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SUPER-SECH SOLITON DYNAMICS IN OPTICAL METAMATERIALS USING COLLECTIVE VARIABLES

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Abstract. *This paper presents collective variable approach for super-sech soliton dynamics in optical metamaterials. The soliton dynamics is governed by the generalized nonlinear Schrödinger's equation. The numerical simulations of pulse width, amplitude, chirp and frequency are given.*

Key words: *solitons, metamaterials, super-sech*

1. INTRODUCTION

Optical metamaterials as novel type of microstructured material have been extensively studied [1–15]. Metamaterials (MMs) are artificial composite structures with both negative permittivity and negative permeability and fascinating physical properties at terahertz and optical frequencies. Different waveguide structures using metamaterials are already demonstrated in optical region [3]. Optical waveguide can be implemented by slab structure with core made of positive-indexed material and claddings of double negative materials. These waveguides are engineered using advanced processing technology. However, the design of microstructured materials is limited by losses. Nevertheless, the development of low-loss metamaterials could be the foundation of switches, modulators and other novel optical devices in all-optical integrated information processing systems.

The transmission of ultrashort pulses through such promising material exhibit unique feature. It is well known that soliton is one of the remarkable nonlinear excitations produced by the balance between nonlinearity and group velocity dispersion [9–11, 13–19]. Recent

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researches point out that ultrashort pulses propagating in MMs can be described by a modified generalized nonlinear Schrödinger equation (GNLSE) in which the linear and nonlinear coefficients can be tailored to attain any combination of signs unachievable in ordinary materials [1–13]. Simply engineering the MMs can tailor linear and nonlinear effective properties. The nonlinear MMs exhibit a rich spatiotemporal dynamics and promising applications which was unthinkable in the past [10–14]. Metamaterials enhance nonlinearity by confining electrical field in a small region, so it is a great challenge to compensate losses and nonlinearity, using metamaterials as waveguides. In metamaterials, linear and nonlinear coefficients of the propagation equation can be set to achieve any combination of signs that is not possible in regular materials. This metamaterials properties allow propagation of a wider variety of solitary waves, efficient phase-matching and modulational instability. Earlier results disclose that similar regular (positive indexed) dielectric material dispersion plays a crucial role in supporting short duration soliton pulses. The dynamics of soliton propagation through these optical metamaterials is governed by the nonlinear Schrödinger'squation (NLSE) with a few perturbation terms. The integrability aspect of this model was studied with various forms of nonlinearity [9-15]. Different algorithms are used to yield solitons, shock waves and other solution to the model that appeared with several integrability conditions.

1.1. Governing equation

The dynamics of solitons in optical metamaterials is governed by the model [4-7]

$$\begin{aligned} iq_z + aq_{tt} + c\,|q|^2\,q = i\alpha q_t + i\lambda(|q|^2\,q)_t + i\nu(|q|^2)_t\,q \\ + \theta_1(|q|^2\,q)_{tt} + \theta_2\,|q|^2\,q_{tt} + \theta_3 q^2 q_{tt}^* \end{aligned} \tag{1}$$

Equation (1) is the nonlinear Schrödinger's equation (NLSE) that is studied in the context of metamaterials. Here in (1), *a* and *b* are the group velocity dispersion and the self-phase modulation terms respectively. This pair produces the delicate balance between dispersion and nonlinearity that accounts for the formation of the stable solitons. On the right hand side λ represents the self-steepening term in order to avoid the formation of shocks and *v* is the nonlinear dispersion, while α represents the intermodal dispersion. Then finally, θ_j for j = 1,2, 3 are the perturbation terms that appears in the context of metamaterials [1]

2. Collective Variable Approach Algorithm

Algorithm of collective variables principle implies that solution of NLSE is split into two components [9, 11, 14]. The first one constitute soliton solution while the second one represents the residual radiation. Decomposition of the original soliton field $q(z,t)$ is made at position z in the fiber and at time t, as follows:

$$q(z,t) \;=\; f(z,t) \;+\; g(z,t) \tag{2}$$

The soliton field f is defined as a function that depends on parameters, symbolically represented by $X_j,\ j = 1,...,n$

$$q(z,t) = f(X_1, X_2, \ldots, X_N, t) + g(z,t), \tag{3}$$

where collection of variables represent soliton amplitude, temporal position, pulse width, chirp, frequency and phase of the pulse.

Introduction of CV in function *f* increases the degrees of freedom resulting in the expansion of available phase space of the system. That is undesirable effect, so there are some constraints and residual free energy given by:

$$E = \int_{-\infty}^{\infty} |g|^2 \, dt = \int_{-\infty}^{\infty} |q - f(X_1, X_2, ..., X_N)|^2 \, dt. \tag{4}$$

should be minimized.

From this definition, let C_j denote the rate of change of residual free energy with respect to the j^{th} CV X_j.

$$C_j = \frac{\partial E}{\partial X_j} = \frac{\partial}{\partial X_j} \int_{-\infty}^{\infty} |g|^2 \, dt = \int_{-\infty}^{\infty} \left(\frac{\partial g}{\partial X_j} g^* + \frac{\partial g^*}{\partial X_j} g \right) dt \tag{5}$$

Second parameter that should be defined is $\dot{C}_j$, the rate of change of C_j with the normalized distance. Using $g(z,t) = q(z,t) - f\{X_1(z,t), X_2(z,t), \ldots, X_N(z,t), t\}$ in the above equation, C_j can be rewritten as:

$$C_j = \left\langle \frac{\partial f^*}{\partial X_j} g \right\rangle + \left\langle \frac{\partial f}{\partial X_j} g^* \right\rangle \tag{6}$$

Now, parameter $\dot{C}_j$ can be presented as:

$$\dot{C}_j = \frac{dC_j}{dz} = 2\Re \left\{ \frac{d}{dz} \left(\int_{-\infty}^{\infty} \frac{\partial f^*}{\partial X_j} g dt \right) \right\} = 2\Re \left(\int_{-\infty}^{\infty} \frac{\partial f^*}{\partial X_j} \frac{\partial g}{\partial z} dt + \sum_{k=1}^{N} \int_{-\infty}^{\infty} \frac{\partial^2 f^*}{\partial X_j \partial X_k} \frac{\partial x_k}{\partial z} g dt \right) \tag{7}$$

The overhead dot represents the derivative with respect to z and the subscripts X_j denote partial derivative. $\Re$ represents the real part and $\langle \cdot \rangle$ means $\int_{-\infty}^{\infty} (\cdot)$. Thus,

$$\dot{C}_j = 2\Re \left(\left\langle \frac{\partial f^*}{\partial X_j}, \frac{\partial g}{\partial z} \right\rangle + \left\langle \frac{\partial^2 f^*}{\partial X_j \partial X_k}, \frac{\partial X_k}{\partial z} g \right\rangle \right) \tag{8}$$

Dirac's principle implies that if a function is approximately zero, it cannot be set equal to zero until its variations with respect to all its parameters are made. Therefore, C_j are minimum and the equations of the constraints are obtained as:

$$C_j \approx 0 \tag{9}$$

$$\dot{C}_j \approx 0 \tag{10}$$

Substituting (2) into (1), we obtain equations of motion of the residual field $g(z, t)$ which upon substitution into (7) gives

$$-\dot{C}_j = 2\Re\sum_{k=1}^{N}\left[\int_{-\infty}^{\infty}\frac{\partial f^*}{\partial X_j}\frac{\partial f}{\partial X_k}dt-\int_{-\infty}^{\infty}\frac{\partial^2 f^*}{\partial X_j\partial X_k}dt\right]\frac{dX_k}{dz}+R_j \tag{11}$$

where

$$\begin{aligned}R_j = &-2\Re\int_{-\infty}^{\infty} iaf_{X_j}^* f_{tt}dt-2\Re\int_{-\infty}^{\infty} iaf_{X_j}^* g_{tt}dt-2\Re\int_{-\infty}^{\infty} ibf_{X_j}^* \mid f+g\mid^2 \ fdt\\
&-2\Re\int_{-\infty}^{\infty} ibf_{X_j}^* \mid f+g\mid^2 \ gdt-2\Re\int_{-\infty}^{\infty}\alpha f_{X_j}^* f_t dt\\
&-2\Re\int_{-\infty}^{\infty}\alpha f_{X_j}^* g_t dt-2\Re\int_{-\infty}^{\infty}\lambda f_{X_j}^*(\mid f+g\mid^2 \ f)_t dt\\
&-2\Re\int_{-\infty}^{\infty}\lambda f_{X_j}^*(\mid f+g\mid^2 \ g)_t dt\\
&-2\Re\int_{-\infty}^{\infty}\nu f_{X_j}^*(\mid f+g\mid^2)_t \ fdt-2\Re\int_{-\infty}^{\infty}\nu f_{X_j}^*(\mid f+g\mid^2)_t \ gdt\\
&+2\Re\int_{-\infty}^{\infty} i\theta_1 f_{X_j}^*(\mid f+g\mid^2 \ f)_{tt}dt+2\Re\int_{-\infty}^{\infty} i\theta_1 f_{X_j}^*(\mid f+g\mid^2 \ g)_{tt}dt\\
&+2\Re\int_{-\infty}^{\infty} i\theta_2 f_{X_j}^* \mid f+g\mid^2 \ f_{tt}dt+2\Re\int_{-\infty}^{\infty} i\theta_2 f_{X_j}^* \mid f+g\mid^2 \ g_{tt}dt\\
&+2\Re\int_{-\infty}^{\infty} i\theta_3 f_{X_j}^*(f+g)^2 \ f_{tt}^* dt+2\Re\int_{-\infty}^{\infty} i\theta_3 f_{X_j}^*(f+g)^2 g_{tt}^* dt\end{aligned}$$

Equation (11) is equivalent to the matrix equation

$$\dot{\mathbf{C}} = \frac{\partial \mathbf{C}}{\partial \mathbf{X}}\dot{\mathbf{X}}+\mathbf{R} \tag{12}$$

$$-\left(\frac{\partial C}{\partial X}\right)=\begin{pmatrix}\frac{\partial C_1}{\partial X_1} & \frac{\partial C_1}{\partial X_2} & \cdots & \frac{\partial C_1}{\partial X_N}\\ \frac{\partial C_2}{\partial X_1} & \frac{\partial C_2}{\partial X_2} & \cdots & \frac{\partial C_2}{\partial X_N}\\ \vdots & \vdots & & \vdots\\ \frac{\partial C_N}{\partial X_1} & \frac{\partial C_N}{\partial X_2} & \cdots & \frac{\partial C_N}{\partial X_N}\end{pmatrix},\ \dot{\mathbf{X}}=\begin{bmatrix}\dot{X}_1\\ \dot{X}_2\\ \cdots\\ \dot{X}_N\end{bmatrix},\ \mathbf{R}=\begin{bmatrix}R_1\\ R_2\\ \cdots\\ R_N\end{bmatrix} \tag{13}$$

with

$$\frac{\partial C_j}{\partial X_k} = 2\Re\left(\int_{-\infty}^{\infty}\frac{\partial f^*}{\partial X_j}\frac{\partial f}{\partial X_k}dt-\int_{-\infty}^{\infty}\frac{\partial^2 f^*}{\partial X_j\partial X_k}gdt\right) \tag{14}$$

3. SUPER-SECH PARAMETER DYNAMICS

In this section soliton parameter dynamics in optical metamaterials will be obtained by CV approach. We assume the desired form of the function *f* is:

$$f = X_1 \operatorname{sech}^m \left[\frac{t - X_2}{X_3} \right] \exp[i \frac{X_4}{2}(t - X_2)^2 + iX_5(t - X_2) + iX_6] \tag{15}$$

where X_1 stands for soliton amplitude, X_2 the center position of the soliton, X_3 the pulse width, X_4 the soliton chirp parameter, X_5 the soliton frequency and X_6 the soliton phase that evolves along with propagation. Also m is the super-sech parameter, where $m > 0$. In this case $N = 6$ and matrices have dimension 6x6.

Equations for all the CV are obtained under lowest order CV theory, *bare approximation*. Applying the bare approximation implies that residual field is set to zero, $g(z,t) = 0$. For $m = 2$ elements of matrix **R** are as follows:

$$R_1 = \frac{32}{35} X_1^3 X_3 X_4 (\theta_1 - \theta_2 \theta_3) \tag{16}$$

$$\begin{aligned} R_2 = {} & \frac{512 \nu X_1^4}{315 X_3} + \frac{64}{35} b X_1^4 X_3 X_5 + \\ & + \frac{2\alpha X_1^2 (336 + 35(-6 + \pi^2) X_3^4 X_4^2 + 420 X_3^2 X_5^2)}{315 X_3} \\ & + \frac{2\lambda X_1^4 (384 + 4(-49 + 6\pi^2) X_3^4 X_4^2 + 288 X_3^2 X_5^2)}{315 X_3} \\ & + \frac{2a X_1^2 (-1008 X_5 - 105(-6 + \pi^2) X_3^4 X_4^2 X_5 - 420 X_3^2 X_5^3)}{315 X_3} \\ & + \frac{2\theta_1 X_1^4 X_5 (1152 + 12(-49 + 6\pi^2) X_3^4 X_4^2 + 288 X_3^2 X_5^2)}{315 X_3} \\ & + \frac{2\theta_2 X_1^4 X_5 (640 + 12(-49 + 6\pi^2) X_3^4 X_4^2 + 288 X_3^2 X_5^2)}{315 X_3} \\ & + \frac{2\theta_3 X_1^4 X_5 (128 + 12(-49 + 6\pi^2) X_3^4 X_4^2 + 288 X_3^2 X_5^2)}{315 X_3} \end{aligned} \tag{17}$$

$$\begin{aligned} R_3 = {} & \frac{4}{45} a (15 - 4\pi^2) X_1^2 X_4 + \frac{4\theta_1 X_1^2 X_4 (-324 X_1^2 + 48(-31 + 6\pi^2) X_1^2)}{2835} \\ & + \frac{16(-205 + 24\pi^2) X_1^4 X_4}{2835} (\theta_2 + \theta_3) \end{aligned} \tag{18}$$

$$\begin{aligned}
R_4 = & -\frac{4}{315} b(-49+6\pi^2) X_1^4 X_3^3 - \frac{1}{9}(-6+\pi^2)\alpha X_1^2 X_3^3 X_5 - \frac{4}{315}(-49+6\pi^2)\lambda X_1^4 X_3^3 X_5 \\
& - \frac{a X_1^2 X_3(-15120(-15+\pi^2) - 945\pi^2(-60+7\pi^2) X_3^4 X_4^2 - 18900(-6+\pi^2) X_3^2 X_5^2)}{170100} \\
& - \frac{\theta_1 X_1^4 X_3(64(58715-7194\pi^2) + 1512(60-35\pi^2+3\pi^4) X_3^4 X_4^2 + 2160(-49+6\pi^2) X_3^2 X_5^2)}{170100} - \\
& - \frac{\theta_2 X_1^4 X_3(5760(-29+3\pi^2) + 1512(60-35\pi^2+3\pi^4) X_3^4 X_4^2 + 2160(-49+6\pi^2) X_3^2 X_5^2)}{170100} \\
& - \frac{\theta_3 X_1^4 X_3(5760(-29+3\pi^2) + 1512(60-35\pi^2+3\pi^4) X_3^4 X_4^2 + 2160(-49+6\pi^2) X_3^2 X_5^2)}{170100}
\end{aligned} \tag{19}$$

$$\begin{aligned}
R_5 = & -\frac{2}{9}(-6+\pi^2)\alpha X_1^2 X_3^3 X_4 - \frac{8}{315}(-49+6\pi^2)\lambda X_1^4 X_3^3 X_4 \\
& + \frac{4}{9} a(-6+\pi^2) X_1^2 X_3^3 X_4 X_5 - \frac{16}{315}(-49+6\pi^2) X_1^4 X_3^3 X_4 X_5 \theta_1 \\
& - \frac{16}{315}(-49+6\pi^2) X_1^4 X_3^3 X_4 X_5 \theta_2 - \frac{16}{315}(-49+6\pi^2) X_1^4 X_3^3 X_4 X_5 \theta_3
\end{aligned} \tag{20}$$

$$\begin{aligned}
R_6 = & -\frac{64}{35} b X_1^4 X_3 - \frac{8}{3}\alpha X_1^2 X_3 X_5 - \frac{64}{35}\lambda X_1^4 X_3 X_5 \\
& - \frac{2a X_1^2(-336-35(-6+\pi^2) X_3^4 X_4^2 - 420 X_3^2 X_5^2)}{315 X_3} \\
& - \frac{2\theta_1 X_1^4(384+4(-49+6\pi^2) X_3^4 X_4^2 + 288 X_3^2 X_5^2)}{315 X_3} \\
& - \frac{2\theta_2 X_1^4(384+4(-49+6\pi^2) X_3^4 X_4^2 + 288 X_3^2 X_5^2)}{315 X_3} \\
& - \frac{2\theta_3 X_1^4(384+4(-49+6\pi^2) X_3^4 X_4^2 + 288 X_3^2 X_5^2)}{315 X_3}
\end{aligned} \tag{21}$$

Finally, the nonlinear dynamical system (DS) reduces to:

$$\dot{X}_1 = -a X_1 X_4 + \frac{4\theta_1(-755+84\pi^2) X_1^3 X_4}{105(-15+4\pi^2)} + \frac{4 X_1^3 X_4(-1025+228\pi^2)}{315(-15+4\pi^2)}(\theta_2+\theta_3) \tag{22}$$

$$\dot{X}_2 = \frac{1}{21}(-21(\alpha - 2a X_5) - 8 X_1^2(3\lambda + 2\nu + 2 X_5(3\theta_1 + \theta_2 - \theta_3))) \tag{23}$$

$$\dot{X}_3 = \frac{2 X_3 X_4(63a(-15+4\pi^2) - 8 X_1^2(3(-89+12\pi^2)\theta_1 + 2(-31+6\pi^2)(\theta_2 - \theta_3)))}{63(-15+4\pi^2)} \tag{24}$$

$$
\begin{aligned}
\dot{X}_4 = &-\frac{312bX_1^2}{7pX_3^2}(b+\lambda X_5)-\frac{2a(-108+pX_3^4X_4^2)}{pX_3^4}\\
&-\frac{2\theta_1X_1^2(r-9qX_3^4X_4^2+7020X_3^2X_5^2)}{315pX_3^4}\\
&-\frac{2\theta_2X_1^2(15840-9qX_3^4X_4^2+7020X_3^2X_5^2)}{315pX_3^4}\\
&-\frac{2\theta_3X_1^2(15840-9qX_3^4X_4^2+7020X_3^2X_5^2)}{315pX_3^4}
\end{aligned}
\tag{25}
$$

$$
\begin{aligned}
\dot{X}_5 = &-\frac{1}{105\left(-6+\pi^2\right)}4X_1^2X_4(3\left(-11+4\pi^2\right)\lambda+\\
&20\left(-6+\pi^2\right)\nu+2X_5\left(3\left(-11+4\pi^2\right)\theta_1+\left(87-8\pi^2\right)\theta_2+\left(207-28\pi^2\right)\theta_3\right))
\end{aligned}
\tag{26}
$$

$$
\begin{aligned}
\dot{X}_6 = &\frac{1}{3780(-45-30\pi^2+4\pi^4)X_3^2}(756a(450+75\pi^2-16\pi^4+5(-45-30\pi^2+4\pi^4)X_3^2X_5^2)+\\
&X_1^2(81(1680-280\pi^2+13\pi^4)X_3^4X_4^2(\theta_1+\theta_2+\theta_3)+\\
&16((352290-111599\pi^2+8328\pi^4)\theta_1+18(-870-305\pi^2+48\pi^4)(\theta_2+\theta_3))+\\
&36X_3^2(3b(-1470-655\pi^2+96\pi^4)+X_5(3(330+545\pi^2-64\pi^4)\lambda-\\
&80(-45-30\pi^2+4\pi^4)\nu+X_5((6390+5235\pi^2-672\pi^4)\theta_1+\\
&(-810+435\pi^2-32\pi^4)\theta_2+(-8010-4365\pi^2+608\pi^4)\theta_3)))))
\end{aligned}
\tag{27}
$$

where $p=4\pi^4-30\pi^2-45$, $q=96\pi^4-1045\pi^2-1050$, $r=16(7464\pi^2-60335)$;

4. RESULTS AND CONCLUSION

Collective variable approach was applied to solve the evolution equation that governs the dynamics of soliton and its propagation through optical metamaterials.

Numerical investigations on the evolution of pulse parameters have been carried out in order to illustrate results of collective variable approach. Results have been obtained using standard fourth order Runge-Kutta method for integration of the system of ordinary differential equations that resulted from the CV analysis. In figure 1 dynamic of the system is presented for the following parameter values: $\alpha = 0.25$, $a = 0.1$, $b = 20$, $\lambda = 0.1$, $\nu = 0.1$, $\theta_1 = -0.1$, $\theta_2 = -0.2$, $\theta_3 = -0.3$. As the pulse propagates, the amplitude (X_1), pulse width (X_3), frequency (X_5) and chirp (X_4) vary periodically.

The control parameter of the soliton solution as it evolves is the total energy Q. The total energy can be expressed as function of the super-sech function parameters

$$Q=\frac{4X_1^2X_3}{3} \tag{28}$$

This expression shows that the total energy strongly depends on amplitude (X_1) and the pulse width (X_3). The collective variables method enables a clear analysis of the equations and reveals the influence of various parameters.

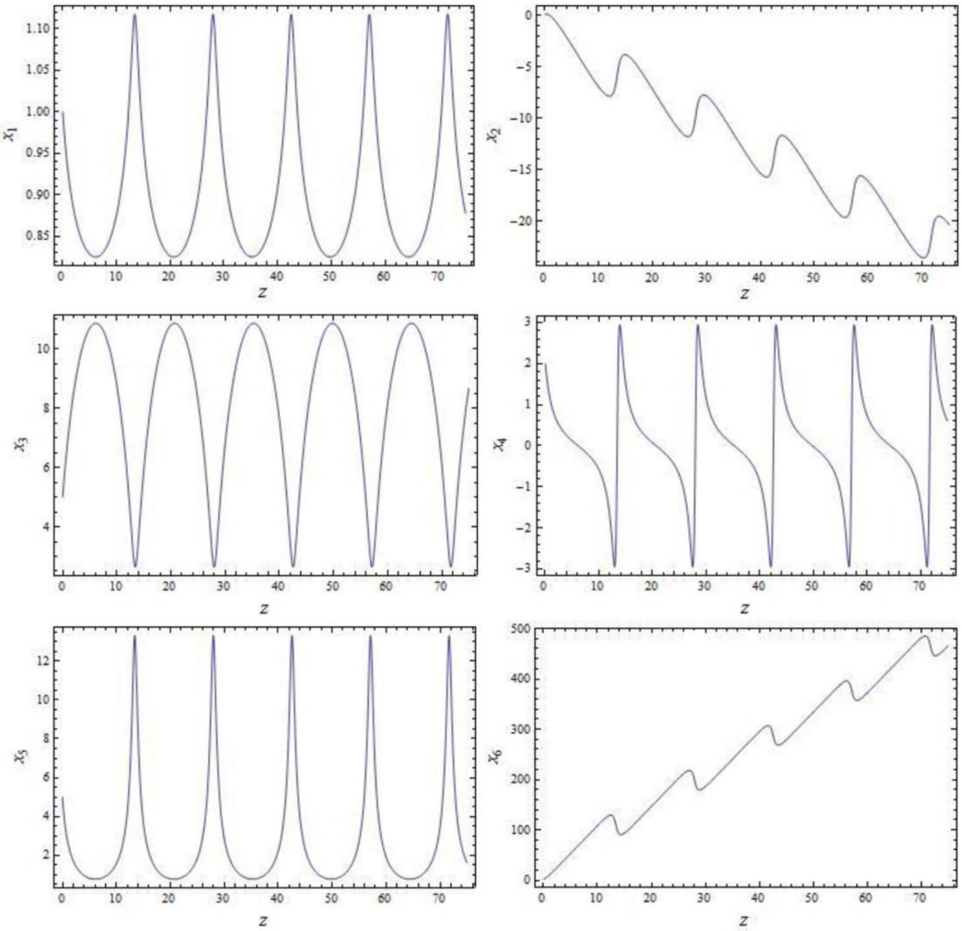

Fig. 1 Variation of pulse parameters (X_1 – soliton amplitude, X_2 – center position of the soliton, X_3 – pulse width, X_4 – soliton chirp, X_5 – soliton frequency, X_6 – soliton phase) with propagation distance.

In conclusion, we have investigated the dynamics of an ultra short pulse in optical fibers, using CV approach.This paper could be used for further investigations of solitons dynamics and the influence of nonlinear parameters on solitons amplitude, temporal position, frequency, phase and chirp.

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References

[1] V. G. Veselago, "The electrodynamics of substances with simultaneously negative values of ε and μ", *Sov. Phys. Usp.*, vol. 10, no. 4, pp. 509-514, 1968.

[2] N. A. Zharova, I. V. Shadrivov, A. A. Zharov, Y. S. Kivshar, "Nonlinear transmission and spatiotemporal solitons in metamaterials with negative refraction". *Optics Express*, vol. 13, no. 14, pp. 1291-1298, 2005.

[3] V. M. Shalaev, Nature Photonics. 1, 41 (2007).

[4] A. Biswas, K. R. Khan, M. F. Mahmood & M. Belic. "Bright and dark solitons in optical metamaterials". *Optik*, vol. 125, Issue 13, pp. 3299-3302, 2014.

[5] A. Biswas, M. Mirzazadeh, M. Savescu, D. Milovic, K. R. Khan, M. F. Mahmood & M. Belic. "Singular solitons in optical metamaterials by ansatz method and simplest equation approach", *Journal of Modern Optics*, vol. 61, Issue 19, pp. 1550-1555, 2014.

[6] A. Biswas, M. Mirzazadeh, M. Eslami, D. Milovic & M. Belic, "Solitons in optical metamaterials by functional variable method and first integral approach", *Frequenz*., vol. 68, Issues 11-12, pp. 525-530, 2014.

[7] G. Ebadi, A. Mojavir, J. Vega-Guzman, K. R. Khan, M. F. Mahmood, L. Moraru, A. Biswas & M. Belic, "Solitons in optical metamaterials by F- expansion scheme". *Optoelectronics and Advanced Materials – Rapid Communications*, vol. 8, no. 9-10, pp. 828-832, 2014.

[8] M. Veljkovic, Y. Xu, D. Milovic, M. F. Mahmood, A. Biswas, and M. R. Belic, "Super-Gaussian Solitons in Optical Metamaterials Using Collective Variables", *Journal of Computational and Theoretical Nanoscience*, vol. 12, no. 12, pp. 5119-5124, 2015.

[9] S. I. Fewo & T. C. Kofane. "A collective variable approach for optical solitons in cubic-quintic complex Ginzburg-Landau equation with third order dispersion", *Optics Communications*, vol. 281, Issue 10, pp. 2893-2906, 2008.

[10] P. Green, D. Milovic, D. Lott & A. Biswas. "Dynamics of Gaussian optical solitons by collective variables method". *Applied Mathematics and Information Sciences*, vol. 2, Issue 3, pp. 259-273, 2008.

[11] E. V. Krishnan, M. Al Gabshi, Q. Zhou, K. R. Khan, M. F. Mahmood, Y. Xu, A. Biswas & M. Belic. "Solitons in optical metamaterials by mapping method", *Journal of Optoelectronics and Advanced Materials*, vol. 17, no 3-4, pp. 511-516, 2015.

[12] A. B. Moubissi, K. Nakkeeran, P. T. Dinda & T. C. Kofane. "Non-Lagrangian collective variable approach for optical soliton in fibers", *Journal of Physics A*., vol. 34, pp. 129-136, 2001.

[13] M. Savescu, K. R. Khan, P. Naruka, H. Jafari, L. Moraru & A. Biswas. "Optical solitons in photonic nano waveguides with an improved nonlinear Schrödinger's equation", *Journal of Computational and Theoretical Nanoscience*, vol. 10, no. 5, pp. 1182-1191, 2013.

[14] S. Shwetanshumala & A. Biswas. "Femtosecond pulse propagation in optical fibers under higher order effects: A collective variables approach", *International Journal of Theoretical Physics*, vol. 47, Issue 6, pp. 1699-1708, 2008.

[15] S. Shwetanshumala. "Temporal solitons in nonlinear media modeled by modified complex Ginzburg-Landau equation under collective variables approach", *International Journal of Theoretical Physics*, vol. 48, Issue 4, pp. 1122-1131, 2008.

[16] Y. Xu, Q. Zhou, A. H. Bhrawy, K. R. Khan, M. F. Mahmood, K. R. Khan & M. Belic. "Bright solitons in optical metamaterials by traveling wave hypothesis", *Optoelectronics and Advanced Materials – Rapid Communications*, vol. 9, no. 3-4, pp. 384-387, 2015.

[17] Q. Zhou, Q. Zhu, Y. Liu, A. Biswas A. H. Bhrawy, K. R. Khan, M. F. Mahmood & M. Belic. "Solitons in optical metamaterials with parabolic law nonlinearity and spatio-temporal dispersion", *Journal of Optoelectronics and Advanced Materials*, vol. 16, no. 11-12, pp. 1221-1225, 2014.

[18] Z. Jakšić, M. Obradov, S. Vuković, M. Belić, "Plasmonic enhancement of light trapping in photodetectors", *Facta Universitatis, Series: Electronics and Energetics*, vol. 27, Issue 2, pp. 183-203, 2014.

[19] E. Suhir, "Fiber optics engineering: physical design for reliability", *Facta Universitatis, Series: Electronics and Energetics*, vol. 27, no. 2, pp. 153-182, 2014.