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A NEURAL NETWORK APPROACH FOR THE ANALYSIS OF LIMIT BEARING CAPACITY OF CONTINUOUS BEAMS DEPENDING ON THE CHARACTER OF THE LOAD

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Abstract. *Being a part of civil engineering, limit state analysis represents a structural analysis with a goal of developing efficient methods to directly estimate collapse load for a particular structural model. As a theoretical foundation, limit state analysis uses a set of bound (limit) theorems. Limit theorems are based on the law of conservation of energy and are used for a direct definition of the limit state function for failure by plastic collapse or by inadaptation. This study proposes an artificial neural network (ANN) model in order to approximate the residual bending moment, limit and the incremental failure force of continuous beams. The neural network structure applied here is a radial-Gaussian network architecture (RGIN) and complementary training procedure. This structure is intended to be used for civil engineering purposes and it is demonstrated on the example of the two-span continuous beam loaded in the middle of the span that the limit and the incremental failure force can be obtained using neural network approach with sufficient precision and is especially suitable in analysis when some of the model parameters are variable.*

Key words: *Continuous beam; incremental force; limit failure force; neural network; radial-Gaussian network architecture*

1. INTRODUCTION

Artificial intelligence can be considered as a field of computer science often defined as "science and engineering of making intelligent machines, especially intelligent computer programs" [1]. After the 50 years of advancement, technology of artificial intelligence is applied in numerous fields: expert systems, knowledge based systems,

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medical diagnosis, remote sensing, intelligent database system, civil engineering and natural language processing. Through years of extensive advancement, delimiting artificial intelligence to a narrower field of research has proven to be capable of offering many significant capabilities and applications [2]. As an example, expert systems are marked as "The Technology of Knowledge Management and Decision Making for the 21st Century" [2]. A broad applicability of artificial intelligence has influenced their usage in the field of civil engineering. It is not an unusual situation for civil engineering researchers to encounter problems influenced by many uncertainties. To resolve some of these problems, civil engineering researchers relay not only on mathematics and mechanics calculations, but also on their experience and practice. However, knowledge and experience do not guarantee that the problem will be solved using traditional procedures. This is the situation where artificial intelligence expresses its supremacy by solving complex problems to the levels of experts by means of imitating experts. This is one of the main reasons why artificial intelligence has a broad application prospects in the practice of civil engineering [3].

In the practice of civil engineering, the broadest interest was shown to artificial neural networks (ANN) [4 - 8], mainly due to their ability to process external data and information based on past experiences. Artificial neural networks represent models of real world problems. ANNs are capable of mapping a set of given patterns to an associated set of a priori known values. They can be observed as non-linear operators that transform input patterns into another set of numerical data at its output. Output values are usually gathered through repeated observations of a particular phenomenon. The ANN is trained with the input data patterns to perform the transformation and to become the numerical model of the observed phenomenon.

ANNs have the ability to learn from examples and to adapt to changing situation. Also, they are capable of bidirectional mappings, e.g. mapping from cause to effect for estimation and prediction and mapping from effect to possible cause [4, 9]. Neural networks can be thought of as models which try to imitate some of the learning activities of the human brain, although they are much simpler. While doing so, the internal structure of the neural operator ANN remains unaltered for a variety of problems. This facilitates the usage of ANN within different applications, including civil engineering applications, because it becomes enough for an application to have and suitably interpret ANN input-output data pairs in order to become able to use the ANN as the numerical model of the particular phenomenon.

Limit state analysis of structures is an analytic procedure which determines the maximum load parameter of load increment parameter, which can be sustained by an elasto-plastic structure. If the structure is exposed to the action of gradually increasing load, at some point it can surpass a certain critical value, which causes the plastic failure of the structure, after which the structure is not capable of receiving any further increase of load. This critical state is called the limit state of structure, and the load that causes it is called the limit load. Determination of the bearing capacity of a structure is very valuable, not only as a simple control of beam bearing capacity, but also as a significant basis and factor in designing of structures.

The beginning of the limit state analysis is related to Kazincy [10], who calculated failure load of the beam fixed at both ends, and confirmed the results through experiments. Even though the static theorem was first proposed by Kist [11], as an intuitive axiom, it is considered that the basic theorem of limit state analysis was first announced by Gvozdev [12]. The limit state analysis theorems were independently developed by Hill for the stiff perfectly plastic material [13], as well as Drucker, Prager and Greenberg [14], for elastic perfectly plastic material. In the meantime, a formal proof of these theorems for beams and frames was derived by Horne [15], as well as Greenberg and Prager [16]. Application of limit theorems in designing of civil engineering structures was later applied by many authors among the following are prominent: Symonds and Neal [17]; Hodge [18]; Baker and Heyman [19]; Zyczkowski [20]; Save [21].

The loading on a structure may vary considerably during its lifetime. For example, apart from dead-loading, a building frame will experience snow loads on the roof and wind loads on each face. The magnitudes of these loads at any particular instant cannot be foreseen, although their characteristic values will be known, so that the sequence of loading is unpredictable. This type of loading is termed variable repeated loading [22].

Generally, the designer's knowledge of the future loadings to which a particular structure will be exposed is usually as follows:

- Types of loads such as live load, wind load, water pressure, snow weight, dead weight, etc. are clearly determined;
- Limits of variations of load intensities of particular load types are also known as supplied by the design codes or they follow from some technological or service conditions;
- Actual future history of the loads, however, is not given explicitly as it is impossible to predict it.

If a structure is deformed elastically, then in the presence of variable loads its strength is determined by the fatigue properties of the material; fracture occurs after a large number of cycles. But if the body experiences elastic-plastic deformation, a load less than limiting can cause the attainment of a critical state with a comparatively small number of cycles [23].

The fact that the collapse loads calculated according to limit state analysis may fail to provide a proper measure of structural safety in the case of variable repeated loads, was pointed out for the first time by Grüning and Bleich in 1932. In 1936 Melan presented a general static shakedown theorem and later extended it to the general case of a continuum [24]. It was Koiter, who formulated a general kinematical shakedown theorem [25].

In the recent years, the shakedown analysis of elasto-plastic structures has been increasingly applied in the analysis of engineering problems due to the increased demands of modern technologies. It is thus successfully applied for many engineering problems, such as designing of nuclear reactors, railways, in civil engineering designing and safety assessment of some building structures.

The aim of this paper is to propose an ANN model aimed to be used for civil engineering purposes in order to approximate the residual bending moment, limit and the incremental failure force of continuous beams. The neural network structure used for these purposes is radial-Gaussian network architecture (RGIN). The rest of the paper will be organized as follows. In section 2 we will present an overview of ANN implementations in civil engineering along with basic postulates of limit and shakedown analysis. Section 3 will present an analysis of the bearing capacity of continuous beams depending on the load character and degree of static indeterminacy. Section 4 will describe the neural network approach used in this research along with detailed description of a neural network structure used for the approximation of the residual bending moment, limit and the incremental failure force of continuous beams. In section 5, we will present an analysis of ANN generated results. The conclusion is presented in section 6 with an outlook of future work and improvement of the research presented in this paper.

2. Related Work

2.1. Artificial neural networks in civil engineering

It seems almost impossible to review all applications of the ANN in civil engineering. A vast amount of research papers presenting results in this science field was published since eighties of the twentieth century. Therefore, we will present a selected part of research results with due respect for all other research results not mentioned in this section. One of the earliest applications of ANN in mechanics was proposed by Ghaboussi, Garrett and Wu [26]. They have used ANN to investigate direct representation of constitutive behavior of concrete. Recently, Arangio and Beck have developed a strategy for the estimation of the integrity of a long-suspension bridge while being influenced by ambient vibrations [27]. Cachim demonstrated the usage of artificial neural networks for calculation of temperatures in timber under fire loading [28]. In this research, a multilayer feed forward network has been used to determine the temperature in the timber as the only output parameter of the neural network. Liu et al. have shown the possibility of using back propagation neural networks (BPNN) as models for predicting the compressive strength of concrete [29]. Evolutionary fuzzy hybrid neural network (EFHNN) was used to enhance the effectiveness of assessing subcontractor performance in the construction industry. This possible usage of EFHNN was demonstrated by Cheng et al., with purpose of to achieving optimal mapping of input factors and subcontractor performance output [30].

Wang et al. have shown a promising perspective of back-propagation neural network usage in cost estimate of construction engineering [31]. The model they presented is based on back-propagation ANN trained to perform estimations on the basis of the large number of past estimation materials. Their test results suggest that the developed model based on ANN successfully extract the relation between the project's features and the estimation of fabrication cost. Gui et al. [32] presented a survey of structural optimization applications in civil engineering. Their aim was to combine different design and development techniques (ANN, expert systems, genetic algorithms) for the bridge project so that the structural design of the system can be optimized. Parhi and Dash have presented an analysis of the dynamic behavior of a beam structure. The analysis was performed upon a beam containing multiple transverse cracks [33]. For these purposes, neural network controller was used. Authors have calculated three natural frequencies and compared results of experimental, theoretical and finite element analysis. The results of the analysis were used to train feed-forward multi-layered neural network controller. After the training process was performed, this controller was capable of predicting crack locations and depths and the results of its predictions were validated against an experimental set-up. Alacali, Akba, and Doran have presented an investigation of a confinement degree for confined concrete using neural network analysis [34]. They have established the neural network algorithm to validate the empirical equations proposed for the confinement coefficient.

Rahman et al. presented the estimation of local scour depth around bridge piers [35]. The estimation was performed using the multi-layer perceptron ANN, ordinary kriging (OK), and inverse distance weighting (IDW) models. They have evaluated results from the sixth test case. The results indicate that the ANN model predicts local scour depth more accurately than the kriging and inverse distance weighting models. In [36], authors present Artificial Neural Network based Heat Convection (ANN-HC) algorithm. They have used an Earth-to-Air Heat Exchanger (ETAHE) component with aim to establish a new thermal modeling method for cooling components. The case study they presented shows working principles of the algorithm they proposed and tested upon ETAHE and its environment. Narasimhan presented a direct adaptive control scheme for the active control of the nonlinear highway bridge benchmark [37]. As a model, author used nonlinearly parameterized neural network. Described ANN contains single hidden layer and is coupled with a proportional-derivative type controller to perform approximation of control force. In this particular case, ANN approximates nonlinear control law and it is not used to model the nonlinearities of the overall system.

Lee, Lin and Lu performed an assessment of highway slope failure using neural network [38]. For these purposes, they have used back-propagation neural networks and demonstrated the effectiveness of ANN in the evaluation of slope failure potential based on five major factors, such as the slope gradient angle, the slope height, the cumulative precipitation, daily rainfall, and strength of materials. Laflamme and Connor used self-tuning Gaussian networks for control of civil structures equipped with magnetorheological dampers [39]. The neural network used for these purposes is an adaptive neural network composed of Gaussian radial functions. Their evaluation results indicate that the neural network is effective for controlling a structure equipped with a magnetorheological damper.

Bilgil and Altun emphasize the importance of prediction of the friction coefficient in hydraulic engineering [40]. Therefore, they propose a method to estimate friction coefficient through means of ANN. Data used for ANN training was obtained experimentally and estimates friction factor in a smooth open channel. Results demonstrate ANN model shows higher efficiency compared to Manning approach in the given environment.

As stated in [41], the civil engineering research community is still in demand for the next generation of applied ANNs that have to be based on sophisticated genetic coding mechanisms in order to develop the required higher-order network structures and utilize development mechanisms observed in nature.

2.2. Basic postulates of limit and shakedown analysis

In the area of elastic behavior of beams, the stresses and strains are proportionally dependant. Due to the increase of load, there is a gradual build-up of stress, until the value of the stress in the most loaded fiber reaches the value of yield stress. The further increase of load causes plastification of the entire cross section, and thus formation of plastic hinge [22].

It is known, that in statically determinate structures, the complete plastification of one cross-section of a beam and transition of the structure into the failure mechanism means loss of load bearing capacity. In statically indeterminate structures, formation of one plastic hinge does not lead to formation of failure mechanism, and the bearing capacity of one n times statically indeterminate structures is fully exhausted when in the structure an n+1 plastic hinge is formed.

If the structure is unloaded prior to formation of failure mechanism, certain residual strain occurs, which causes occurrence of residual bending moments. By applying the limit state analysis it is not possible to include the residual bending moments in the calculations. This is possible by applying the shakedown analysis. In the shakedown analysis all the assumptions of the limit state analysis are also valid, whereby this method makes possible the analysis of the behavior of the structure exposed to repeated load.

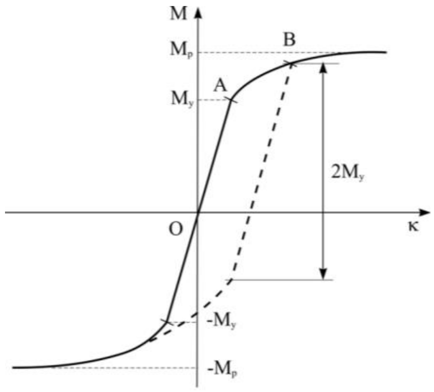


Fig. 1 Relation moment–curve in shakedown theory

The shakedown theorem can be established for a material having the more general moment-curvature relationship of Fig. 1. The basic curve (OAB) is assumed to be the same whichever way the bending moment is applied, so that first yield occurs at a moment M_y in either sense, and the full plastic moment has value M_p, again in either sense. The linear elastic range thus extends for a total $2M_y$. The assumption is made that this range of $2M_y$ is not affected by any partially plastic deformation that might occur. Thus if a moment corresponding to the value at point B in Fig. 1 is applied to the cross-section, followed by unloading, then the behaviour will be linear for a total decrease of moment $2M_y$ [19].

If a structure, made of an elasto-plastic material, is exposed to variable loads, then, the following situations are possible [42]:

- If the load intensities remain sufficiently low, the structural response is perfectly elastic;
- If the load intensities become sufficiently high, the instantaneous load-carrying capacity of the structure becomes exhausted; plastic, unconstrained flow mechanism develops and the structures collapse;
- If the plastic strain increments in each load cycle are of the same sign then, after a sufficient number of cycles, the total strains (and therefore displacements) become so large that the structure departs from its original form and becomes unserviceable. For sufficiently high load amplitude (although below the load-carrying capacity) the deflection grows in each cycle. This phenomenon is called ***incremental collapse***;
- If the strain increments change sign in every cycle, they tend to cancel each other out and the total deformation remains small (alternating plasticity). In this case, however, after a sufficient number of cycles, material at the most stressed points begins to break due to low-cycle fatigue;
- It may also occur that, after some plastic deformation in the initial load cycles, the structural behavior becomes eventually elastic, for lower load amplitudes. Such stabilization of plastic deformations is called ***shakedown*** or ***adaptation***.

2.3. Theorems of limit state analysis

The basic theorems of limit state analysis consist of:

- static theorem or the theorem of the lower border of limit load and,
- kinematic theorem or the theorem of the upper border of limit load.

Static theorem is based on the static equilibrium of the observed system. A large number of distributions of bending moments meeting the equilibrium conditions as a result of the given external load can be assumed for one statically indeterminate system. Greenberg and Prager [16] named such distribution statically admissible. If the bending moment has not exceeded the appropriate value it is claimed that it is also safe.

The static theorem can be expressed in the following way: if there exists any distribution of bending moment throughout structure which is simultaneously safe and statically admissible under the load λP, then the value λ must be less or equal to the factor of failure load λ_C, ($\lambda_C > \lambda$). The actual limit load ($\lambda_C P \leq Pp$) can be equal or higher than the given one.

The kinematic theorem relates to the possible failure mechanism. The failure mechanism comprises a kinematically unstable system which a beam becomes after the plastic hinges are formed in the cross sections where there are conditions for this [16]. The factor of failure load λ_C, i.e. the limit load ($\lambda_C P$), is determined the equalizing the work of external forces with the work absorbed in plastic hinges for each assumed failure mechanism.

The kinematic theorem can be expressed in the following way: for the given static system, subjected to the set of loads λP, the value of λ which corresponds to any assumed failure mechanism must be higher or equal to the factor of failure load λ_C, that is, $\lambda_C \geq \lambda$.

2.4. Theorems of shakedown analysis

As well as in the limit state analysis, in the shakedown analysis there are static and kinematic theorems, on whose basis it is possible to determine the safe limit load depending on the type of variable repeated load.

The static shakedown or Melan's theorem is as follows: shakedown occurs if it is possible to find a field of fictitious residual stress $\overline{\sigma}_{ij}$, independent of time, such that for any variations of loads within the prescribed limits the sum of this field with the stress field σ_{ij}^* in a perfectly elastic body is safe (sufficient condition). Shakedown cannot occur if there does not exist any time-independent field of residual stresses $\overline{\sigma}_{ij}$ such that the sum $\overline{\sigma}_{ij} + \sigma_{ij}^*$ is admissible (necessary condition) [43].

The kinematic shakedown or Koiter's theorem is as follows: shakedown does not occur if it is possible to find an admissible cycle of plastic strain rates and some programme of load variations between prescribed limits for which

$$\int_0^\tau \mathrm{d}t \int X_{ni} \upsilon_{io} dS_F > \int_0^\tau \mathrm{d}t \int \dot{A}(\xi_{ij0}^p) \mathrm{d}V \tag{1}$$

where $\dot{A}(\xi_{ij0}^p) = \sigma_{ij0} \xi_{ij0}^p$ is the rate of work of the plastic strain on the admissible rates [43].

3. Analysis of the Bearing Capacity of Continuous Beams Depending on the Load Character and Degree of Static Indeterminacy

Applying the adequate method based on upper and lower limit and shakedown analysis [44], and depending on the character of the load, an analysis of the limit load of continuous beam displayed on the Fig. 2 was conducted. The span of the beams affects the distribution of internal forces, and therefore on the relevant condition of failure, that is, the value of the failure force. On the example of the continuous beam, a procedure of the failure force calculation has been previously conducted and presented in [44], depending on the change of beam span, which is defined by the coefficients α and β, as well as depending on moment of plasticity M_p.

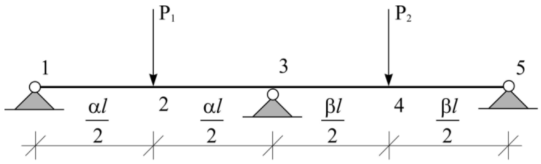
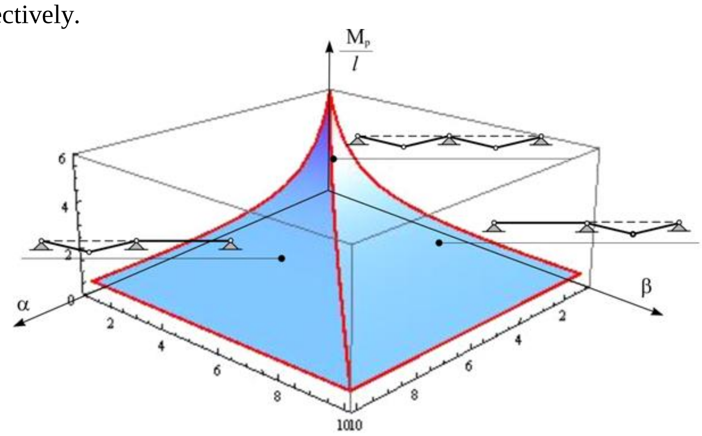
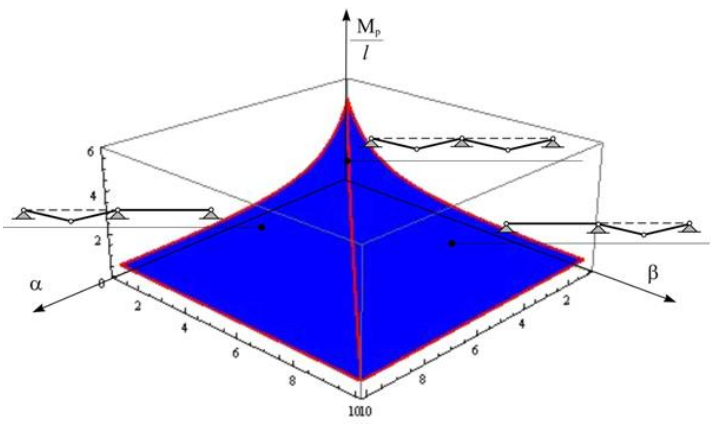


Fig. 2 Continuous two–span beam loaded by concentrated forces in the middle of the span

The limit force of failure in one-parameter form, the incremental failure force and residual bending moment depending on the change of span length are shown in Fig. 3 and Fig. 4 respectively.



a



b

Fig. 3 a) Change of the limit failure force depending on α and β
b) Change of incremental failure force depending on α and β

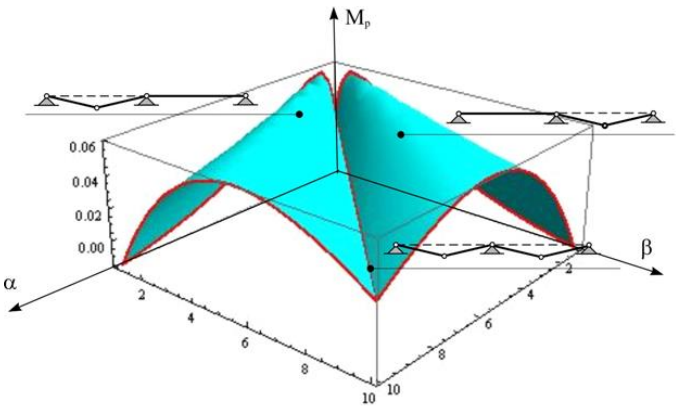


Fig. 4 Change of residual bending moment depending on α and β

4. The Neural Network Approach

The most obvious reason why ANN models are gaining popularity is their ability to adapt to changing situation and to learn from a predefined set of examples. These characteristics facilitate the development of a model of the observed phenomenon is situations where there is limited theory describing the cause-effect relationship between the beam configuration and its performances (such as failure force, the incremental failure forces and the residual bending moment). Also, neural networks are capable of performing fast processing which makes them particularly relevant to frame performance analysis because this type of analysis tends to be highly computationally intensive.

In cases where a neural network will be applied to perform an analysis of a particular phenomenon, an issue that needs special attention is the determination of the neural network architecture and training method. There are other alternative approaches to choose from, such as ones described in [45 - 47]. For the analysis presented in this paper, we have adopted RGIN architecture and complementary training procedure [47]. RGIN architecture was chosen mainly because of its performance characteristics. RGIN system is capable of produce very precise models of a function [45, 46]. It circumvents the issue of how many hidden nodes to incorporate in a network and provide the possibility to perform a rapid training process including the usage of large training sets (containing thousands of training data sets [48]). Finally, if the training set is validated (e.g. there is a confirmation that the training set does not contain ambiguous data), then there is a guarantee RGIN system will converge on a solution during training process.

Radial-Gaussian networks represent a specialized form of the radial basis function (RBF) networks [49]. As illustrated in Figure 5, RGINs comprise three layers of neurons connected in a feed-forward manner. These networks perform mapping from a vector of inputs to a vector of outputs. Vector of inputs represents the observed phenomenon, e.g. the problem that should be solved, while a vector of outputs represents the solution of the problem provided by the neural network. In this study, the input vector would represent the frame configuration, while the output vector would be an estimate of the performance of the frame, such as its the limit failure force, the incremental failure forces and the residual bending moment.

The data is passed through RGIN network in single direction - forward only. As the data flows through the network forward, simple processing is applied upon it. Processing is applied both within the neurons and along the links connecting the neurons. The equation 2 summarizes the way of functioning of this type of network:

$$O_k = \sum_{j=1}^{J}\left(a_{k,j} \cdot \exp\left(-s_j \sum_{i=1}^{I}\left(\alpha_i \cdot I_i - o_{j,i}\right)^2 \right)\right) \quad (2)$$

where the value of the *i*th element of the input vector is represented by I_i; $o_{i,j}$ represents an offset on the connection between the *i*th input neuron to the *j*th hidden neuron; α_i is a normalizing term at the *i*th input neuron; $a_{k,j}$ is an amplitude term connecting the *j*th hidden neuron and *k*th output neuron; s_j is a spread term coupled with the *j*th hidden neuron; and O_k is the output value from the *k*th neuron in the output layer.

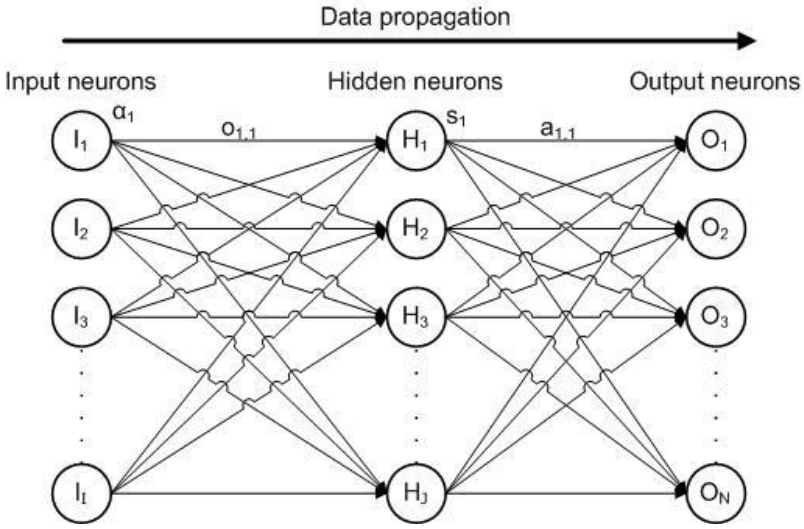


Fig. 5 RGIN neural network

The process of training a RGIN network is used to configure a suitable set of values for network parameters so that the ANN may perform the required function. A more detailed description of RGIN networks and their method of training are provided in [50]. The training scheme used for RGINs training process is supervised. The network is provided with a data set of training sample problems and corresponding answers. This data set represents a mapping between input and output vectors. The network is expected to learn from these correspondences to within a predefined error tolerance. Once the training process is finished, the performance of the network should be tested against a data set comprising problem samples not used within the training process. Error is calculated by determining the sum of the absolute difference between the actual output given by ANN and the expected output, calculated over all training data examples.

While training a RGIN, hidden neurons are added (one at a time) to a middle layer in network. The ANN can be thought of as implementing an overall function and every hidden neuron adds a radial-Gaussian function to it. A correction term is generated by each radial-Gaussian function belonging to each of the hidden neurons in the middle layer. As training process progresses, each of the three radial-Gaussian function parameters, $o_{j,i}$, $a_{k,j}$, and s_j, are adjusted.

5. Discussion of the Results

As previously stated, ANN used for the research presented in this paper a RGIN and complementary training procedure [47] was adopted. The input layer consists of 4 neurons representing the structure of the two-span continuous beam loaded in the middle of the span that the residual bending moment, limit and the incremental failure force are obtained using the developed ANN. In particular, these 4 neurons represent the following beam configuration parameters: L, α, β and M_p. The input layer transmits information from the outside into the hidden layer and the process continues up to reach the output layer. The structure of the hidden layer is a result of the training process, as described in [47]. In our approach, it consists of 8 neurons. The output layer consists of 3 neurons representing the limit failure force, the incremental failure force and the residual bending moment. For the training and validation purposes, two sets with the same number of samples were generated (training set and validation set), each containing 233 samples. Both sets were populated with data calculated according to postulates of limit and shakedown analysis. As input values, in both sets parameter L is an integer ranging from 1 to 10, parameter α is a floating-point number ranging from 1 to 10, parameter β is a floating-point number ranging from 1 to 10 and parameter M_p is a floating-point value ranging from 1.5 to 75. After the training process, ANN outputs were compared to the validation set (expected output calculated according to postulates of limit and shakedown analysis) and the results of the validation process for the incremental failure force, the residual bending moment and the limit failure force are displayed on Figs. 6, 7 and 8, respectively. On each figure, curves that describe the change of expected (theoretical) values are shown in blue while the output generated by ANN is shown in red.

Fig. 6 visualizes a comparison between the expected (theoretical) values compared to the values obtained using the ANN in the case of incremental failure force. As shown in Fig. 6, developed ANN shows a high level of precision of the output results. For a majority of the samples that were subjected to validation (in particular, 68% of validation samples), the mean absolute percentage error was less than 8%. The largest discrepancy in the results was detected in the case of a training samples subset in which value of the input parameter α has a steep growth. In these cases, the precision of ANN is expected to be achieved by increasing the number of training samples within this subset of training samples which will be used for the additional ANN training.

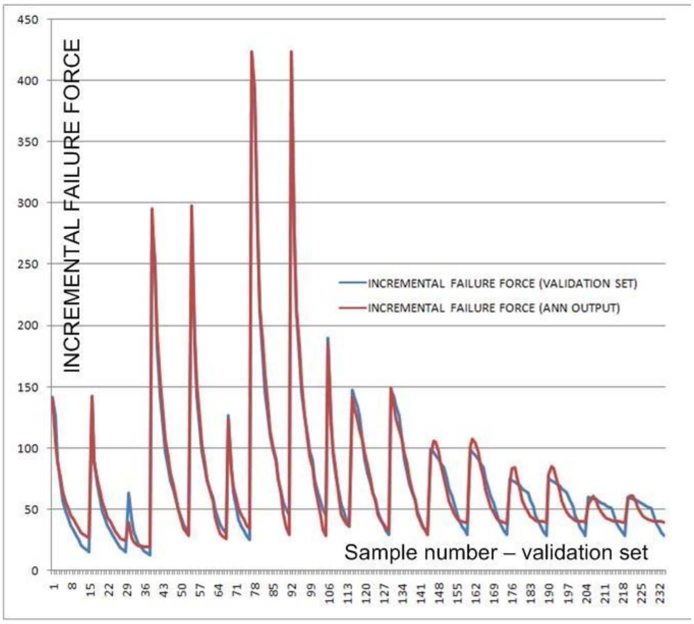


Fig. 6 Comparison of theoretical and RGIN generated results – incremental failure force

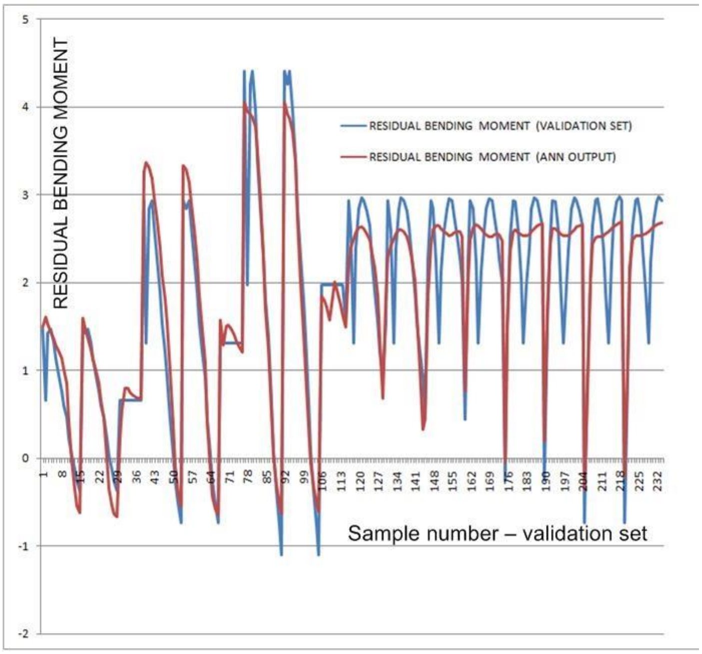


Fig. 7 Comparison of theoretical and RGIN generated results – residual bending moment

In the case of residual bending moment values validation, the obtained results are shown in Figure 7. As shown in Fig. 7, in this case ANN outputs the highest percentage of discrepancy with the theoretical results. To adjust the network to be fully able to determine the residual bending moment, the training set should be expanded along with an increase of the number of training epochs. Despite the increased results discrepancy, for 59% of the validation samples the mean absolute percentage error was less than 15%, which indicates a real opportunity to improve network quality through proposed changes.

Figure 8 visualizes a comparison between the expected (theoretical) values compared to the values obtained using the ANN in the case of limit failure force. As shown in Fig. 8, developed ANN shows a high level of precision of the output results. For a majority of the samples that were subjected to validation (in particular, 68% of validation samples), the mean absolute percentage error was less than 10%. The largest discrepancy in the results was detected in the case of a training samples subset in which value of the input parameter α has a steep growth. In these cases, the precision of ANN is expected to be achieved by adding additional training samples to ANN training sample set in order to cover better critical parts of the input space. .

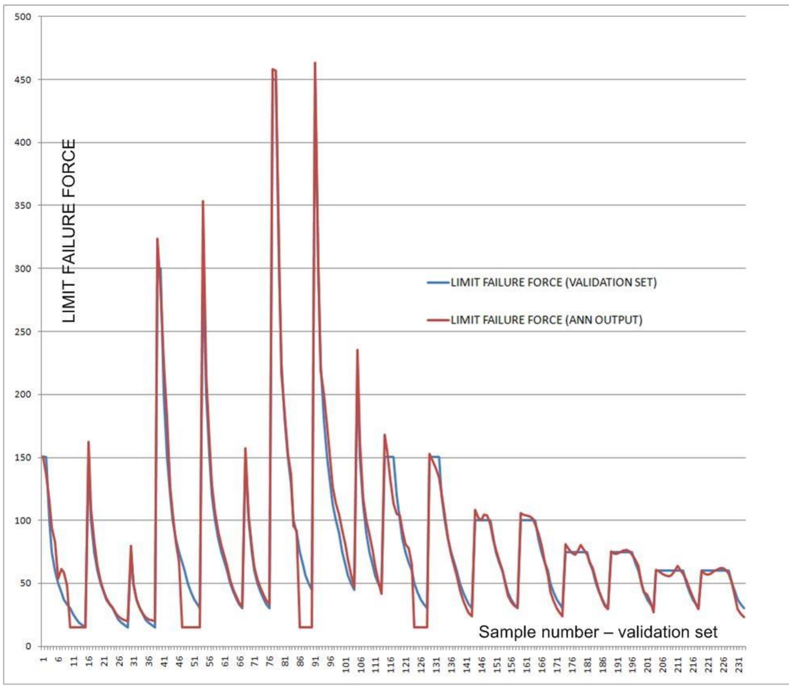


Fig. 8 Comparison of theoretical and RGIN generated results – limit failure force

6. Conclusion

Modern technologies applied in civil engineering are in constant demand for new solutions during the analysis of different problems. One of the widely accepted approaches used in civil engineering is the ANN approach because of its ability to adapt to changing situation and to learn from example. Since the shakedown analysis of elasto-plastic structures has been increasingly applied in the analysis of engineering problems, the potential usage of ANN approach in these analyses should be devoted significant attention.

In this paper, we have proposed an ANNmodel in order to approximate the residual bending moment, limit and the incremental failure force of continuous beams. ANN model we have developed, trained and evaluated was designed as radial-Gaussian network architecture. Our analysis of the developed ANN model, on the example of the two-span continuous beam loaded in the middle of the span, indicates that the residual bending moment, limit and the incremental failure force can be obtained using the proposed neural network approach with sufficient precision. The proposed ANN model was trained with 233 training samples calculated on the basis of theorems of limit and shakedown analysis. The testing process resulted with a mean absolute percentage error less than 15% which indicates that ANN used during the test is a feasible tool for an approximation of the residual bending moment, limit and the incremental failure force of continuous beams.

The presented ANN model still exposes some weaknesses that could be eliminated with additional effort. In case of the approximation of residual bending moment, the accuracy of the ANN model output can be improved by introducing additional training samples in the critical zones that expose the highest mean absolute percentage error. The training set modification performed in this manner would positively affect all ANN aspects which will lead towards other ANN outputs becoming even more precise. Also, as a part of the future research and development, ANN structure will be modified in terms of increasing the number of neurons in the hidden layer. We expect this change to decrease mean absolute percentage errors across the whole validation set. Another possibility which will be considered in future is splitting existing ANN into three ANNs, one for each of the estimated forces (residual bending moment, limit and the incremental failure force). It is our aim to use different ANN structures for different forces to achieve better results.

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