PERFORMANCE ANALYSIS OF DUAL-BRANCH SELECTION DIVERSITY SYSTEM USING NOVEL MATHEMATICAL APPROACH

Aleksandra Golubović[1], Nikola Sekulović[2], Mihajlo Stefanović[1], Dejan Milić[1]

[1]University of Niš, Faculty of Electronic Engineering, Niš, Republic of Serbia
[2]College of Applied Technical Sciences, Niš, Republic of Serbia

Abstract. *In this paper, novel mathematical approach for evaluation of probability density function (PDF) of instantaneous signal-to-interference ratio (SIR) at the receiver output in interference-limited environment is proposed. Dual-branch selection combining (SC) receiver operating over correlated Weibull fading channels applying SIR algorithm is considered. Analytical expression for joint PDF of desired signal and interference at the receiver output is derived and used for evaluation of PDF of instantaneous SIR. The expression for PDF of SIR is used for system performance analysis via outage probability, average bit error probability (ABEP) and average output SIR as system performance measures. Numerical results are graphically presented showing the effects of fading severity, average SIR at the input and level of correlation on the diversity receiver performance. In addition, results obtained for the PDF of instantaneous SIR in this paper, are compared to the results when the PDF of instantaneous SIR is directly calculated.*

Key words: *Cochannel interference, correlated channels, decision algorithms, selection diversity, Weibull fading channels.*

1. INTRODUCTION

The main performance limitations in wireless communications systems are fading and cochannel interference (CCI). Fading emerges due to multipath propagation while CCI develops as a side effect of frequency reuse. In order to make as accurate system design as possible, depending on propagation environment, several models are used to describe the statistical behaviour of the multipath fading envelopes. The most frequently used in literature are Rayleigh, Rice, Nakagami-*m* and Weibull. This paper focuses on Weibull distribution since it is simple and flexible yet not exploited as much as the other models. It represents an

excellent fit to experimental fading channel measurements for indoor [1], [2] and outdoor [3]-[5] environments.

Wireless communication system performance can be improved at relatively low cost by diversity techniques. Basic idea behind diversity systems is simultaneous reception of the same radio signal over two or more paths in order to increase the overall signal-to-noise ratio (SNR) [6]. The diversity paths can be separated by space, frequency or time and in all cases some redundancy in time, frequency and/or spatial domain is required [7]. Compared with other diversity techniques, space diversity is power- and bandwidth-efficient that makes it the most commonly used diversity technique [8]. If the best of the received signals is selected or if they are properly combined, the outage time can be substantially reduced [9]. Depending on the communication system complexity restrictions and the amount of channel state information (CSI) available at the receiver, space diversity has several principal types of combining techniques. Combining techniques like maximal ratio combining (MRC) and equal-gain combining (EGC) require some amount of the channel state information of received signal and separate receiver chain for each branch of the diversity system that results in system complexity increase. On the other hand, selection combining (SC) receiver processes only one of the diversity branches at the time and it is much simpler and cheaper for practical realization [6].

In interference-limited environment, where the level of CCI is sufficiently high compared to noise, SC receiver can employ one of the combining algorithms: the desired signal (DS) algorithm, the signal-to-interference ratio (SIR) algorithm and the total signal (TS) algorithm [10]. SIR algorithm is based on selecting the diversity branch that has the highest SIR and it usually provides the best results in the case of interference-limited systems.

L-branch EGC and MRC receivers operating over non identical Weibull fading channels have been considered in [11]. Performance analysis of digital communications receivers over Weibull fading channels that employ SIR algorithm was thoroughly investigated in [12]-[14]. The performance of SC diversity system operating over correlated Weibull fading channels that applies SIR decision algorithm is studied in [12] for dual-branch system, in [13] for triple-branch system and in [14] for *L*-branch system. A system that uses DS algorithm, where both desired signal and interference are correlated and under Weibull fading, is presented in [15] for dual-branch and [16] and [17] for triple branch system.

This paper presents novel mathematical approach for deriving an expression for the probability density function (PDF) of instantaneous SIR at the output of a selection combining diversity system with two correlated Weibull fading channels that applies SIR algorithm. The mathematical approach used in [12] for the same system, directly calculates PDF of instantaneous SIR at the system output while this paper calculates joint PDF of desired signal and interference at the output first and then the result is used for calculation of PDF of instantaneous SIR at the output. Finally, the results obtained in this paper are compared to the results obtained in [12] and [15].

2. SYSTEM AND CHANNEL MODEL

We consider a SC diversity system with two branches in interference-limited Weibull fading environment. In practice, diversity systems are applied in small-size terminals and complete independence between branches can not be achieved resulting in diversity gain

degradation. In such case, desired signal envelopes (x_1, x_2) and CCI envelopes (y_1, y_2) experience correlative Weibull fading with joint PDFs [18, eq. (11)]

$$p_{x_1 x_2}(x_1, x_2) = \frac{\beta_1 \beta_2}{(1-\rho)} I_0 \left(\frac{2\sqrt{\rho} x_1^{\beta_1/2} x_2^{\beta_2/2}}{(1-\rho)\sqrt{\Omega_{d_1} \Omega_{d_2}}} \right) \times \frac{x_1^{\beta_1-1} x_2^{\beta_2-1}}{\Omega_{d_1} \Omega_{d_2}} \exp\left[-\frac{1}{1-\rho} \left(\frac{x_1^{\beta_1}}{\Omega_{d_1}} + \frac{x_2^{\beta_2}}{\Omega_{d_2}} \right) \right], \quad (1)$$

$$p_{y_1 y_2}(y_1, y_2) = \frac{\beta_1 \beta_2}{(1-\rho)} I_0 \left(\frac{2\sqrt{\rho} y_1^{\beta_1/2} y_2^{\beta_2/2}}{(1-\rho)\sqrt{\Omega_{c_1} \Omega_{c_2}}} \right) \times \frac{y_1^{\beta_1-1} y_2^{\beta_2-1}}{\Omega_{c_1} \Omega_{c_2}} \exp\left[-\frac{1}{1-\rho} \left(\frac{y_1^{\beta_1}}{\Omega_{c_1}} + \frac{y_2^{\beta_2}}{\Omega_{c_2}} \right) \right], \quad (2)$$

where ρ represents branch correlation coefficient ($0 \le \rho \le 1$), β is Weibull fading parameter which expresses fading severity ($\beta > 0$). As the value of Weibull fading parameter increases, fading severity decreases. $\Omega_{di} = \overline{x_i^{\beta_i}}$ and $\Omega_{ci} = \overline{y_i^{\beta_i}}$ are the average powers of desired and interference signal at i-th branch ($i=1,2$), respectively. $I_n(\cdot)$ is the modified Bessel function of the first kind and n-th order [19, eq. (8.445)].

Instantaneous values of SIR on the first and second diversity branch are defined as $z_1 = x_1/y_1$ and $z_2 = x_2/y_2$, respectively. The joint PDF of these random variables is

$$p_{z_1 z_2}(z_1, z_2) = \int_0^\infty \int_0^\infty y_1 y_2 p_{x_1 x_2}(z_1 y_1, z_2 y_2) p_{y_1 y_2}(y_1, y_2) dy_1 dy_2. \quad (3)$$

SC receiver based on SIR algorithm chooses and outputs the branch with larger SIR, i.e. $z = \max \{z_1, z_2\}$. Applying the concepts of probability, the PDF of instantaneous SIR at the output of SC combiner can be obtained as

$$p_z(z) = \int_0^z p_{z_1 z_2}(z, z_2) dz_2 + \int_0^z p_{z_1 z_2}(z_1, z) dz_1. \quad (4)$$

The approach described by (3) and (4) is used in previously published papers which study SC receivers. In this work, we propose mathematical approach based on calculation of the joint PDF of desired and interference signal envelopes.

The joint PDF of desired and interference signal envelopes on input diversity branches can be easily expressed as

$$p_{x_1 y_1 x_2 y_2}(x_1, y_1, x_2, y_2) = p_{x_1 x_2}(x_1, x_2) p_{y_1 y_2}(y_1, y_2). \quad (5)$$

When a dual-branch SC diversity system uses SIR algorithm, one of two conditions have to be fulfilled:

1. $\dfrac{x_1}{y_1} > \dfrac{x_2}{y_2}$ => $x_1 = x$, $y_1 = y$ => $y_2 > \dfrac{y}{x} x_2$

2. $\dfrac{x_2}{y_2} > \dfrac{x_1}{y_1}$ => $x_2 = x$, $y_2 = y$ => $y_1 > \dfrac{y}{x} x_1$.

In that case, the joint PDF of desired signal and interference envelopes at the output of dual-branch SC receiver based on SIR algorithm can be obtained as

$$p_{xy}(x, y) = \int_0^\infty \int_{\frac{y}{x}x_2}^\infty p_{x_1 y_1 x_2 y_2}(x, y, x_2, y_2)dy_2 dx_2 + \int_0^\infty \int_{\frac{y}{x}x_1}^\infty p_{x_1 y_1 x_2 y_2}(x_1, y_1, x, y)dy_1 dx_1, \qquad (6)$$

which by substituting (5) and after some mathematical manipulations yields

$$
\begin{aligned}
p_{xy}(x, y) = {} & \beta_1^2 \exp\left[-\frac{1}{1-\rho}\left(\frac{x^{\beta_1}}{\Omega_{d_1}} + \frac{y^{\beta_1}}{\Omega_{c_1}}\right)\right] \\
& \times \sum_{i,j=0}^\infty \frac{\rho^{i+j}}{(1-\rho)^{i+j}(i!\,j!)^2(i+1)} \frac{x^{\beta_1(i+1)-\beta_2(1+j)-1}y^{\beta_1(j+1)+\beta_2(1+j)-1}}{(\Omega_{d_1}\Omega_{d_2})^{i+1}(\Omega_{c_1}\Omega_{c_2})^{j+1}} \\
& \times \frac{\Gamma(i+j+2)}{\left(\frac{1}{\Omega_{c_2}}\left(\frac{y}{x}\right)^{\beta_2} + \frac{1}{\Omega_{d_2}}\right)^{i+j+2}} \\
& \times {}_2F_1\left(1, i+j+2; i+2; \left(\frac{\Omega_{d_2}}{\Omega_{c_2}}\left(\frac{y}{x}\right)^{\beta_2} + 1\right)^{-1}\right) \\
& + \beta_2^2 \exp\left[-\frac{1}{1-\rho}\left(\frac{x^{\beta_2}}{\Omega_{d_2}} + \frac{y^{\beta_2}}{\Omega_{c_2}}\right)\right] \\
& \times \sum_{m,n=0}^\infty \frac{\rho^{n+m}}{(1-\rho)^{m+n}(m!n!)^2(m+1)} \frac{x^{-\beta_1(n+1)+(1+m)\beta_2-1}y^{\beta_1(n+1)+(n+1)\beta_2-1}}{(\Omega_{d_1}\Omega_{d_2})^{m+1}(\Omega_{c_1}\Omega_{c_2})^{n+1}} \\
& \times \frac{\Gamma(m+n+2)}{\left(\frac{1}{\Omega_{c_1}}\left(\frac{y}{x}\right)^{\beta_1} + \frac{1}{\Omega_{d_1}}\right)^{m+n+2}} \\
& \times {}_2F_1\left(1, m+n+2; m+2; \left(\frac{\Omega_{d_1}}{\Omega_{c_1}}\left(\frac{y}{x}\right)^{\beta_1} + 1\right)^{-1}\right),
\end{aligned}
\qquad (7)
$$

where ${}_2F_1(\alpha,\beta;\gamma;z)$ represents Gaussian hypergeometric function [19, eq. (9.100)] and $\Gamma(\cdot)$ represents gamma function [19, eq. (8.310.1)]. To the best of the authors' knowledge, the above presented expression for the joint PDF of desired and interference signal envelopes at the SIR based SC receiver output is novel in the open technical literature.

The PDF of instantaneous SIR at the SC output can be calculated using following equation

$$p_z(z) = \int_0^\infty y\, p_{xy}(zy, y)dy. \qquad (8)$$

By substituting (7) in (8) and after integration, final expression for the PDF of instantaneous SIR at the receiver output is derived as

$$p_z(z) = \beta_1(1-\rho)^2 \sum_{i,j=0}^{\infty} \frac{\rho^{j+i} z^{(i+1)\beta_1-(1+j)\beta_2-1}}{(i!\,j!)^2 (i+1)(\Omega_{d_1}\Omega_{d_2})^{i+1}(\Omega_{c_1}\Omega_{c_2})^{j+1}}$$

$$\times \frac{\Gamma^2(i+j+2)}{\left(\left(\dfrac{z^{\beta_1}}{\Omega_{d_1}}+\dfrac{1}{\Omega_{c_1}}\right)\left(\dfrac{1}{\Omega_{c_2}}\left(\dfrac{1}{z}\right)^{\beta_2}+\dfrac{1}{\Omega_{d_2}}\right)\right)^{i+j+2}}$$

$$\times {}_2F_1\left(1,i+j+2;i+2;\left(\dfrac{\Omega_{d_2}}{\Omega_{c_2}}\left(\dfrac{1}{z}\right)^{\beta_2}+1\right)^{-1}\right)$$

$$+\beta_2(1-\rho)^2 \sum_{m,n=0}^{\infty} \frac{\rho^{n+m} z^{(m+1)\beta_2-(1+n)\beta_1-1}}{(m!\,n!)^2 (m+1)(\Omega_{d_1}\Omega_{d_2})^{m+1}(\Omega_{c_1}\Omega_{c_2})^{n+1}}$$

$$\times \frac{\Gamma^2(m+n+2)}{\left(\left(\dfrac{z^{\beta_2}}{\Omega_{d_2}}+\dfrac{1}{\Omega_{c_2}}\right)\left(\dfrac{1}{\Omega_{c_1}}\left(\dfrac{1}{z}\right)^{\beta_1}+\dfrac{1}{\Omega_{d_1}}\right)\right)^{m+n+2}}$$

$$\times {}_2F_1\left(1,m+n+2;m+2;\left(\dfrac{\Omega_{d_1}}{\Omega_{c_1}}\left(\dfrac{1}{z}\right)^{\beta_1}+1\right)^{-1}\right), \tag{9}$$

The PDF of instantaneous SIR at the output of the same system obtained using mathematical approach described by (3) and (4) is presented in [12] by (11).

Table 1 Comparison of number of terms of (9) and (11) in [12] to achieve accuracy at the fourth significant digit ($\beta_1=2$, $\beta_2=3$, $S_1=S_2=10$dB)

	z=5		z=25	
	(9) in this paper	(11) in [12]	(9) in this paper	(11) in [12]
$\rho=0.2$	4	6	5	6
$\rho=0.5$	12	13	13	15
$\rho=0.8$	34	34	34	41

Considering that convergence represents significant problem in infinite-series expressions, Table 1 summarizes the number of terms that need to be summed in the expressions for the PDF of instantaneous SIR at the SC output obtained in this paper and paper [12] to achieve accuracy at the 4th significant digit after the truncation of the infinite series. Instead of individual signal and interference powers, as it was presented in equation (9), the table considers their ratio at the input of i-th branch of selection combiner $S_i=\Omega_{di}/\Omega_{ci}$, $i=1,2$. The results show that the expression obtained in this paper converges more rapidly than the expression (11) in [12], making it more manageable for system analysis.

3. System Performance Analysis

The performance of dual-branch SC system operating over correlated Weibull fading channels is analysed using analytically obtained expression for the PDF of instantaneous SIR at the output. Performance indicators that are considered in this section are outage probability, average bit error probability (ABEP) and average output SIR. The influence of fading severity, correlation coefficient and average powers is studied. Moreover, numerical results are compared to numerical results in [12] to verify mathematical approach proposed in this work.

3.1. Outage probability

Being a basic system performance measure in interference-limited environment, outage probability, P_{out}, can be defined as the probability that the output SIR drops below a specified threshold z_{th}

$$P_{out} = \int_0^{z_{th}} p_z(z)dz. \tag{10}$$

Fig. 1 depicts outage probability of balanced ($S_1=S_2=S$) dual-branch SC receiver as a function of outage threshold for different system parameters. The results obtained in this paper match perfectly the results obtained in [12]. The outage probability decreases for lower values of outage threshold and higher Weibull fading parameters. For higher values of outage threshold, when desired signal is dominant, the system performance deteriorates as Weibull fading parameter increases. When fixed values of Weibull fading parameters are observed, it is obvious that for higher correlation coefficient system performance deteriorates.
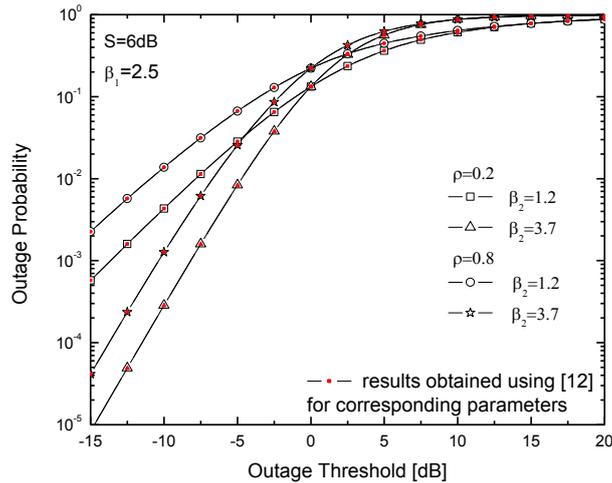
Fig. 1 Outage probability of dual-branch SC system

Comparison of the results for outage probability when DS algorithm [15] and SIR algorithm are used for different fading severity is illustrated in Fig. 2. The branches of the

receiver are correlated and balanced. It can be seen that system with SIR algorithm shows slightly better performance in terms of outage probability compared to DS algorithm.
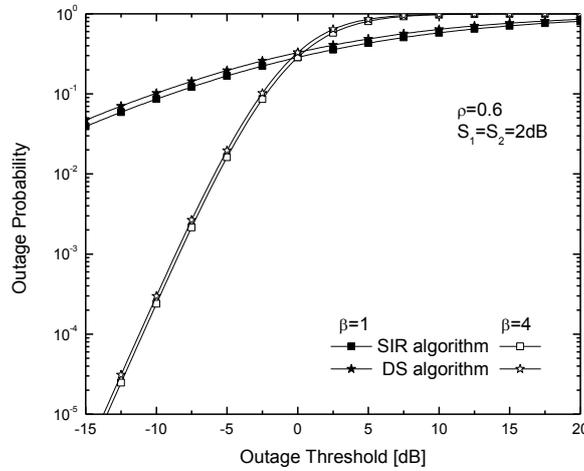
Fig. 2 Result comparison of outage probability for SIR and DS decision algorithms

3.2. Average bit error probability

ABEP represents one of the important first order performance measures. It is often used for system performance evaluation because it is the most revealing of the nature of the system behaviour. ABEP is calculated using conditional bit error probability (BEP), which is a function of the modulation/detection scheme employed by the system. In this paper, two modulations are considered, BDPSK and BFSK. For these two cases, the conditional BEP for a given SIR is

$$P_e(z) = \frac{1}{2} e^{-gz^2}, \qquad (11)$$

where g represents modulation constant and the values are, for BDPSK $g=1$ and BFSK $g=1/2$. ABEP at the SC output can be evaluated directly by averaging the conditional BEP over the PDF of z

$$P_e = \int_0^\infty P_e(z) p_z(z) dz. \qquad (12)$$

Fig. 3 illustrates ABEP of balanced dual-branch SC receiver for BFSK and BDPSK signalling for different correlation coefficient. The results obtained in [12] perfectly match the results obtained in this paper. The system performance is better for lower values of correlation coefficient which means that the system performance is better as the distance between the antennas increases. For the case when correlation is too high, it is possible for deep fades in the branches to occur simultaneously resulting in low improvement degree of considered space diversity. It is obvious from the figure that system with BDPSK signalling shows better performance than system with BFSK signalling which is in compliance with conclusion presented in [6].
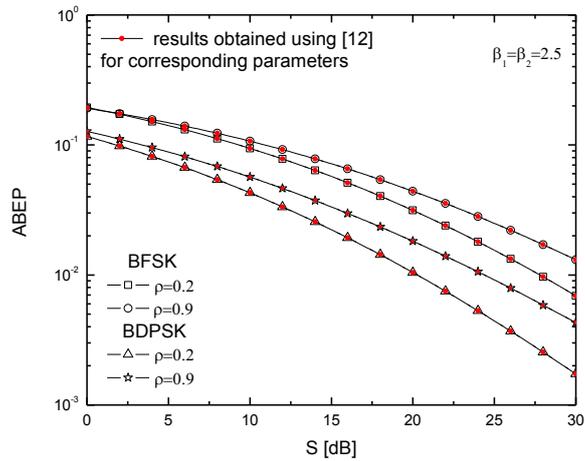
Fig. 3 The influence of correlation coefficient on ABEP of dual-branch SC system
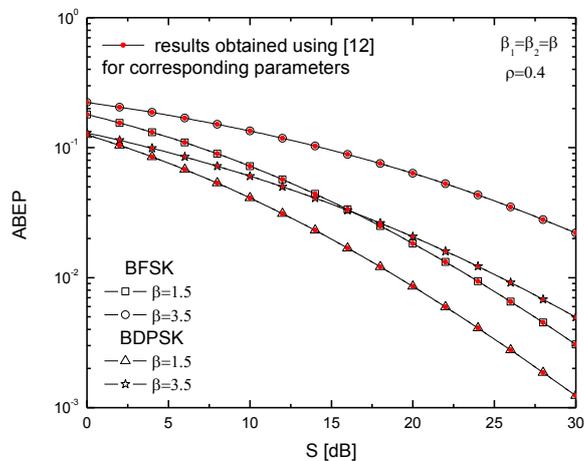
Fig. 4 The influence of fading severity on ABEP of dual-branch SC system

In Fig. 4, ABEP of balanced dual-branch SC receiver for BFSK and BDPSK signalling for different fading intensity is presented. It is obvious that system performance is better in the environment with lower fading parameter. It is interesting to note that for lower values of S, BFSK signalling with lower value of β, shows worse system performance than BDPSK signalling with higher value of β while for the case when higher values of S are observed, the situation is vice versa. It can be explained by the fact that in the considered scenario desired signal and CCI, which is inferior for higher values of S, are exposed to the same fading severity.

3.3. Average output SIR

Average output SIR is one more useful parameter that is used in wireless communications in the case when CCI is present. It can be calculated by

$$\bar{z}_{sc} = \int_0^\infty z p_z(z) dz. \tag{13}$$

Based on (9) and (13), Fig. 5 is plotted. It shows that the results obtained using (9) match perfectly with the results obtained using mathematical approach presented in [12].

The figure shows that the average output SIR degrades rapidly for higher values of correlation coefficient. It is also obvious that the system performance is better for higher values of S, which is more significant in the case of lower values of fading parameters.
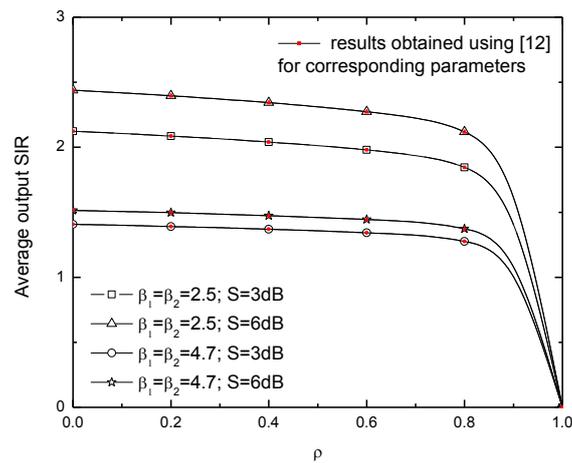
Fig. 5 Average output SIR as a function of correlation coefficient

4. CONCLUSION

This paper studies the performance of dual-branch SC receiver operating over correlated Weibull fading channels in the presence of Weibull distributed CCI for the case when SIR algorithm is applied. The PDF of instantaneous SIR at the system output was derived using mathematical approach based on calculation of the joint PDF of desired signal and interference signal envelopes at the output. Using the PDF of instantaneous SIR at the system output, outage probability, ABEP and average output SIR were evaluated as efficient system performance measures. Numerical results were graphically presented describing the influence of correlation coefficient, fading severity and average SIR at the input on overall system performance. In addition, obtained results were compared to the results in [12] which proved the perfect match, as it was expected. It was shown that the expression for PDF of instantaneous SIR obtained in this paper converges faster than the expression in [12] therefore the novel expression derived in this paper can be used more efficiently. Moreover, the joint PDF of desired signal and interference signal envelopes at the system

output can be used to calculate other important distributions. For example, the PDF of sum of desired signal and interference signal envelopes can be obtained and applied in performance analysis of system with micro and macrodiversity when macrodiversity combiner uses total power signal algorithm. Motivated by these facts, the subject of our future work will be generalization of the mathematical approach for arbitrary order of diversity and macrodiversity system based on TS algorithm.

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