

NOVEL APPROACH TO MODELLING OF LIGHTNING CURRENT DERIVATIVE

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Abstract. *A new approach to mathematical modelling of lightning current derivative is proposed in this paper. It builds on the methodology, previously developed by the authors, for representing lightning currents and electrostatic discharge (ESD) currents waveshapes. It considers usage of a multi-peaked form of the analytically extended function (AEF) for approximation of current derivative waveshapes. The AEF function parameters are estimated using the Marquardt least-squares method (MLSM), and the framework for fitting the multi-peaked AEF to a waveshape with an arbitrary number of peaks is briefly described. This procedure is validated performing a few numerical experiments, including fitting the AEF to single- and multi-peaked waveshapes corresponding to measured current derivatives.*

Key words: *analytically extended function, lightning current derivative, lightning current function, lightning stroke, Marquardt least-squares method*

1. INTRODUCTION

Besides different parameters of lightning electromagnetic field and lightning discharge currents, which greatly endanger the functionality of power systems, electrical equipment and electronic devices, lightning current derivative signal is often measured at tall instrumented towers, towers at elevated terrain and at rocket-triggered stations, [1]-[8]. Current derivatives approximation is important for calculation of lightning induced overvoltages and for further improvements of lightning discharge models [1], [5], [9].

Generalizing the function for representing lightning currents from [10]-[12], the proposed multi-peaked analytically extended function (AEF) has been applied by the authors to modelling of different lightning currents, including those defined in the IEC Standard 62305-1 [13], slow and fast-decaying ones, as well as measured ones, see e.g. [14]-[16]. Furthermore, it has been recently used in [9] and [17] for representation of the electrostatic discharge (ESD) current corresponding to the IEC Standard 61000-4-2 waveshape as given

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in [18]-[19]. The AEF's parameters were fitted to the desired current waveshapes using the Marquardt least-squares method (MLSM), [20].

In this paper we explore the possibility of reproducing the waveshape of the lightning current first derivative using the AEF and adjusting its non-linear parameters employing the MLSM. The validity of approximation and this methodology is tested by performing a few numerical experiments related to modelling of lightning current derivative signals measured at the CN Tower [5]. Since installation, simultaneous measurements of currents and current derivatives by Rogowski coils, corresponding electromagnetic field values detected by sensors and high-speed cameras at a few km distance from the tower have been providing useful data for analysis, [1]-[5]. Reflection coefficients are estimated for the CN Tower and employed for magnetic field calculation in [5]. Reflections occur from the tip of this tower, top and bottom of its restaurant and from the ground, so as at the upward-propagating lightning return-stroke channel front, and produce peaks in the current derivative waveshape.

In this paper, lightning current derivative approximation is done taking into account the initial peak and subsequent peaks in the derivative waveshape, regardless of their cause. The same procedure may be used in the case when measured current derivatives have multi-peaked waveshapes for other reasons, e.g. due to various configurations of the terrain and some tall structures, or due to lightning current channel discontinuities and branching.

2. MODELLING OF THE LIGHTNING CURRENT DERIVATIVE

2.1. Analytically extended function (AEF) and some of its properties

The basic building block of the multi-peaked AEF is, as referred to in [18], the power exponential function (PEF) given by

$$x(\beta; t) = (te^{1-t})^\beta, 0 \leq t, \quad (1)$$

where the β -parameter determines the steepness of both its rising and its decaying part.

The AEF is constructed as a function consisting of piecewise linear combinations of PEFs that have been scaled and translated to ensure that the resulting function is continuous. In [18], it is defined as

$$\frac{di(t)}{dt} = \begin{cases} \sum_{k=1}^{q-1} I_{Dm_k} + I_{Dm_q} \sum_{k=1}^{n_q} \eta_{q,k} x_{q,k}(t), & t_{m_{q-1}} \leq t \leq t_{m_q}, 1 \leq q \leq p, \\ \sum_{k=1}^p I_{Dm_k} \sum_{k=1}^{n_{p+1}} \eta_{q,k} x_{q,k}(t), & t_{m_p} \leq t, q = p+1, \end{cases} \quad (2)$$

where:

- $I_{Dm_1}, I_{Dm_2}, \dots, I_{Dm_p}$ - the difference in height between each pair of peaks,
- $t_{m_1}, t_{m_2}, \dots, t_{m_p}$ - the times corresponding to these peaks,
- $n_q > 0$ - the number of terms in each time interval,
- $\eta_{q,k}$ - real values so that $\sum_{k=1}^{n_q} \eta_{q,k} = 1$, and
- $x_{q,k}(t)$ - PEFs defined by $\beta_{q,k}$ parameters in the following way:

$$x_{q,k}(t) = \begin{cases} \left(\frac{t - t_{m_{q-1}}}{\Delta t_{m_q}} \exp\left(\frac{t_{m_q} - t}{\Delta t_{m_q}}\right) \right)^{\beta_{q,k}^2 + 1} & 1 \leq q \leq p, \\ \left(\frac{t}{t_{m_q}} \exp\left(1 - \frac{t}{t_{m_q}}\right) \right)^{\beta_{q,k}^2} & q = p + 1, \end{cases} \quad (3)$$

where $\Delta t_{m_q} = t_{m_q} - t_{m_{q-1}}$.

Expression (2) can be written more compactly as

$$\frac{di(t)}{dt} = \begin{cases} \sum_{k=1}^{q-1} I_{Dm_k} + I_{Dm_q} \boldsymbol{\eta}_q^T \mathbf{x}_q(t), & t_{m_{q-1}} \leq t \leq t_{m_q}, 1 \leq q \leq p, \\ \sum_{k=1}^p I_{Dm_k} \boldsymbol{\eta}_q^T \mathbf{x}_q(t), & t_{m_p} \leq t, q = p + 1, \end{cases} \quad (4)$$

after introducing

$$\boldsymbol{\eta}_q = [\eta_{q,1} \quad \eta_{q,2} \quad \dots \quad \eta_{q,n_q}]^T,$$

$$\mathbf{x}_q(t) = [x_{q,1}(t) \quad x_{q,2}(t) \quad \dots \quad x_{q,n_q}(t)].$$

The first derivative of the multi-peaked AEF corresponds to the second derivative of the lightning discharge current, $i(t)$, and can be easily found since the AEF consists of elementary functions. Compact form is given by

$$\frac{d^2i(t)}{dt^2} = \begin{cases} I_{Dm_q} \frac{t_{m_q} - t}{t - t_{m_{q-1}}} \frac{x_q(t)}{\Delta t_{m_q}} \boldsymbol{\eta}_q^T \mathbf{B}_q \mathbf{x}_q(t), & t_{m_{q-1}} \leq t \leq t_{m_q}, 1 \leq q \leq p, \\ I_{Dm_q} \frac{t_{m_q} - t}{t_{m_q}} \frac{x_q(t)}{\Delta t_{m_q}} \boldsymbol{\eta}_q^T \mathbf{B}_q \mathbf{x}_q(t), & t_{m_p} \leq t, q = p + 1, \end{cases} \quad (5)$$

where \mathbf{B}_q are diagonal matrices:

$$\mathbf{B}_q = \begin{cases} \text{diag}(\beta_{q,1}^2 + 1, \beta_{q,2}^2 + 1, \dots, \beta_{q,n_q}^2 + 1), & 1 \leq q \leq p, \\ \text{diag}(\beta_{q,1}^2, \beta_{q,2}^2, \dots, \beta_{q,n_q}^2), & q = p + 1. \end{cases}$$

Based on this expression, it is easy to see that the current's second derivative is also continuous since it will be zero at each t_{m_q} .

The integral of the AEF corresponds to the lightning discharge current $i(t)$ and is also relatively straightforward to find, since the integral of the PEF can be written using the lower incomplete gamma function ([21]) i.e.

$$\int_{t_0}^{t_1} x(\beta; t) dt = \frac{e^\beta}{\beta^{\beta+1}} (\gamma(\beta+1, \beta t_1) - \gamma(\beta+1, \beta t_0)) = \frac{e^\beta}{\beta^{\beta+1}} \Delta\gamma(\beta, t_1, t_0), \quad (6)$$

where $\gamma(\beta, t) = \int_0^t \tau^{\beta-1} e^{-\tau} d\tau$ is the incomplete gamma function.

Combining (6) and (2) we obtain the integral of the rising part of the AEF

$$\begin{aligned} \int_{t_a}^{t_b} \frac{di(t)}{dt} dt &= (t_{m_a} - t_a) \left(\sum_{k=1}^{a-1} I_{Dm_k} \right) + I_{Dm_a} \sum_{k=1}^{n_a} \eta_{a,k} g_a(t_{m_a}, t_a) + \\ &+ \sum_{q=a+1}^{b-1} \left(\Delta t_{m_q} \left(\sum_{k=1}^{q-1} I_{Dm_k} \right) + I_{Dm_q} \sum_{k=1}^{n_q} \eta_{q,k} \hat{g}(\beta_{q,k}^2 + 1) \right) + \\ &+ (t_b - t_{m_b}) \left(\sum_{k=1}^{b-1} I_{Dm_k} \right) + I_{Dm_b} \sum_{k=1}^{n_b} \eta_{b,k} g_b(t_b, t_{m_b-1}), \quad 0 \leq t_a \leq t_b \leq t_{m_p}, t_{m_{q-1}} \leq t_a \leq t_{m_q}, t_{m_{b-1}} \leq t_b \leq t_{m_b}, \end{aligned} \quad (7)$$

$$\text{with } g_q(t_1, t_0) = \frac{e^{\beta_{q,k}^2+1}}{(\beta_{q,k}^2+1)^{\beta_{q,k}^2+2}} \Delta\gamma \left(\beta_{q,k}^2 + 1, \frac{t_1 - t_{m_{q-1}}}{\Delta t_{m_q}}, \frac{t_0 - t_{m_{q-1}}}{\Delta t_{m_q}} \right) \text{ and } \hat{g}(\beta) = \frac{e^\beta}{\beta^{\beta+1}} \gamma(\beta+1, \beta).$$

The integration formula corresponding to the decaying part is

$$\begin{aligned} \int_{t_0}^{t_1} \frac{di(t)}{dt} dt &= \sum_{k=1}^p I_{Dm_k} \sum_{k=1}^{n_{p+1}} \eta_{p+1,k} g_{p+1}(t_1, t_0), \quad t_{m_p} \leq t_0 \leq t_1 < \infty, \text{ i.e.} \\ \int_{t_{m_p}}^{\infty} \frac{di(t)}{dt} dt &= \sum_{k=1}^p I_{Dm_k} \sum_{k=1}^{n_{p+1}} \eta_{p+1,k} \tilde{g}(\beta_{p+1,k}^2), \end{aligned} \quad (8)$$

$$\text{with } \tilde{g}(\beta) = \frac{e^\beta}{\beta^{\beta+1}} (\Gamma(\beta+1) - \gamma(\beta+1, \beta)) \text{ and } \Gamma(\beta) = \int_0^\infty \tau^{\beta-1} e^{-\tau} d\tau, \text{ the Gamma function, [21].}$$

2.2. Marquardt least-squares method (MLSM)

Detailed explanation of the MLSM algorithm is given in [15], [16], here we just go over the parts specific for the multi-peaked AEF.

The MLSM is used for estimating β -parameters, and from these, the corresponding η -parameters are calculated. In each iteration step, η -parameters are obtained using the regular least-square method since for fixed β -parameters the AEF is linear in η . Based on these η -parameters, a new set of β -parameters is found.

The MLSM uses a Jacobian matrix, denoted by \mathbf{J} , containing partial derivatives of the residuals. The least square fitting of the multi-peaked AEF to a set of data points can be done separately between each peak (and after the final one), and the corresponding \mathbf{J} matrix is

$$\mathbf{J} = \begin{bmatrix} p_{q,1}(t_{q,1}) & p_{q,2}(t_{q,1}) & \cdots & p_{q,n_q}(t_{q,1}) \\ p_{q,1}(t_{q,2}) & p_{q,2}(t_{q,2}) & \cdots & p_{q,n_q}(t_{q,2}) \\ \vdots & \vdots & \ddots & \vdots \\ p_{q,1}(t_{q,k_q}) & p_{q,2}(t_{q,k_q}) & \cdots & p_{q,n_q}(t_{q,k_q}) \end{bmatrix}, \quad (9)$$

where k_q is the number of data points between the $(q-1)$ th and q th peak, and $t_{q,r}$ is the time corresponding to these data points, and

$$p_{q,r}(t_{q,r}) = \left. \frac{\partial \left(\frac{di(t)}{dt} \right)}{\partial \beta_{q,r}} \right|_{t=t_{q,r}} = \begin{cases} 2I_{Dm_q} \eta_{q,k} \beta_{q,k} h_q(t) x(\beta_{q,k}^2 + 1), & 1 \leq q \leq p, \\ I_{Dm_q} \eta_{q,k} \beta_{q,k} h_q(t) x(\beta_{q,k}^2), & q = p + 1, \end{cases}$$

with

$$h_q(t) = \begin{cases} \ln \left(\frac{t - t_{m_{q-1}}}{\Delta t_{m_q}} \right) + \frac{t - t_{m_{q-1}}}{\Delta t_{m_q}} + 1, & 1 \leq q \leq p, \\ \ln \left(\frac{t}{t_{m_q}} \right) + \frac{t}{t_{m_q}} + 1, & q = p + 1. \end{cases}$$

3. AEF REPRESENTING MEASURED LIGHTING CURRENT DERIVATIVE - EXAMPLES

In this section we validate our model by attempting to represent measured lightning current derivatives data obtained at the 553m CN Tower, Toronto, Canada, [5]. Time and current values corresponding to AEF peaks were chosen manually and the rest of the AEF parameters were obtained using the framework briefly described in Section 2.2. The number of time intervals and terms in each of them vary from example to example. General notation, $AEF_p(n_1, \dots, n_p)$ for $n_q, q=1, \dots, p$, is used to denote an AEF with p peaks and chosen number of terms n_q in each time interval q .

3.1. Single-peaked waveshape

The first example illustrates the application of a single-peaked AEF to representation of the measured initial current derivative impulse occurring in the first 0.5 μ s given in [5, Fig. 4]. The best fitting was obtained choosing two terms in each of the two time intervals: $0-t_m$ and $t_m-0.5 \mu$ s (the moment t_m corresponds to the maximum current derivative). Current derivative value at $t=0$ is treated as the first point of approximation, so there are 4 terms in total, for these 2 intervals. Obtained $AEF_2(2,2)$ model is illustrated in Fig. 1a along with the measured data, data points used for the MLSM fitting, and the locations of peaks observed in this waveshape. Using the expressions (7) and (8) we also obtained the AEF's integral, i.e. the lighting discharge current. It can be observed in Fig. 1b along with the numerically integrated measured data.

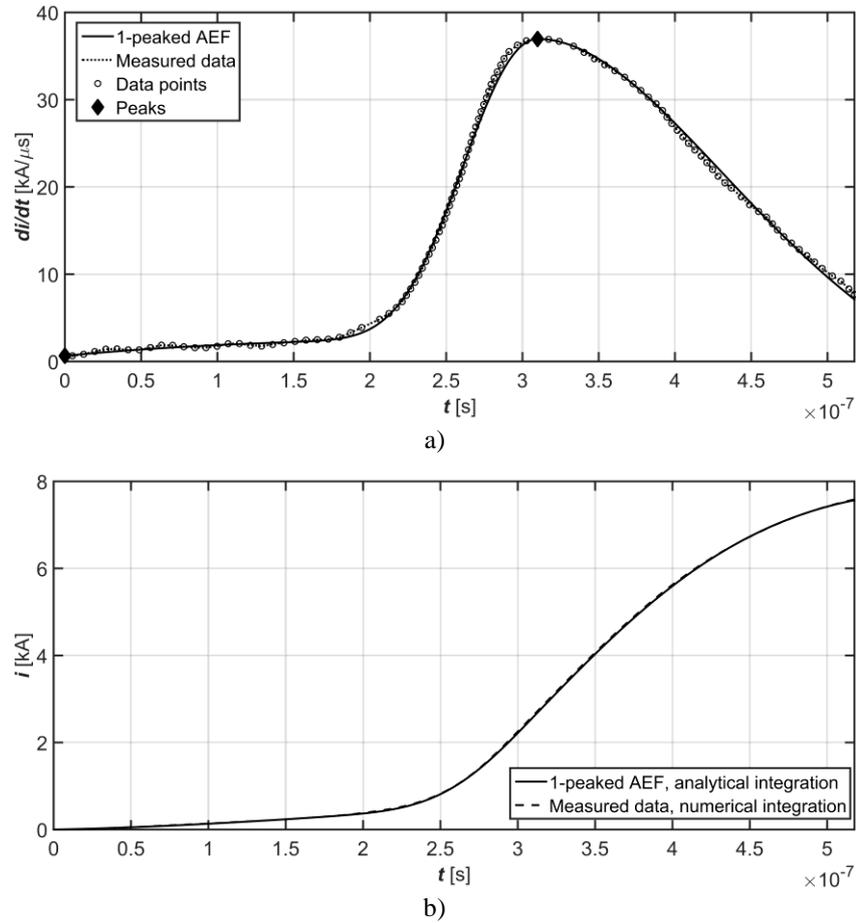


Fig. 1 a) $\text{AEF}_2(2,2)$ representing measured lightning current derivative from [5, Fig. 4], and b) the corresponding lightning current

3.2. Multi-peaked waveshapes

In this part we attempt modeling of the measured current derivatives data that include the initial and a number of subsequent impulses. The recorded waveshapes have great number of peaks and therefore is harder to model them using standard functions, but these are more suitable for modelling by the multi-peaked AEF.

The first example corresponds to an event of lightning discharge measured at the CN Tower, using the Rogowski coil positioned at 474 m, illustrated in [5, Fig. 2] in $10 \mu\text{s}$. Such current derivative waveshape corresponds to typical fast-rising negative lightning discharge which occurs in about 80% of the registered cases (in 126 flashes out of 160 [5]). The complexity of the AEF used for modelling of such multi-peaked waveshapes depends on the desired level of accuracy of the data representation.

In Fig. 2 are presented two AEFs with different number of peaks, including the starting current derivative value at $t = 0$ and other peaks which are chosen such that they correspond to local maxima only: a) $AEF_6(1,2,2,2,2,2)$ with 6 intervals and 11 terms in total, and b) $AEF_8(1,2,2,2,2,2,2,2)$ with 8 intervals and 15 terms in total. The increased number of time intervals fixes representation of the waveshape part corresponding to the period between the fourth and fifth peak of AEF_6 , and also after its sixth peak, so that the total number of intervals in AEF_8 is increased by 2, whereas the number of terms by 4.

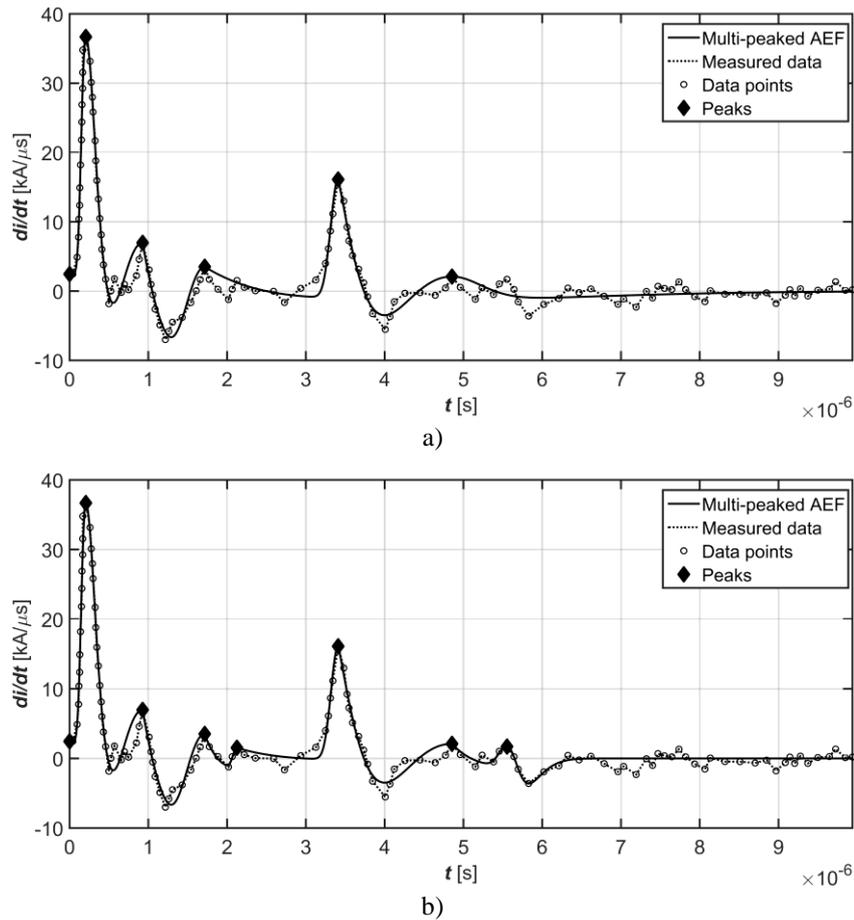


Fig. 2 Multi-peaked AEFs (using starting point and maxima only) representing measured lightning current derivative from [5, Fig. 2]: a) $AEF_6(1,2,2,2,2,2)$ with 6 peaks, b) $AEF_8(1,2,2,2,2,2,2,2)$ with 8 peaks

Additional improvement is needed and could be achieved by further segmentation and including also local minima, so as by increasing the number of terms. Two such AEF models are illustrated in Figs. 3a and 3b, both with thirteen peaks, but for different number of terms chosen to represent some of its intervals. Thirteen peaks in AEF_{13} include 4 minima added to AEF_8 and also one more maximum at its ending part, so that the number of peaks is increased from 8 (in Fig.2b) to 13 (in Figs. 3a and 3b). These two AEFs are denoted by a) $AEF_{13a}(1,1,1,1,1,2,1,1,1,2,2,1)$ with 13 intervals and 16 terms in total, and b) $AEF_{13b}(1,1,2,1,2,2,1,2,1,2,2,2,2)$ with 13 intervals and 21 terms in total, where the bold numbers in brackets point out to the changed number of terms, in some intervals increased from **1** to **2**.

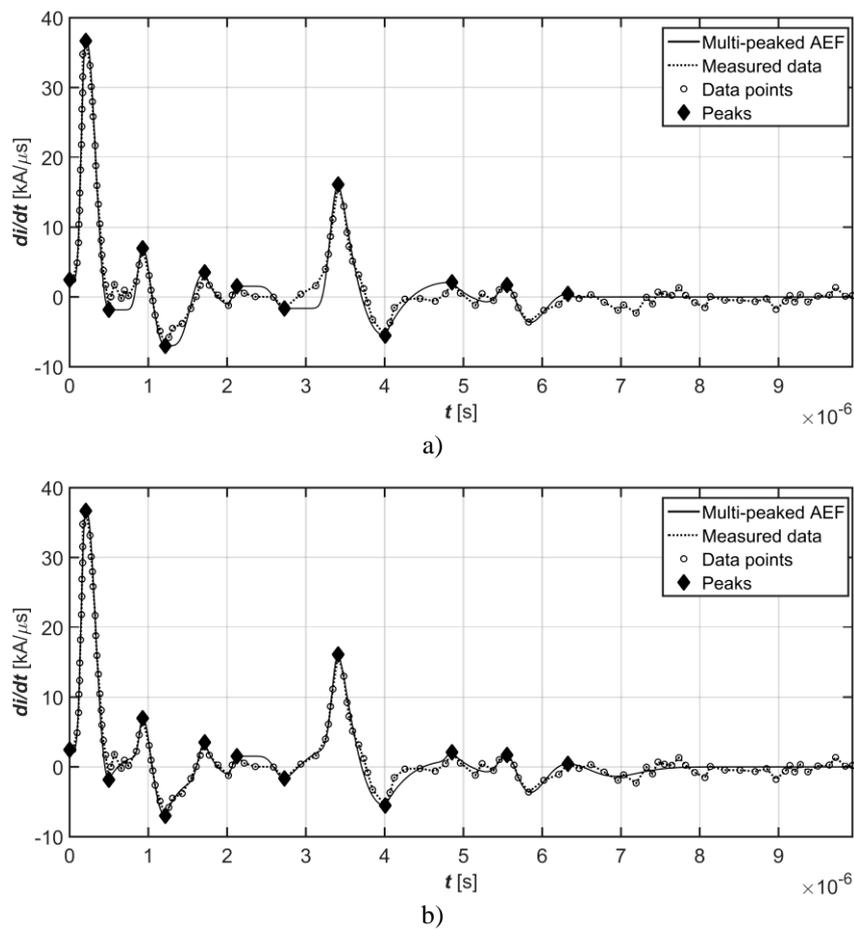


Fig. 3 Multi-peaked AEFs with 13 peaks (using starting point, 8 maxima and 4 minima) representing measured lightning current derivative from [5, Fig. 2]:
a) $AEF_{13a}(1,1,1,1,1,2,1,1,1,2,2,1)$, b) $AEF_{13b}(1,1,2,1,2,2,1,2,1,2,2,2,2)$

Results for the same lightning current derivative measured at CN Tower are given in first 7 μ s in Figs. 4a and 4b for fitting by AEFs corresponding to data from [5, Fig. 6]. Model AEF₇(1,2,2,2,2,2,2) with 7 peaks (starting point and maxima only) and 13 terms is presented in Fig. 4a, able to capture the initial impulse and subsequent peaks due to reflections at the tower discontinuities. AEF₇ has one more peak added at the end of AEF₆, and 2 more terms in total.

In Fig. 4b, AEF_{13c}(1,2,2,2,2,2,2,2,2,1,1,2,2) model is presented with the total of 13 peaks (the starting point, 8 maxima and 4 minima), which almost perfectly models measured set of data. It has 23 terms in total, 4 terms added and 2 excluded compared to AEF_{13b}. The difference between those two is that 1 peak was added for AEF_{13c} between tenth and eleventh peak of AEF_{13b}, which improved significantly the approximation, and the thirteenth peak was excluded from the end of AEF_{13b}.

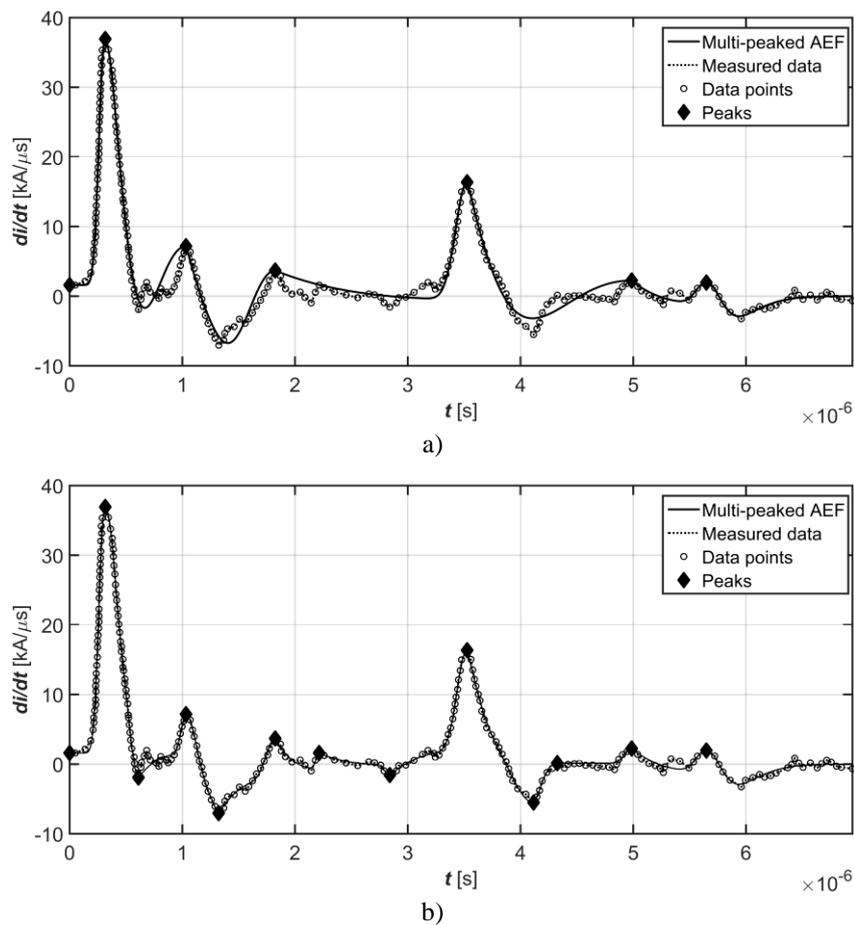
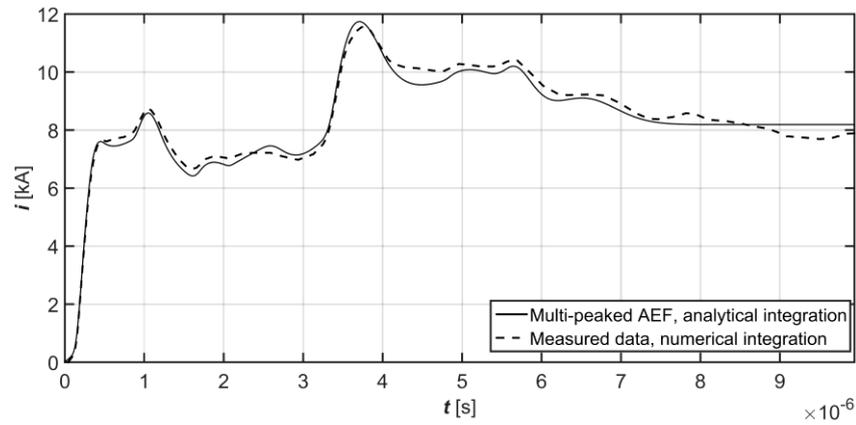
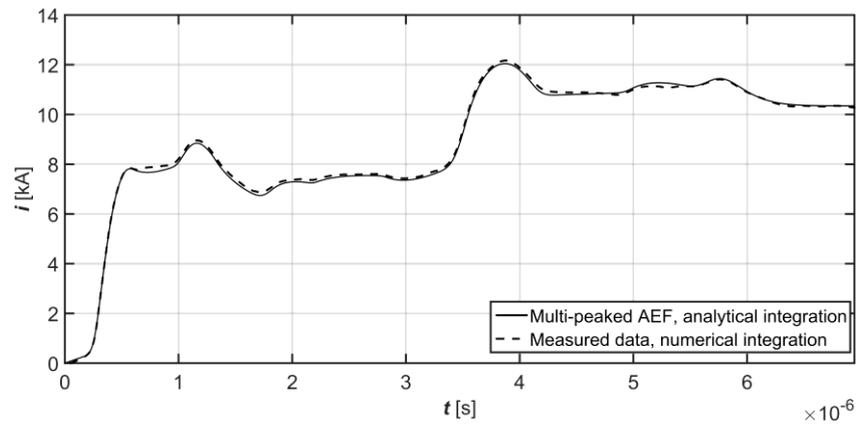


Fig. 4 Multi-peaked AEF representing measured current derivative from [5, Fig. 6]:
 a) AEF₇(1,2,2,2,2,2,2) with 7 peaks (using starting point and maxima only),
 b) AEF_{13c}(1,2,2,2,2,2,2,2,2,1,1,2,2) with 13 peaks (starting point, 8 maxima&4 minima)

Figure 5 illustrates lightning discharge currents corresponding to above modelled multi-peaked current derivative waveshapes. Again, expressions (7) and (8) are employed to calculate them, and the numerically integrated measured data is also given for comparison. Fig. 5a corresponds to $AEF_{13b}(1,1,2,1,2,2,1,2,1,2,2,2,2)$ model shown in Fig. 3b, while Fig. 5b relates to model $AEF_{13c}(1,2,2,2,2,2,2,2,2,1,1,2,2)$ from Fig. 4b.



a)



b)

Fig. 5 Lightning currents corresponding to derivatives modelled by AEFs:

- a) $AEF_{13b}(1,1,2,1,2,2,1,2,1,2,2,2,2)$ from Fig. 3b,
- b) $AEF_{13c}(1,2,2,2,2,2,2,2,2,1,1,2,2)$ from Fig. 4b

4. CONCLUSIONS

Approximation of lightning current derivatives is needed for calculation of lightning induced effects and for improvements of lightning discharge models. Suitability of the multi-peaked AEF to represent lightning current derivatives is presented in this paper through a few examples.

AEF's non-linear parameters are calculated using Marquardt least-squares method (MLSM), so that the measured current derivatives signals [5] are well approximated. The approximation by AEFs in this paper is done for single- and multi-peaked current derivative waveshapes. Increasing the number of maxima and minima, so as the number of terms in total, improves the approximation of the current derivative by AEF. The lightning current waveshape is obtained with great accuracy as analytically integrated AEF representation of the measured derivative.

Multi-peaked lightning current derivatives are characteristic for lightning discharges to tall towers and high structures at elevated terrain, but also for subsequent lightning strokes and lightning current channels with discontinuities and branching. Further work should be aimed at including such current and its derivative function into lightning stroke models in order to obtain measured lightning electromagnetic field at certain distances.

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