

NUMERICAL COMPACT MODELING APPROACH OF DISPERSIVE MAGNETOELECTRIC MEDIA BASED ON SCATTERING PARAMETERS

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Abstract: *Z-TLM based compact modeling approach for dispersive media exhibiting magnetoelectric coupling is presented in this paper. Scattering parameters based representation of considered medium is created in a form of compact model by extracting effective electromagnetic parameters using a retrieval method, and implementing them into a non-uniform TLM grid. Proposed approach is illustrated here on the example of dispersive isotropic chiral medium modeling.*

Key words: *Dispersive media, Compact models, Scattering parameters, Z-TLM method, Retrieval method, Non-uniform mesh.*

1. INTRODUCTION

Numerical modeling techniques nowadays represent important tools in a research process of complex materials especially when it is not possible or worthwhile to solve a problem with analytical approach. Two most used discrete time domain numerical techniques, the Finite Difference Time Domain (FD-TD) method [1] and Transmission Line Matrix (TLM) method [2] are very suitable for solving problems of electromagnetic (EM) wave propagation through complex structures and media. Even though the FD-TD method is often favored by researchers, the TLM method offers in some cases a more straightforward approach for describing and modeling different discontinuities, internal boundaries, propagation in dispersive media etc. This is a result of TLM feature that both electric and magnetic field components are solved in center of the TLM cell simultaneously without a need for temporal and spatial averaging.

Modifying and extending TLM method with Z transformation techniques create valuable means for an efficient time domain modeling of linear isotropic and anisotropic, bi-isotropic, nonlinear, quantum, chiral materials and metamaterials [3-7]. This so-called

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Z-TLM method supports implementation of the Debye, Drude, Lorentz and other dispersion models along with specific methods which allow for describing materials with complex frequency dependencies.

Compact models allow for complex structure, artificial or multilayered material to be represented as one effective material block via scattering parameters which to some degree simplifies numerical analysis and modeling process. In addition, compact models can be also used to reduce computational and time costs of the simulation by using a much coarser mesh for modeling of thin material panel, where instead of direct modeling by a fine mesh the material is replaced with single interface between two TLM cells [8,9].

In this paper, a formulation based on nonlinear constitutive relations and discretization of Maxwell's equations which allows implementation of most general properties of dispersive and anisotropic materials into the Z-TLM non-linear grid, is described. Procedures for applying TLM method in modeling of dispersive and general anisotropic media inside of non-uniform mesh are given in [9-11]. Z-TLM based approach presented in [12,13] is here expanded to allow modeling of dispersive materials with magnetoelectric coupling characteristics while preserving the advantage of including the arbitrary frequency dependencies of modeled material EM parameters, i.e. these dependences do not have to necessarily follow some of the known dispersion models. Effective EM parameters which are used to characterize materials with magnetoelectric coupling are extracted from S parameters through retrieval procedure [14-16], approximated through the vector fitting (VF) method [17-19] and then used to form a compact model after applying the bilinear Z transforms, which is later included into the TLM scattering algorithm. Created model efficiently describes studied material based on provided S parameters and enables analyzing and observing EM field propagation throughout the medium. Proposed approach is demonstrated by modeling dispersive isotropic chiral material slab exhibiting magnetoelectric coupling [20].

2. FORMULATION OF MAXWELL'S EQUATIONS FOR NON-UNIFORM MESH

Non-uniform mesh generally enables a proper resource handling especially while modeling materials and structures with nonlinear characteristics. It allows for a usage of smaller size cells in areas which are physically small but have a greater EM importance and also bigger cells in less complex or less EM important areas of the structure which improves an overall efficiency of the simulations.

In a non-uniform TLM cell (see Fig. 1) one or more directional space steps (Δx , Δz and Δy) are not the equal. Relations between incident V^i and reflected V^r voltage wave components of the twelve-port non uniform hybrid TLM cell [2] can be presented as:

$$\begin{aligned}
 V_1^r &= V_z - I_z Z_1 - V_2^i & V_7^r &= V_z + I_x Z_7 - V_8^i \\
 V_2^r &= V_z + I_z Z_2 - V_1^i & V_8^r &= V_z - I_x Z_8 - V_7^i \\
 V_3^r &= V_x + I_y Z_3 - V_{10}^i & V_9^r &= V_y - I_x Z_9 - V_4^i \\
 V_4^r &= V_y + I_x Z_4 - V_9^i & V_{10}^r &= V_x - I_y Z_{10} - V_3^i \\
 V_5^r &= V_z + I_y Z_5 - V_6^i & V_{11}^r &= V_y - I_z Z_{11} - V_{12}^i \\
 V_6^r &= V_z - I_y Z_6 - V_5^i & V_{12}^r &= V_y + I_z Z_{12} - V_{11}^i
 \end{aligned} \tag{1}$$

where impedance of each link line can be calculated based on link line inductance, time and space steps:

$$Z_k = L_k \Delta i / \Delta t, \quad k \in (1, 2, \dots, 12) \quad (2)$$

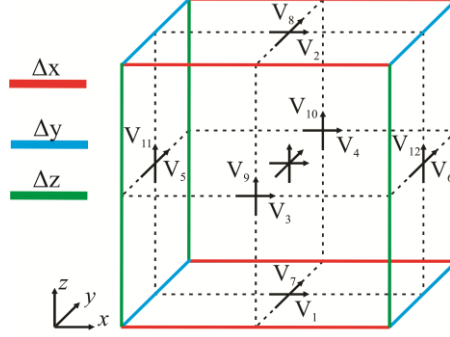


Fig. 1 Non-uniform TLM cell

Using notations for fields, current and flux densities Maxwell's curl equation can be written as

$$\begin{aligned} \nabla \times \underline{H} &= \underline{J}_{ef} + \frac{\partial}{\partial t} \underline{D}, \\ -\nabla \times \underline{E} &= \underline{J}_{mf} + \frac{\partial}{\partial t} \underline{B} \end{aligned} \quad (3)$$

After given constitutive relations for electric and magnetic current and flux densities $\underline{J}_{ef}, \underline{J}_{mf}, \underline{D}, \underline{B}$, (3) can be written as:

$$\begin{aligned} \nabla \times \underline{H} - \underline{J}_{ef} &= \frac{\partial}{\partial t} \epsilon_0 \underline{E} + \underline{\sigma}_e * \underline{E} + \frac{\partial}{\partial t} (\epsilon_0 \underline{\chi}_e * \underline{E} + \underline{\xi}_r / c * \underline{H}), \\ -\nabla \times \underline{E} - \underline{J}_{mf} &= \frac{\partial}{\partial t} \mu_0 \underline{H} + \underline{\sigma}_m * \underline{H} + \frac{\partial}{\partial t} (\zeta_r / c * \underline{E} + \mu_0 \underline{\chi}_m * \underline{H}) \end{aligned} \quad (4)$$

where sign * denotes time domain convolution, $\underline{\sigma}_e, \underline{\sigma}_m$ and $\underline{\chi}_e, \underline{\chi}_m$ are electric and magnetic conductivity and susceptibility matrices respectively, ϵ_0, μ_0 are free space permittivity and permeability, respectively, and $\underline{\xi}_r, \underline{\zeta}_r$ are dimensionless matrices describing magnetoelectric coupling coefficients given as

$$\underline{\xi}_r = \frac{1}{c} \begin{bmatrix} \xi_r^{xx} & \xi_r^{xy} & \xi_r^{xz} \\ \xi_r^{yx} & \xi_r^{yy} & \xi_r^{yz} \\ \xi_r^{zx} & \xi_r^{zy} & \xi_r^{zz} \end{bmatrix}, \quad \underline{\zeta}_r = \frac{1}{c} \begin{bmatrix} \zeta_r^{xx} & \zeta_r^{xy} & \zeta_r^{xz} \\ \zeta_r^{yx} & \zeta_r^{yy} & \zeta_r^{yz} \\ \zeta_r^{zx} & \zeta_r^{zy} & \zeta_r^{zz} \end{bmatrix} \quad (5)$$

Formulation (4) is extended further with transformations and introduced additional compact notations to create normalized form of Maxwell's equations (6):

$$\begin{aligned}\underline{\underline{C}} \cdot \underline{i} - \underline{i}_{ef} &= 2 \frac{\partial}{\partial T} \underline{V} + 2 \frac{\partial}{\partial T} \underline{\chi}_b \cdot \underline{V} + \underline{g}_e * \underline{V} + \left(\frac{2\alpha_t}{\Delta\ell} \cdot \frac{\partial}{\partial T} \cdot \underline{A} \cdot \underline{\Delta}^{-1} (\underline{\chi}_e * \underline{V} + \underline{\xi}_r * \underline{i}) \right), \\ -\underline{\underline{C}} \cdot \underline{V} - \underline{V}_{mf} &= 2 \frac{\partial}{\partial T} \underline{i} + 2 \frac{\partial}{\partial T} \underline{\chi}_b \cdot \underline{i} + \underline{r}_m * \underline{i} + \left(\frac{2\alpha_t}{\Delta\ell} \cdot \frac{\partial}{\partial T} \cdot \underline{A} \cdot \underline{\Delta}^{-1} (\underline{\chi}_m * \underline{i} + \underline{\zeta}_r * \underline{V}) \right)\end{aligned}\quad (6)$$

with $\underline{\underline{\chi}}_b, \underline{A}, \underline{\underline{C}}, \underline{\Delta}^{-1}$ representing background susceptibility matrix, matrix of inverse cell areas, normalized curl matrix and matrix of inverse cell length, respectively [9].

Previous form can be further transformed into the traveling wave format by using following relation where vector \underline{V}^i represents sum of appropriate incident wave intensities [9]:

$$\begin{aligned}\underline{\underline{C}} \cdot \underline{i} - 2 \frac{\partial}{\partial T} \underline{V} &\xrightarrow{TLM} 2\underline{V}^i - 4\underline{V}, \\ -\underline{\underline{C}} \cdot \underline{V} - 2 \frac{\partial}{\partial T} \underline{i} &\xrightarrow{TLM} -2\underline{V}^i - 4\underline{i}\end{aligned}\quad (7)$$

This leads to:

$$\begin{aligned}2\underline{V}^i - \underline{i}_{ef} &= 4\underline{V} + \underline{g}_e * \underline{V} + 2 \frac{\partial}{\partial T} \left(\underline{\chi}_b \cdot \underline{V} + \frac{\alpha_t}{\Delta\ell} \cdot \underline{A} \cdot \underline{\Delta}^{-1} \cdot (\underline{\chi}_e * \underline{V} + \underline{\xi}_r * \underline{i}) \right), \\ -2\underline{V}^i - \underline{V}_{mf} &= 4\underline{i} + \underline{r}_m * \underline{i} + 2 \frac{\partial}{\partial T} \left(\underline{\chi}_b \cdot \underline{i} + \frac{\alpha_t}{\Delta\ell} \cdot \underline{A} \cdot \underline{\Delta}^{-1} \cdot (\underline{\chi}_m * \underline{i} + \underline{\zeta}_r * \underline{V}) \right)\end{aligned}\quad (8)$$

If reflected voltage wave \underline{V}^r and reflected free current \underline{i}^r are introduced as in [9], and matrix \underline{P} is defined:

$$\underline{V}^r = 2\underline{V}^i - \underline{i}_{ef}, \quad (9)$$

$$\underline{i}^r = -2\underline{V}^i - \underline{V}_{mf}$$

$$\underline{P} = \frac{\alpha_t}{\Delta\ell} \cdot \underline{A} \cdot \underline{\Delta}^{-1} \quad (10)$$

while $\underline{\underline{\chi}}_b = \underline{P}^{-1}$, (8) can be written as:

$$\begin{aligned}2\underline{V}^r &= 4\underline{V} + \underline{g}_e * \underline{V} + 2 \frac{\partial}{\partial T} (\underline{\chi}_b \cdot \underline{V} + \underline{P} \cdot (\underline{\chi}_e * \underline{V} + \underline{\xi}_r * \underline{i})), \\ -2\underline{i}^r &= 4\underline{i} + \underline{r}_m * \underline{i} + 2 \frac{\partial}{\partial T} (\underline{\chi}_b \cdot \underline{i} + \underline{P} \cdot (\underline{\chi}_m * \underline{i} + \underline{\zeta}_r * \underline{V}))\end{aligned}\quad (11)$$

3. MATERIAL MODELING BASED ON EXTRACTED EFFECTIVE PARAMETERS

Modeling process based on proposed approach (see Fig. 2) begins by applying appropriate retrieval method [14-16] on the scattering matrix parameters of considered material, obtained either analytically, numerically or experimentally, in order to acquire effective permittivity, permeability and magnetoelectric coupling coefficients in the frequency range of interest.

In order to calculate effective parameters from S parameters the index of refraction n and characteristic impedance of considered medium z_{red} need to be first determined [15]. Based on calculated n and z_{red} effective parameters of the material can be determined as:

$$\varepsilon = \frac{n}{z_{red}}, \quad \mu = nz_{red}, \quad \xi = \sqrt{\varepsilon\mu} - n \quad (12)$$

After straightforward conversion of effective permittivity, permeability and magnetoelectric coupling coefficients to appropriate effective susceptibilities, (electric–e, magnetic–m and magnetoelectric–r which is related to (5)), they are then further approximated using the VF method [17-19] in order to represent each susceptibility in the form of rational function:

$$\chi_{[e,m,r]}(s) = \sum_{i=0}^{NP_{[e,m,r]}-1} \frac{C_{[e,m,r]i}}{s - s_{[e,m,r]pi}} \quad (13)$$

In (13) $NP_{[e,m,r]}$ stands for the number of poles, $s_{[e,m,r]pi}$ represents the set of complex pole frequencies, and $C_{[e,m,r]i}$ are the pole residues of VF approximated susceptibilities.

Next, the bilinear Z-transform is applied:

$$s \xrightarrow{z} \frac{2}{\Delta t} \left(\frac{1 - z^{-1}}{1 + z^{-1}} \right) \quad (14)$$

to obtain a discrete-time model:

$$\chi_{[e,m,r]}(z) = \frac{\sum_{i=0}^{NP_{[e,m,r]}} B_{[e,m,r]i} z^{-i}}{\sum_{i=0}^{NP_{[e,m,r]}} A_{[e,m,r]i} z^{-i}} \quad (15)$$

where $A_{[e,m,r]i}$ and $B_{[e,m,r]i}$ are real coefficients and z is time-shift operator.

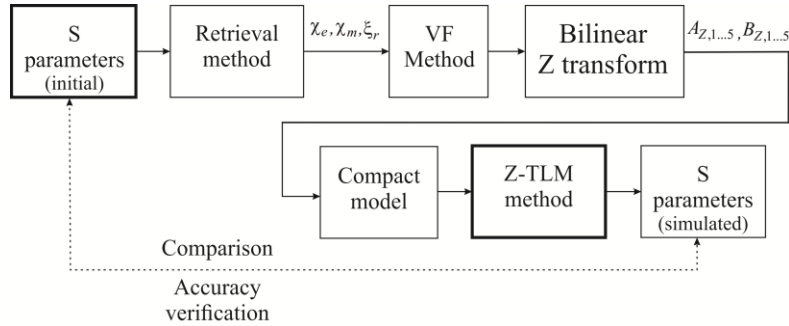


Fig. 2 Z-TLM compact modelling approach

A and B coefficients are used to define compact model which is incorporated into TLM scattering procedure where electric and magnetic fields are calculated as:

$$\begin{bmatrix} \underline{V} \\ \underline{i} \\ \underline{S}_e \\ \underline{S}_m \end{bmatrix} = \begin{bmatrix} \underline{T}_e \cdot (2\underline{V}^r + z^{-1} \underline{S}_e) \\ \underline{T}_m \cdot (-2\underline{i}^r + z^{-1} \underline{S}_m) \\ 2\underline{V}^r + k_e \underline{V} + \underline{S}_{e1} + \underline{S}_{rm} \\ -2\underline{i}^r + k_m \underline{i} + \underline{S}_{m1} + \underline{S}_{re} \end{bmatrix} \quad (16)$$

where $\underline{T}_e = (\underline{4} + \underline{g}_{e0} + 4\underline{\chi}_{e0} + 4\underline{\zeta}_{r0})^{-1}$, $\underline{T}_m = (\underline{4} + \underline{r}_{m0} + 4\underline{\chi}_{m0} + 4\underline{\zeta}_{r0})^{-1}$, $\underline{k}_e = -(\underline{4} + \underline{g}_{e0} - 4\underline{\chi}_{e1})$, $\underline{k}_m = -(\underline{4} + \underline{r}_{m0} - 4\underline{\chi}_{m1})$ and $\underline{S}_{e1}, \underline{S}_{m1}, \underline{S}_{re}, \underline{S}_{rm}$ represent additional material accumulator vectors. For simplicity, it is assumed that electric conductivity and magnetic resistivity are not dependent on frequency.

4. NUMERICAL RESULTS

Proposed approach is illustrated here for an efficient modeling of dispersive isotropic $d = 200 \text{ mm}$ wide chiral material slab placed between two isotropic free-spaces [20].

In this case due to isotropic nature of the material slab, magnetoelectric susceptibilities have the form

$$\underline{\underline{\zeta}}_r = \frac{1}{c} \begin{bmatrix} \underline{\zeta}_r & 0 & 0 \\ 0 & \underline{\zeta}_r & 0 \\ 0 & 0 & \underline{\zeta}_r \end{bmatrix}, \quad \underline{\underline{\zeta}}_r = -\underline{\underline{\zeta}}_r \quad (17)$$

By using the retrieval method, effective susceptibilities are first extracted from S parameters analytically obtained in [20] (Figs. 3 - 6, marked with triangle). The retrieved electric, magnetic and magnetoelectric susceptibilities are shown in Figs. 7, 8 and 9, respectively (marked with red triangle).

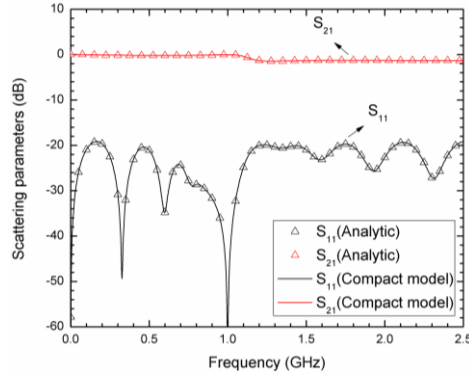


Fig. 3 Magnitude of scattering parameters S_{11} and S_{21}

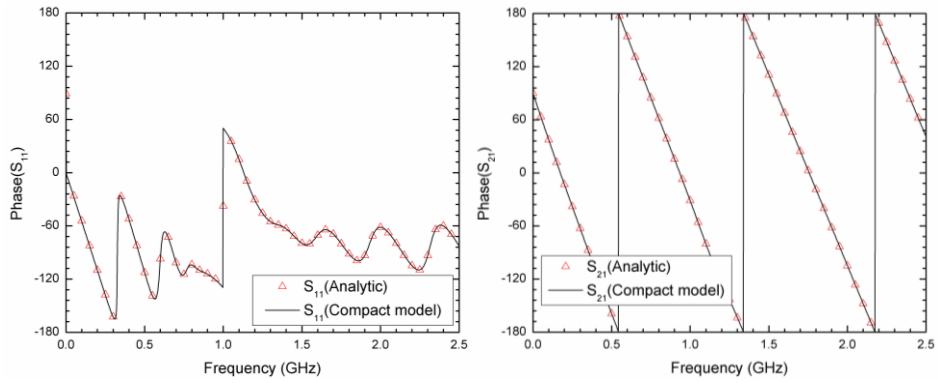


Fig. 4 Phase of scattering parameters S_{11} and S_{21}

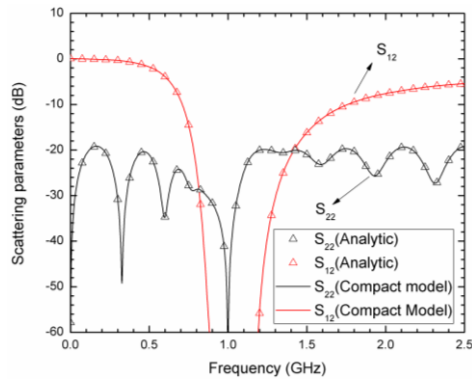


Fig. 5 Magnitude of scattering parameters S_{12} and S_{22}

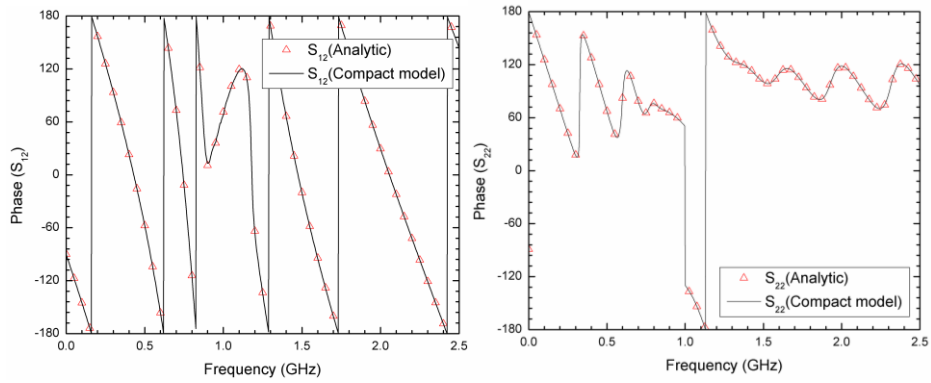


Fig. 6 Phase of scattering parameters S_{12} and S_{22}

Retrieved susceptibilities are approximated with the VF method [17-19] with fourth order rational function ($NP = 4$) (12). Obtained A's and B's coefficients for electric and magnetic susceptibility as well as magnetoelectric susceptibility are presented in Table 1. Accuracy of approximation is confirmed through perfectly matched comparison of retrieved parameters values (red triangles on the graph) and values calculated based on A and B coefficients (solid line) (see Figs. 7-9).

Discrete time-models described by (14) are further incorporated into the Z-TLM scattering algorithm in (15) taking into consideration magnetoelectric coupling characteristic of isotropic chiral slab. Total of 800 TLM cells are used in x direction, while chiral slab itself is modeled with 200 cells and simulation is executed within 2000 time steps. Model was excited with initial z polarized *Gaussian* pulse which propagates along $+x$ direction.

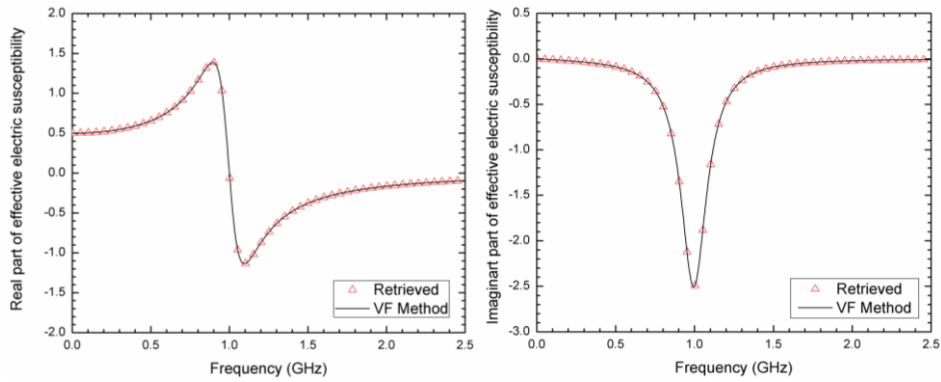


Fig. 7 Real and imaginary parts of effective electric susceptibility of isotropic chiral medium

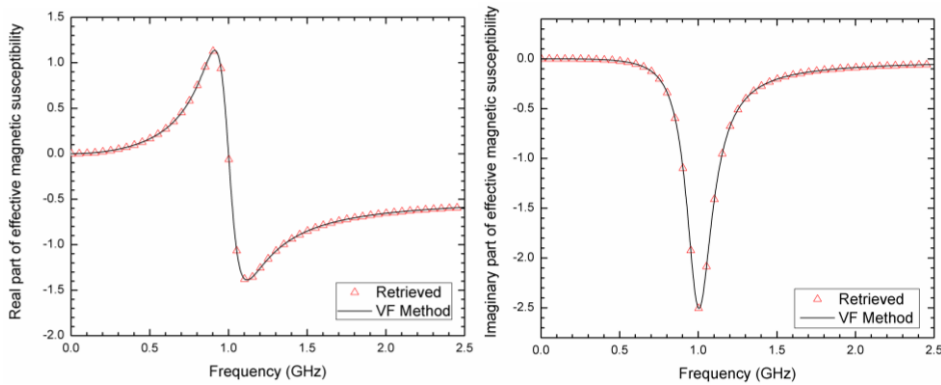


Fig. 8 Real and imaginary parts of effective magnetic susceptibility of isotropic chiral medium

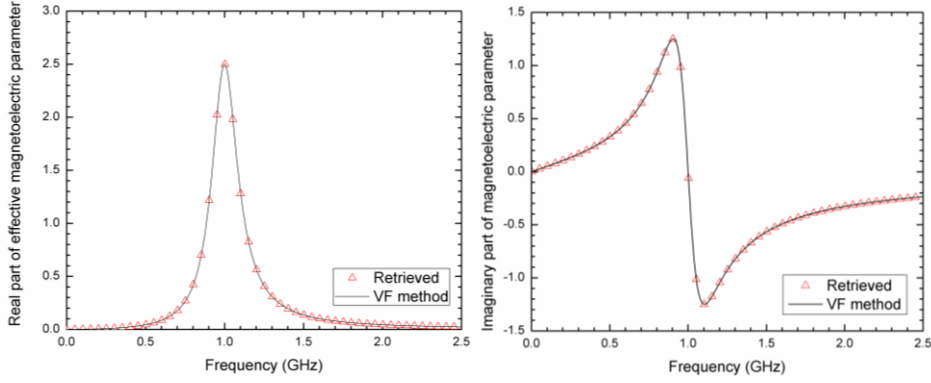


Fig. 9 Real and imaginary parts of effective magnetoelectric susceptibility of isotropic chiral medium

Two simulations are performed, first where the chiral medium was not considered present inside of the mesh in order to obtain incident field values, and second simulation with chiral medium included in the mesh in order to obtain the total field at the first interface, free space-chiral slab, and transmitted fields at the second interface, chiral slab-free space. Reflected field was calculated by deducting incident field from total field values.

Accuracy of the approach is confirmed through a comparison of magnitudes and phases of scattering parameters from [20] and simulated scattering parameters shown with solid lines in Figs. 3 - 6.

4. CONCLUSION

In this paper, an approach for compact modeling of dispersive media exhibiting magnetoelectric coupling, described with effective parameters extracted from scattering matrix, is presented. Since there is a wide variety of dispersive media exhibiting magnetoelectric coupling the compact models presented here provide efficient tools for characterization and inside view of EM waves propagation through these media. In future research this approach will be used in the design process of devices based on these media because it allows optimization and fine tuning of their characteristics in the frequency range of interest.

Table 1 Coefficients of discrete-time model in (15)

Coefficient	χ_e	χ_m	$\xi_r = -\zeta_r$
	1	1	1
A	-3.994898e+00	-1.971846e+00	-3.994897e+00
	5.985618e+00	-2.760262e-02	5.985617e+00
	-3.986539e+00	1.971845e+00	-3.986539e+00
	9.958199e-01	-9.723764e-01	9.958197e-01
B	5.471056e-05	-4.988997e-01	5.224473e-03
	2.651185e-08	9.860555e-01	-1.044641e-02
	-1.093681e-04	1.174398e-02	-3.898947e-09
	2.649748e-08	-9.860555e-01	1.044641e-02
	5.471051e-05	4.871557e-01	-5.224469e-03

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