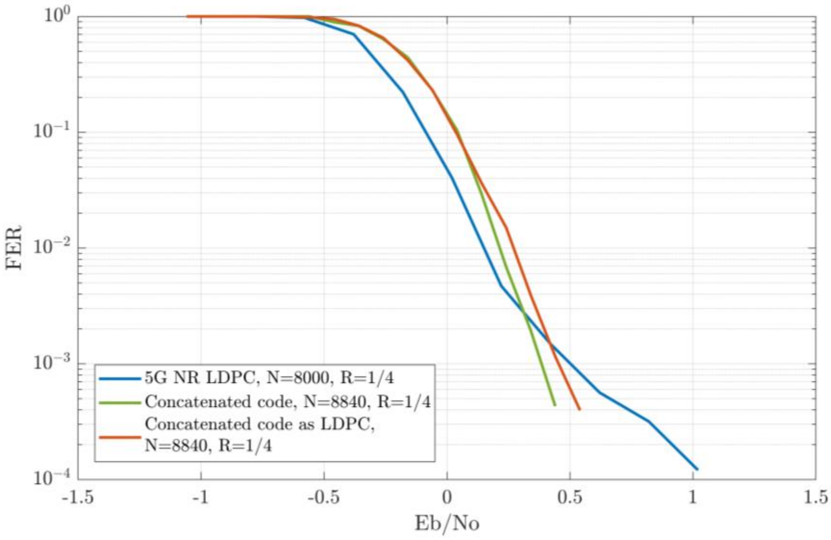


**Fig. 5** Simulation results for N=8840 and R≈1/4

It can be noticed that the LDPC code using the composite parity-check matrix starts to outperform LDPC code from the 5G standard on Eb/No above 0.4 dB. A crucial observation is that the new code does not exhibit an error floor, unlike the 5G LDPC code. At the same time, concatenated code decoded using woven-like decoder demonstrates better performance and steeper slope of FER characteristic than code with composite parity-check matrix on Eb/No levels above 0.15 dB. The complexity of its concatenated decoder is 35% lower than that of the BP decoder using the same number of iterations, as calculated using formula in section 4.

## 7. Simulation Results

In this section, we present simulation results that demonstrate the effectiveness of our proposed methods. The simulation setup that we used is as follows:

- AWGN channel, BPSK modulation.
- For the classic LDPC codes, 50 iterations of the BP algorithm were used. For construction of two LDPC codes – 5 iterations were used for the concatenated iterative woven-like decoder, and 5 BP iterations were performed on the component decoders. Therefore, the summary number of BP iterations for this concatenated decoder is 50.
- The ACE algorithm was used as an outer LDPC code lifting method, and the lifting algorithm described in [22] was applied for the inner LDPC code.
- SCL decoder with list size 32 was used for decoding classic polar code, and list size 4 was used for decoding PAC code in concatenated construction.
- Reed Muller (RM) rate-profiling [23] was used as frozen set for polar code with middle code length and PAC code in presented simulations.

Through multiple simulations and comparisons, we found that CC consisting of two LDPC is more effective at longer code lengths, while CC with outer PAC code demonstrates better performance at short and middle code length. This fact can be explained by the similar behavior of LDPC and polar codes underlying our two constructions.

In this way, we compare performance of the proposed concatenated codes with length $N$=500 with polar code and LDPC code from the 5G standard. After that, error-correcting performance of concatenated constructions with length $N$=4000 is demonstrated. These schemes are also compared with LPDC and polar code with corresponding code parameters. It is worth noting that the ratios between coding rates of inner and outer components in the suggested coding scheme were chosen experimentally from the point of view of better performance and lower decoding complexity.

We perform a simulation of the initial proposed CC of two LDPC codes and the same concatenated scheme with puncturing of some code symbols of outer LDPC code. (The puncturing procedure consists in setting LLRs from a certain pattern to zero at the receiver end. We perform this procedure in a similar way to that proposed in the 5G standard).

Concatenated schemes consisting of two LDPC codes have the following parameters:

- $N$=4420, $R$≈1/4, $R_i$≈0.29, $R_o$≈0.8
- $N$=4320 with 100 punctured symbols, $R$=0.28, $R_i$≈0.30, $R_o$≈0.92
- $N$=480 with 24 punctured symbols, $R$=1/4, $R_i$≈0.36, $R_o$≈0.77

Concatenated construction consisting of outer PAC code and inner error-reducing LDPC code considered in simulation results has following parameters:

- $N$=3840, $R$=1/4, $R_i$=8/30, $R_o$=30/32
- $N$=480, $R$=1/4, $R_i$=8/30, $R_o$=30/32

5G NR LDPC code of rate $R$=1/4 and lengths $N$=480 and $N$=5120 was chosen for comparison with the developed constructions. Moreover, we compare our concatenated codes with polar codes of rate $R$=1/4 and lengths $N$=512 and $N$=4096. In the first case RM rate-profiling was used. But as for the length of 4096, polar code constructed with RM has very poor performance. Therefore, the Gaussian approximation (GA) technique proposed in [24] was utilized to develop an information set of polar code for such length.

It should be noted that concatenated code consisting of PAC and LDPC has poor flexibility in code parameters (code length and rate) which is associated with the length of outer PAC code limited to powers of 2. This leads to a limited set of lifting sizes of inner LDPC code. We selected the closest possible parameters to other considered codes to compare the numerical results taking into account the comparison in Eb/No.

We demonstrate the simulation results of the proposed coding methods, LDPC and polar codes of code length equal to 480 in Fig. 6.

We can note the superiority of construction with outer PAC code to polar code both in error correcting properties (almost over the entire range of Eb/No) and in complexity (gain is 35%, according to the tables in fourth section). At the same time, LDPC code with the same parameters outperforms concatenation of PAC and LDPC, but it should be noticed that the decoder of the proposed construction has gain equals to 58.4% compared to BP50 decoder of LDPC code. Concatenated code consisting of two LDPC has no noticeable performance advantage at these parameters, with a small gain in complexity.

The simulation results for proposed concatenated codes, LDPC and polar codes with length 4000 are presented in Fig. 7.

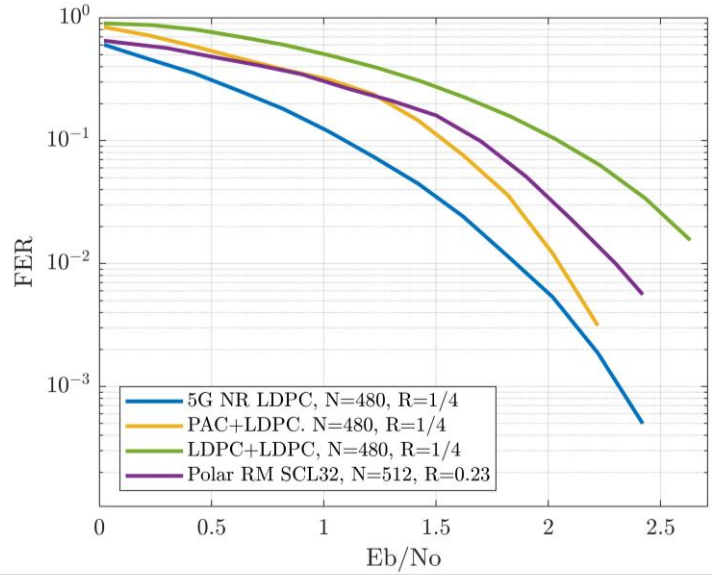


**Fig. 6** Simulation results for N=480 and R=1/4

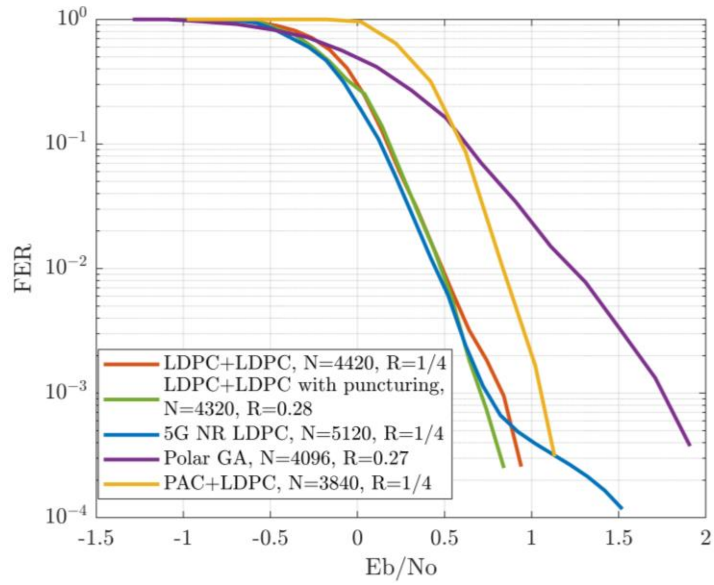


**Fig. 7** Simulation results for N=4000 and R≈1/4

As it can be found, the performance of CC with outer PAC code and inner LDPC code is better than polar code in the Eb/No range from 0.55 dB and above. Moreover, the gain in complexity for these parameters is 34.8%.

The proposed construction consisting of two LDPC codes without puncturing outperforms the LDPC code from the 5G standard on Eb/No above 0.9 dB, while CC with outer puncturing demonstrates the same performance as 5G LDPC before Eb/No of 0.6 dB and after that the error probability of CC with puncturing is lower than the FER of 5G LDPC. Construction with puncturing, as well as without it, has better error probability compared to polar code with noticeably gap despite 13.8% loss in complexity. In addition, both suggested concatenated constructions have a sharp slope of error correcting characteristic in contrast to polar code and have no error floor unlike LDPC.

## 8. CONCLUSION

Our paper focuses on the development of serial concatenated code constructions based on PAC and LDPC codes. Each of this code, being a component of concatenated construction, serves a specific purpose in the entire concatenated system. The outer LDPC or PAC code acts as error-correcting code, while the inner LDPC code becomes error-reducing code after a special optimization algorithm. This algorithm finds a new base matrix that achieves the target FER at the lowest possible SNR. Consequently, the inner code improves the transmission channel for the outer decoder of concatenated construction. We also applied the optimization process to the base matrix of outer code (in the case of concatenation of two LDPC codes), which made it possible to enhance the characteristics of our construction. The decoder of woven-like codes was utilized as the decoding algorithm of the proposed method. Such a decoding method reduces the complexity of the proposed solutions significantly. The simulation results show that the error correcting properties of suggested concatenated codes are comparable to 5G LDPC codes and polar codes and are noticeably superior to the latter in some cases, while the concatenated decoder has less complexity. The main disadvantage of the explored method is the necessity of using an optimization algorithm when using new combinations of codes. Additionally, proposed constructions demonstrate better performance mainly at low coding rates. In addition, we demonstrated a method for constructing LDPC codes using matrices of initial outer code of our construction and optimized inner code. The performance of such code outperforms LDPC from the 5G standard as well, but this method has a greater decoding complexity compared to concatenated decoder.

The proposed concatenated schemes can be applied in the different low-rate scenarios such as massive Machine-Type Communication (mMTC) and Ultra-Reliable Low Latency Communication (URLLC). For future research, we plan to explore concatenated schemes with inner SPARC code and apply optimization and woven decoding techniques to different combinations with SPARC code. SPARC codes are considered as channel coding in multi-user communication scenarios as well, which can extend the application area of the considered constructions. Moreover, we will explore the performance of the suggested methods with fading channel model for better match with realistic wireless systems. Universal algorithms proposed in this research can be applied to various channel models and combinations of codes.

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