FACTA UNIVERSITATIS
Series: **Electronics and Energetics** Vol. 32, N° 1, March 2019, pp. 65-74
https://doi.org/10.2298/FUEE1901065M

COMPARISON OF MEMRISTOR MODELS FOR MICROWAVE CIRCUIT SIMULATIONS IN TIME AND FREQUENCY DOMAIN

Ivo Marković[1], Milka Potrebić[1], Dejan Tošić[1], Zlata Cvetković[2]

[1]University of Belgrade, School of Electrical Engineering, Belgrade, Serbia
[2]University of Niš, Faculty of Electronic Engineering, Niš, Serbia

Abstract. *As reported in the open literature, there are many memristor models for the circuit-level simulations. Some of them are not particularly suitable for microwave circuit simulations. At RF/microwave frequencies, the memristor dynamics become an important issue for the transition process. In this paper we present a number of different SPICE memristor model groups. Each group is explained using representative models, which are analysed and compared from the microwave circuit analysis viewpoint. We consider the model behaviour at RF/microwave frequencies and the memristance setting issues. Results are compared and the best models are recommended.*

Key words: *Memristor models, microwave circuit, transition process.*

1. INTRODUCTION

In 1971, Leon Chua theoretically predicted [2] the existence of memristor, the fourth fundamental element. He claims that memristor fills the gap in the relation that connects magnetic flux and electric charge: $M(t)=\mathrm{d}\phi(t)/\mathrm{d}q(t)$. In 2008, Strukov et al. [3] published a paper in which the authors claim that the component was produced in HP's laboratory. The conclusion was made on the basis of matching behavioural properties that Chua predicted with the measured results.

To the contrary, there are papers such as [4], in which authors claim that memristor is not a new element. They claim that HP laboratory did not create a new component, and that Chua's prediction was not correct. This discussion should be classified as theoretical, since it is not relevant for the component's implementation. Presently, there are two companies that provide commercially available memristors [5, 6], and a lot of researchers are trying to apply memristors in various fields of electrical engineering. There are plenty of reasons for memristors implementations, such as: small dimensions, small power consumption, fast switching time (of the order of seconds or less) from ON to OFF state and

---

Received February 26, 2018; received in revised form July 31, 2018
Corresponding author: Milka Potrebić
University of Belgrade, School of Electrical Engineering, Bulevar kralja Aleksandra 73, Belgrade, Serbia.
(e-mail: milka_potrebic@etf.rs)
* An earlier version of this paper was presented at the 13th International Conference on Applied Electromagnetics (ПЕС 2017), August 30 - September 01, 2017, in Niš, Serbia [1].

vice versa, no mechanical parts are included, etc. All of the mentioned characteristics are relative to the commercially available devices in use for implementations for RF/microwave systems and devices.

In order to implement memristor-based circuits, appropriate models for circuit-level simulations are required. In this paper, we analyse models of memristors that operate at high frequencies. First, we display models for the transition process and display results achieved using the models. Next, models for the frequency analysis are discussed and compared. Finally, we summarize results in short and suggest which models are most suitable for simulations at RF/microwave frequencies.

In the new volume of IEEE Circuits and Systems Magazine, there are two papers which understanding could improve researches in both modelling area and implementation area. Biolek et al. [7] point out at important fingerprints of memristors, which could be good guidelines. Ascoli et al. [8] discuss about dynamics of real world memristors.

Some of the models were recently used to realize memristor-based circuits. Different filter realizations [9] and reconfigurable microwave filters [10,11] were analysed using these models. Transition of memristor states in reconfigurable microwave filter was analysed in [12]. The use of memristors in power divider, coupled resonator bandpass filters, and a low-reflection quasi-Gaussian lowpass filter with lossy elements, was discussed in [13]. Realisation of phase shifter was presented in [14]. Modelling and simulation of large memristive networks was reported in [15]. Potential applications of memristors in RF/microwave circuits were presented in [16]. There is also a chapter in the book [17] regarding memristors, in which authors summarized possible applications using memristors in passive microwave circuits.

2. Models for Transition Process

In this chapter a couple of memristor model types are presented. We analyse these specific models which seem to be the most effective, to the best of our knowledge.

2.1. Biolek's model

Based on HP's device that was fabricated back in 2008, Biolek et al. came with an initial memristor model in 2009 [18], and with improved models in 2013 [19]. The physical model of the memristor from [3] is shown in Fig. 1. It consists of two-layer thin film, sandwiched between two platinum electrodes. The first layer is doped with oxygen vacancies, so it behaves as a semiconductor. The other region, which is undoped, has an insulating property.

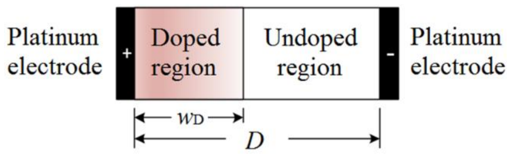

Fig. 1 Graphical representation of memristor model

The total resistance of the device is a sum of the doped and undoped regions. By applying adequate voltage at platinum electrodes, it is possible to change the width of the doped and undoped region. This fact implies that it is possible to change the total resistance of the memristor. After some calculations, we can present a simplified equation for the memristor resistance:

$$M(q) = \frac{d\Phi}{dq} = R_{\mathrm{off}}\left(1 - \frac{\mu_{\mathrm{v}} R_{\mathrm{on}}}{D^2} q\right), \quad (1)$$

where M represents memristance (memristor's resistance), Φ is the flux, q is charge, μ_v is dopant mobility, D is the distance between platinum electrodes, R_{on} and R_{off} are resistances of the doped and undoped regions.

The total memristance depends on initial values of R_{on} and $R_{off,}$ as well as the width of the doped region x referenced to the total width of the memristor D. The speed of the movement of the boundary between the doped and undoped regions may be calculated as:

$$\frac{dx(t)}{dt} = \frac{\mu_{\mathrm{v}} R_{\mathrm{on}}}{D^2} i_{\mathrm{M}}(t) f(x) = k i(t) f(x) \quad (2)$$

where k is a constant: $k = \mu_v R_{on} / D^2$, i_M is the current through the memristor, and $f(x)$ is a so-called window function. The purpose of the window function is to describe nonlinear dopant drift. It is a phenomenon that manifests when small voltages yield enormous electric fields, which can produce significant nonlinearities in ionic transport. These nonlinearities manifest themselves particularly at the thin film edges, where the speed of the boundary between the doped and undoped regions gradually decreases to zero. This model is graphically presented in Fig. 2. The memristor memory effect is modelled using a feedback-controlled integrator. It stores the effects of the passing current, and controls the memristance.

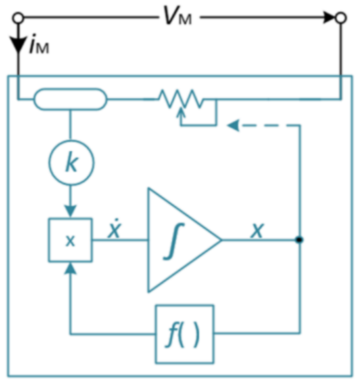

Fig. 2 Graphical representation of the memristor model

On the basis of this reasoning, it becomes apparent that the problem of modelling the memristor is reduced to the modelling of the window function. The problem is not just to provide one formula for all the micro effects, but also the limitations that exist in the software for circuit analysis. In light of this, it becomes clear why so many papers on this topic were published in the last decade. Some of the most important papers from this group, alongside Biolek's, are Joglekar [20], Yakopcic [21]. There are also new papers by Biolek et. al [22, 23], in which they solve some problems caused by SPICE programs, in terms of representation of very large numbers.

Here, we discuss the resistive model reported in [19]. The model is based on the minimal resistances in ON state and the maximal resistance in OFF state - R_{ON}, R_{OFF} initial resistance R_{ini}; mobility of charge μ_V; and the distance between the platinum electrodes d.

Memristance could be calculated as:

$$R = R_{on}x + R_{off}(1-x), \tag{3}$$

where x is the width of the doped region, and x ∈ [0,1]. So R_{ini} = R for some initial state of x, $R = R_{OFF}$ when $x = 0$, and $R = R_{ON}$ when $x = 1$.

Those are the only values that need to be set up for the model to work properly. R_{ON}, R_{OFF}, μ_V and d are fabrication dependent values. R_{ON} and R_{OFF} differ from R_{On} and R_{Off} from Eq. (1), and should not be mixed: $R_{On} = R_{ON}\,x$, $R_{Off} = R_{OFF}\,(1-x)$. This model provides theoretically predicted results in case of I-V curve analysis, as shown in Fig. 3. Results are obtained using LTspice [24].

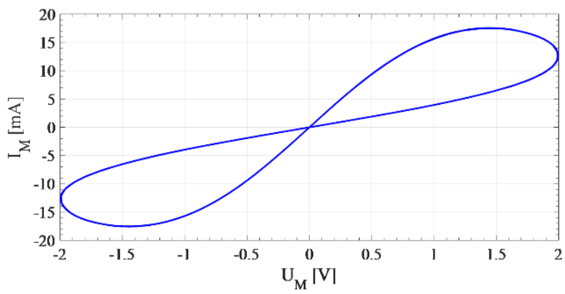

Fig. 3 I-V curve for Biolek's model, at f = 10 Hz.

Additionally, it is an excellent model of an ideal memristor. The model is resistive, so it might be used for the proof-of-concept transition process of idealized microwave circuits. For switching from the OFF-to-ON and from the ON-to-OFF state analysis, a simple circuit containing serial connection of a generator and a memristor is used.

Generator produces PWM signal, with voltage amplitude of ±2 V, at the frequency of 100 kHz, and the high voltage level lasts 70% of the signal period.

Simulation result suggests that the transition time is of the order of seconds, for both ON to OFF and from OFF to ON transitions. Increase of voltage amplitude would shorten the transition time, but we considered that memristor cannot handle very high voltages. Increase of frequency would also shorten the transition time.

2.2. Mazady's model

Mazady et al. proposed models for both transition process and frequency [26] analysis. They are based on the TiO_2 device. We can classify these models as a new category, comparing to Biolek's, Yakopcic's, Joglekar's etc. In this case, model for time-domain analysis are defined by 24 ideal switches (exact moments of switching ON or switching OFF); currents through 6 parallel branches. Model for frequency-domain analysis consists of resistances in ON and OFF states R_{HC}, R_{LC}; parasitic inductances L_1 and L_2; capacitances C and C_b. R_{HC} and C are time dependent variables. I-V curve matches theoretically predicted results, as shown in Fig. 4. Results are obtained using LTspice [24]. All of the circuit parameters are the same as in the original paper [26].

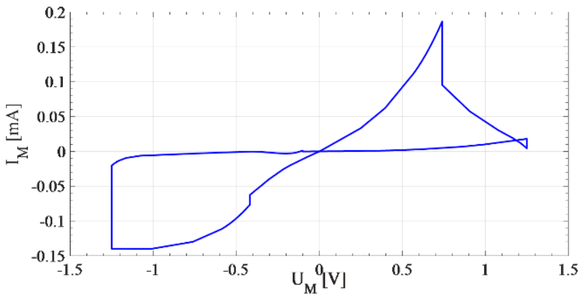

Fig. 4 I-V curve for Mazady's model at 50 Hz.

Switching time is very short. Authors claim [16] that it takes only 210 ps for switching from OFF to ON state, and vice versa. Since the switching time is very short, it is possible to use a short rectangular impulse for programming memristor's states.

The particularity of these models lies in the fact that they depend on excitations and the actual circuit parameters. Consequently, no stand-alone self-contained memristor model is available, which is based only on the port description equations.

2.3. Amirsoleimani's model

Amirsoleimani et al. also proposed a memristor model based on the TiO_2 device [27]. This kind of model could be classified as the third category. It includes plenty of fabrication details and specific calculations. Amirsoleimani's approach is based on the tunnelling effect, and the model is very similar to the Shottky diode model. It is based on many physical properties, which are directly used in calculations for the current of the tunnelling effect and the Ohmic currents. The memristor I-V curve obtained from this model matches the theoretically predicted results for different excitations, as shown in Fig. 5. Results are obtained using NI AWR Microwave Office [28].

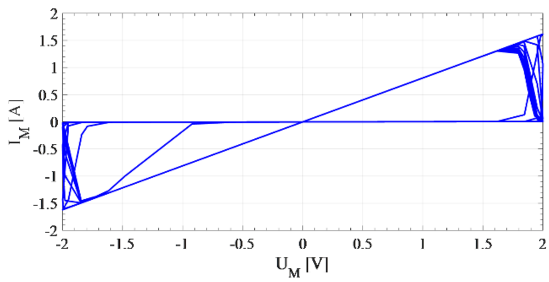

Fig. 5 I-V curve for Amirsoleimani's model, at 100 Hz.

Switching time depends on parameters, but it is of the order of magnitude of a few seconds, as reported in [27]. This model does not depend on excitation and circuit parameters. The downside of this model is that it requires many fabrication parameters to be measured and set in order to properly illustrate memristors behaviour. Also, complicated equations require more time to be calculated. Simulation time is longer. Signal for programming could be the same as for Biolek's model.

It is important to mention another category of models – analytical. The purpose of these kind of models is to provide simplified mathematical (analytical) apparatus, which will facilitate design of electrical circuits. This means that it will improve modelling of memristors, but also facilitate the design of memristors programming circuitries. To the best of our knowledge, there is only one published model [29] at this time. These kind of models are still in development, and perhaps there will be some useful results in near future.

Another group of memristor models that is important to mention is based on Verilog models. One of the best models of this kind was presented in series of papers [30-32]. Since our focus is on SPICE models and circuits, we will not analyse this model, but leave Verilog models for some further researches.

3. Models for RF/Microwave Simulations

From the RF/microwave simulation viewpoint, Mazady's model [26], Pi's model [25] and improved Pi's model [24] may be particularly interesting. Mazady's model [26] is actually the Laplace transform of the model (equations) for the transition process. However, this model strongly depends on the actual circuit and its excitations.

3.1. Pi's model

Pi et al. [25] fabricated series of devices and provided obtained results. Authors claim that it was fabricated on an intrinsic silicon wafer with a 380-nm-thick thermally grown silicon dioxide. They used silver (Ag) layer as a terminal for one side, and gold (Au) with a thin titanium (Ti) adhesion layer as a terminal from the other side. The terminals are separated by a 35-nm-wide air gap. This structure is graphically presented in Fig. 6.

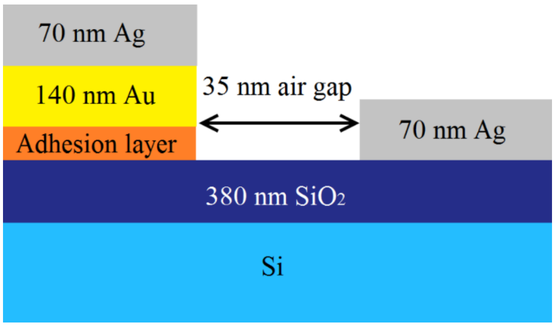

Fig. 6 Pi's device graphical model.

Switching is related to the formation/rupture of multiple conductive filaments between two electrodes. In the ON state, there is always at least one conductive filament, which leads to the low resisting state. In the OFF state, ruptures are dominant, so this state is dominantly defined by capacitances of the gaps between the filaments.

According to the measurements on fabricated devices, authors proposed an RF memristor model of the memristor switch that behaves as a resistor in ON state, and a capacitor in OFF state, as shown in Fig. 7. Averaged values for resistance and capacitance of the fabricated series of devices are: R_{ON} = 3.6 Ω, C_{OFF} = 1.37 fF. The capacitance is predominantly associated with the capacitance of the air gap, which depends on the effective dielectric constant of the substrate–air interface. It is constant over the frequency range of [10 MHz, 110 GHz]. Frequency and power ratings are also provided in the paper. Model was tested in the range of 10 MHz – 110 GHz. At 40 GHz, in the ON state, measured insertion loss was 0.3 dB, and in the OFF state, measured isolation was 30 dB. Memristor is operative up to about 20 dBm (17 dBm measured; 24 dBm calculated). The ON state is critical, since the device is limited by fusing. As the memristance increases, memristor's power handling also increases.

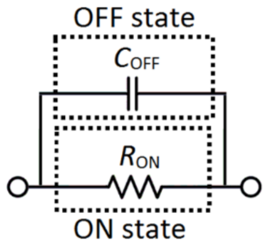

Fig. 7 Pi's RF memristor model.

3.2. Weinstein's model

Weinstein et al. [33] analysed behaviour of Pi's memristor model, and presented some improvements. They take into account the following effects: SiO_2 parasitic capacitance; Si substrate capacitance and the fringe capacitance between the signal line and ground planes. Inductance of the filament and the electrodes are also taken into account. The model is displayed in Fig. 8. Values for the proposed model are: R_{ON} = 2.56 Ω, C_{OFF} = 1.168 fF, L = 52 fH, L_{ω} = 3.1 pH, C_p = 1.15 fF.

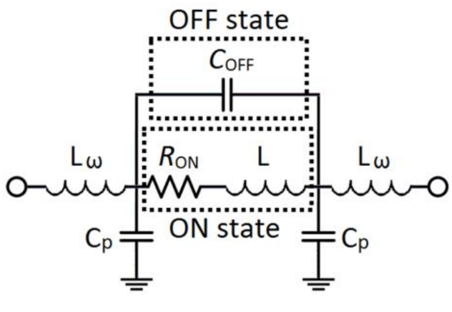

Fig. 8 Weinstein's RF memristor model.

4. COMPARISON OF MEMRISTOR MODELS FOR TRANSITION PROCESS

In this section we compare the results obtained by using different memristor models. Biolek's model [19] is based on HP's device [3]. This model's parameters are initial resistance, minimal resistance in ON state, maximal resistance in OFF state, memristor width, dopant drift speed. Mazady's model [26] was also based on TiO_2 device. In this model, parameters are switching times. Amirsoleimani [27] based his model on accurate charge transport calculation, and it could be applied on TiO_2 as well. In this model, there is a lot of parameters to set, such as: temperature, electron mobility, ion mobility, barrier height, density of states in valence and conduction bands, activation energy, some circuit parameters, and others.

Our goal is to have a RF memristor model. SPICE memristor models available to the present day do not accurately describe the transient process of the corresponding RF devices. Basic parameters of Pi's RF device are: resistance in ON state, capacitance in OFF state, resistance in OFF state, memristor width, conductor atoms mobility…

Main model properties and results are presented in Table 1, for models used for time-domain analysis:

Table 1 Result comparison

	Biolek	Mazady	Amirsoleimani
Matches Chua's theoretical prediction	Yes	Yes	Yes
Excitation independent	Yes	No	Yes
Setting simplicity	High	Medium	Low
Calculation complexity	Medium	Low	High
Matching real physical behaviour	Medium	Low	High
Order of magnitude of transition time	~s	~ps	~s

Comparing with the other two models, Biolek's model has the longest transition time. Transition time is longer in this case because Biolek's model is model of an ideal resistive memristor. Other parameters / properties are equal or better. It is simple to set, and there is no great calculation complexity. Mazady's model main issue is that it is not excitation independent, so model should be changed as the excitation changes, which requires calculations. Amirsoleimani's model is the most precise model. On the other hand, it is not easy to set, and calculation complexity is big in comparison with the other two models.

In case of the frequency-domain models, Pi's and Weinstein's model provide very similar results in circuits such as phase shifters and filters. Weinstein's model could possibly be treated as more precise, since there is more parameters taken into consideration. On the other hand, in a complex circuitry Pi's model would faster provide simulation results.

5. CONCLUSION

A lot of researchers are working on the memristor fabrication and modelling. There are many different models of memristors available in the open literature. Not all of them are suitable for RF/microwave analysis.

In this paper we compared several SPICE models from several categories used for transition process and frequency analysis. The best results for transition process are achieved using models based on window functions. Biolek's model seems to be one of the most precise in this category, as well as in general. It is excitation independent and simple to use and adjust. Equations are not of great complexity, so the simulation is not time-consuming. Mazady's models provide good matching with theoretical expectations. These models are dependent on the circuit parameters and excitations. Amirsoleimani's model is probably the most accurate when considering physical processes inside the TiO_2 device. It is excitation independent, circuit independent, and matching with theory is good. On the other hand, the main issue with this kind of models is its complexity of setting all the parameters. Simulation time is a little bit longer comparing with Biolek's model. Analytical models are still in development.

In the case of frequency analysis, we analysed two models. To be more precise, we discuss one device, its model and improved version of the model. Original Pi's model was experimentally verified. Authors provided information on average values in ON and OFF states, power handling, isolation in OFF state and insertion loss in ON state. The improved model was based on the original model, plus some side effects were taken in consideration. It is only theoretical and no measurement was involved. These two models provide very similar simulation results when applied in the phase shifter circuits.

REFERENCES

[1] I. Marković, M. Potrebić, D. Tošić, Z. Cvetković, "Comparison of memristor models for microwave circuit simulations", In Proceedings of the 13th International Conference on Applied Electromagnetics - ΠΕC 2017. Niš, Serbia, 2017, pp. P10 1-3.

[2] L. Chua, "Memristor – 'The missing circuit element'", *IEEE Transactions on Circuit Theory*, vol. 18, no. 5, pp. 507-519, 1971.

[3] D. Strukov, G. Snider, D. Stewart, R. Williams, "The missing memristor found", *Nature*, vol. 453, no. 7191, pp. 80-83, 2008.

[4] S. Vongehr, X. Meng, "The missing memristor has not been found", *Scientific Reports*, vol. 5, pp. 1-7, 2015.

[5] Knowm, Inc. http://knowm.org, Santa Fe, NM, 87502–4698, USA. [Accessed June 2017]

[6] Bio Inspired Technologies, LLC. http://bioinspired.net, 720 W. Idaho St. Suite 30 Boise, Idaho 83702 USA. [Accessed June 2017]

[7] D. Biolek, Z. Biolek, "About Fingerprints of Chua's Memristors", *IEEE Circuits and Systems Magazine*, vol. 18, no. 2, pp. 35-47, 2018.

[8] A. Ascoli, R. Tetzlaff, S. Menzel, " Exploring the Dynamics of Real-Word Memristors on the Basis of Circuit Theoretic Model Predictions", *IEEE Circuits and Systems Magazine*, vol. 18, no. 2, pp. 48-76, 2018.

[9] I. Marković, M. Potrebić, D. Tošić, "Possibility of applying memristors in microwave filters", *Tehnika*, vol. 71, no. 6, pp. 853-860, 2016. [in Serbian]

[10] M. Potrebić, D. Tošić, D. Biolek, "Reconfigurable microwave filters using memristors", *International Journal of Circuit Theory and Applications*, vol. 46, no.1, pp. 113-121, 2018.

[11] I. Marković, "Implementation of reconfigurable bandpass - band stop filter using memristors", In Proceedings of the 24th Telecommunications Forum – TELFOR 2016, Belgrade, Serbia, 22-23 Nov. 2016, pp. 11.11. 1-4.

[12] I. Marković, M. Potrebić, D. Tošić, "Memristor state transition in reconfigurable microwave filter", In Proceedings of the 2017 IEEE 30th International Conference on Microelectronics (MIEL), IEEE, Niš, Serbia, 9-11 Oct. 2017, pp. 71-74.

[13] M. Potrebić, D. V. Tošić, "Application of Memristors in Microwave Passive Circuits", *Radioengineering*, vol. 24, no. 2, pp. 408-419, 2015.

[14] I. Marković, M. Potrebić, D. Tošić, "Main-line memristor mounted type loaded-line phase shifter realization", *Microelectronic Engineering*, vol. 185-186, pp. 48-54, 2018.

[15] D. Biolek, Z. Kolka, V. Biolková, Z. Biolek, M. Potrebić, D. Tošić, "Modeling and simulation of large memristive networks", *International Journal of Circuit Theory and Applications*, vol. 46, no. 1, pp. 50-65, 2018.

[16] M. Potrebić, D. Tošić, "Potential applications of memristors in microwave circuits", invited plenary paper and talk, In Proceedings of full papers13th International Conference on Applied Electromagnetics, ΠΕC 2017, Faculty of Electronic Engineering of Niš, Niš, Serbia, August 30-September 01, 2017, pp. I2 1-4.

[17] M. Potrebić, D. Tošić, D. Biolek, "RF/Microwave Applications of Memristors", Chapter, pp. 159-185, S. Vaidyanathan, C. Volos (Editors), Advances in Memristors, Memristive Devices and Systems, Studies in Computational Intelligence, vol. 701, Springer, 2017.

[18] Z. Biolek, D. Biolek, V. Biolkova, "SPICE Model of Memristor with Nonlinear Dopant Drift", *Radioengineering*, vol. 18, no. 2, pp. 210-214, 2009.

[19] D. Biolek, M. Di Ventra, Y. V. Pershin, "Reliable SPICE simulations of memristors, memcapacitors and meminductors", *Radioengineering*, vol. 22, no. 4, pp. 945-968, 2013.

[20] Y. N. Joglekar, S. J. Wolf, "The elusive memristor: properties of basic electrical circuits." *European Journal of Physics*, vol. 30, no. 4, pp. 661-685, 2009.

[21] C. Yakopcic, T. M. Taha, G. Subramanyam, R. E. Pino, S. Rogers, "A memristor device model", *IEEE Electron Device Letters*, vol. 32, no. 10, pp. 1436-1438, 2011.

[22] Z. Biolek, D. Biolek, V. Biolkova, Z. Kolka, A. Ascoli, R. Tetzlaff, "Analysis of memristors with nonlinear memristance versus state maps", *International Journal of Circuit Theory and Applications*, vol. 45, no. 11, pp.1814-1832, 2017.

[23] D. Biolek, V. Biolkova, Z. Kolka, "Modified MIM model of titanium dioxide memristor for reliable simulations in SPICE", In Proceedings of the 14^{th} International Conference on. IEEE Synthesis, Modeling, Analysis and Simulation Methods and Applications to Circuit Design (SMACD), 2017, pp. 1-4.

[24] M. Engelhardt, LTspice IV version 4.23i. Linear Technology Corporation, http://linear.com. [Accessed June 2018]

[25] S. Pi, M. Ghadiri-Sadrabadi, J. Bardin, Q. Xi: "Nanoscale memristive radiofrequency switches", Nature Communications, vol. 7519, no. 6, pp. 1-9, 2015.

[26] A. Mazady, M. Anwa: "Memristor: part II–DC, transient, and RF analysis", *IEEE Transactions on Electron Devices*, vol. 61, no. 4, pp. 1062-1070, 2014.

[27] A. Amirsoleimania, J. Shamsib, M. Ahmadia, A. Ahmadia, S. Alirezaeea, K. Mohammadib, M. A. Karamib, C. Yakopcic, O. Kaveheid, S. Al-Sarawie: "Accurate charge transport model for nanoionic memristive devices", *Microelectronics Journal*, vol. 65, pp. 49-57, 2017.

[28] NI AWR Design Environment, National Instruments, Inc., El Segundo, CA 90245, USA. http://ni.com/awr (Accessed June 2018).

[29] A. Ascoli, V. Ntinas, R. Tetzlaff, G.C. Sirakoulis, "Closed-form analytical solution for on-switching dynamics in a TaO memristor", *Electronics Letters*, vol. 53, no. 16, pp. 1125-1126, 2017.

[30] S. Kvatinsky, E. G. Friedman, A. Kolodny, U. C. Weiser, "TEAM: Threshold adaptive memristor model", *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 60, no. 1, pp. 211-221, 2013.

[31] S. Kvantinsky, M. Ramadan, E. G. Friedman, A. Kolodny, "VTEAM: Voltage threshold adaptive memristor model", *IEEE Transactions on Circuit and System-II*, vol. 62, pp. 786-790, 2015.

[32] N. Wainstein, S. Kvantinsky: "A Lumped RF Model for Nanoscale Memristive Devices and Non-Volatile Single-Pole Double-Throw Switches", *IEEE Transactions on Nanotechnology*, 2018.

[33] N. Wainstein, S. Kvantinsky: "An RF Memristor Model and Memristive Single-Pole Double-Throw Switches", In Proceedings of the IEEE International Symposium on Circuits and Systems (ISCAS), Baltimore, MD, USA, May 28-31, 2017, pp. 1-4.