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ON SOME COMMON FIXED POINT THEOREMS FOR GENERALIZED INTEGRAL TYPE F-CONTRACTIONS IN PARTIAL METRIC SPACES

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Abstract. In this article, we prove some common fixed point theorems for generalized integral type F-contractions in the setting of complete partial metric spaces and give some consequences of the main result. Also we give an example in support of the result. Our result extends and generalizes several results from the existing literature.
Keywords: Common fixed point, generalized integral type F-contraction, partial metric space.

1. Introduction

Banach contraction principle [7] is one of the milestones in the development of fixed point theory. Its significance lies in the vast applicability to a great number of branches of mathematical sciences, for example, theory of existence of f solutions for nonlinear differential, integral and functional equations, variational inequalities and optimization and approximation theory. There are many generalizations of this principle. These generalizations are made either by using different contractive conditions or by imposing some additional condition on the ambient spaces.

A mapping $S{:}\mathcal{U} \to \mathcal{U}$, where $\mathcal{U}$ is a nonempty set and $(\mathcal{U}, d)$ is a metric space, is said to be a contraction if there exists $c \in [0,1)$ such that for all $y, z \in \mathcal{U}$,

$$d(S(y), S(z)) \leq c\, d(y, z). \tag{1.1}$$

If the metric space $(\mathcal{U}, d)$ is complete then the mapping satisfying (1.1) has a unique fixed point. Inequality (1.1) implies continuity of S. Many authors generalized this

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famous result in different ways. Indeed, one of those ways is integral type contraction which was introduced by Branciari [8] in 2002 and proved a fixed point result for mappings defined on a complete metric space satisfying a general contractive type condition of integral type.

Matthews [14] introduced the concept of partial metric space as a part of the study of denotational semantics of dataflow networks [13, 14, 16, 23]. It is widely recognized that partial metric spaces play an important role in constructing models in the theory of computation. In partial metric spaces the distance of a point in the self may not be zero. Introducing partial metric space, Matthews extended the Banach contraction principle [7] and proved the fixed point theorem in this space.

Wardowski [22] *(Fixed Point Theory Appl. 2012, Article ID 94(2012))* introduced a new type of contraction called F-contraction and proved a new fixed point theorem related to F-contraction and gave an example showing that the obtained extension is significant. Later, a large number of researchers have proved many results in this direction (for more details see, [2], [3], [4], [5], [10], [12], [17], [18], [19], [20] and many others).

In this paper, we establish a common fixed point theorem for generalized integral type F-contraction in partial metric spaces and give some consequences of main result as corollaries. We also give an example in support of the result. Our result extends and generalizes many comparable results in the existing literature.

2. Preliminaries

Now, we give some basic properties and auxiliary results on the concept of partial metric space (PMS) and F-contraction.

Definition 2.1. ([14]) Let $\mathcal{U}$ be a nonempty set and $p: \mathcal{U} \times \mathcal{U} \to \mathbb{R}^+$ be such that for all $y, z, w \in \mathcal{U}$ the followings are satisfied:

$(P1)$ $y = z \Leftrightarrow p(y, y) = p(y, z) = p(z, z)$,

$(P2)$ $p(y, y) \leq p(y, z)$,

$(P3)$ $p(y, z) = p(z, y)$,

$(P4)$ $p(y, z) \leq p(y, w) + p(w, z) - p(w, w)$.

Then p is called partial metric on $\mathcal{U}$ and the pair $(\mathcal{U}, p)$ is called partial metric space (in short PMS).

Remark 2.1. It is clear that if $p(y, y) = 0$, then $y = z$. But, on the contrary $p(y, y)$ need not be zero.

Example 2.1. ([6]) Let $\mathcal{U} = \mathbb{R}^+$ and $p: \mathcal{U} \times \mathcal{U} \to \mathbb{R}^+$ given by $p(y, z) = \max\{y, z\}$ for all $y, z \in \mathbb{R}^+$. Then $(\mathbb{R}^+, p)$ is a partial metric space.

Example 2.2. ([6]) Let $\mathcal{U} = \{[a, b] : a, b \in \mathbb{R}, a \leq b\}$. Then $p\Big([a, b], [c, d]\Big) = \max\{b, d\} - \min\{a, c\}$ defines a partial metric p on $\mathcal{U}$.

Various applications of this space has been extensively investigated by many authors (see [11], [21] for details).

Remark 2.2. ([9]) Let $(\mathcal{U}, p)$ be a partial metric space.

(1) The function $d_w: \mathcal{U} \times \mathcal{U} \to \mathbb{R}^+$ defined as $d_w(y, z) = 2p(y, z) - p(y, y) - p(z, z)$ is a (usual) metric on $\mathcal{U}$ and $(\mathcal{U}, d_w)$ is a (usual) metric space.

(2) The function $d_s: \mathcal{U} \times \mathcal{U} \to \mathbb{R}^+$ defined as $d_s(y, z) = \max\{p(y, z) - p(y, y), p(y, z) - p(z, z)\}$ is a (usual) metric on $\mathcal{U}$ and $(\mathcal{U}, d_s)$ is a (usual) metric space.

Note also that each partial metric p on $\mathcal{U}$ generates a T_0 topology τ_p on $\mathcal{U}$, whose base is a family of open p-balls $\{B_p(y, \varepsilon) : y \in \mathcal{U}, \varepsilon > 0\}$ where $B_p(y, \varepsilon) = \{z \in \mathcal{U} : p(y, z) \leq p(y, y) + \varepsilon\}$ for all $y \in \mathcal{U}$ and $\varepsilon > 0$.

On a partial metric space the notions of convergence, the Cauchy sequence, completeness and continuity are defined as follows [13].

Definition 2.2. ([13]) Let $(\mathcal{U}, p)$ be a partial metric space. Then

(a) a sequence $\{y_n\}$ in $(\mathcal{U}, p)$ is said to be convergent to a point $y \in \mathcal{U}$ if and only if $p(y, y) = \lim_{n\to\infty} p(y_n, y)$;

(b) a sequence $\{y_n\}$ is called a Cauchy sequence if $\lim_{m,n\to\infty} p(y_m, y_n)$ exists and finite;

(c) $(\mathcal{U}, p)$ is said to be complete if every Cauchy sequence $\{y_n\}$ in $\mathcal{U}$ converges to a point $y \in \mathcal{U}$ with respect to τ_p. Furthermore,

$$\lim_{m,n\to\infty} p(y_m, y_n) = \lim_{n\to\infty} p(y_n, y) = p(y, y).$$

(d) A mapping $R: \mathcal{U} \to \mathcal{U}$ is said to be continuous at $z_0 \in \mathcal{U}$ if for every $\varepsilon > 0$, there exists $\delta > 0$ such that $R\Big(B_p(z_0, \delta)\Big) \subset B_p\Big(R(z_0), \varepsilon\Big)$.

Definition 2.3. ([15]) Let $(\mathcal{U}, p)$ be a partial metric space. Then

$(e1)$ a sequence $\{y_n\}$ in $(\mathcal{U}, p)$ is called 0-Cauchy if $\lim_{m,n\to\infty} p(y_m, y_n) = 0$;

$(e2)$ $(\mathcal{U}, p)$ is said to be 0-complete if every 0-Cauchy sequence $\{y_n\}$ in $\mathcal{U}$ converges to a point $y \in \mathcal{U}$, such that $p(y, y) = 0$.

Lemma 2.1. *([13, 14]) Let $(\mathcal{U}, p)$ be a partial metric space. Then* *(f1) a sequence $\{y_n\}$ in $(\mathcal{U}, p)$ is a Cauchy sequence if and only if it is a Cauchy sequence in the metric space $(\mathcal{U}, d_w)$;*

(f2) $(\mathcal{U}, p)$ is complete if and only if the metric space $(\mathcal{U}, d_w)$ is complete;

(f3) a subset E of a partial metric space $(\mathcal{U}, p)$ is closed if a sequence $\{y_n\}$ in E such that $\{y_n\}$ converges to some $y \in \mathcal{U}$, then $y \in E$.

Lemma 2.2. *([1]) Assume that $y_n \to u$ as $n \to \infty$ in a partial metric space $(\mathcal{U}, p)$ such that $p(u, u) = 0$. Then $\lim_{n\to\infty} p(y_n, y) = p(u, y)$ for every $y \in \mathcal{U}$.*

Remark 2.3. (see [9]) Let $(\mathcal{U}, p)$ be a PMS. Therefore, for all $y, z \in \mathcal{U}$

(i) if $p(y, z) = 0$, then $y = z$;

(ii) if $y \neq z$, then $p(y, z) > 0$.

The definition of F-contraction is due to Wardowski [22], which can be stated as follows.

Let $\mathcal{F}$ be the family of all functions $F: [0, \infty) \to \mathbb{R}$ satisfying the following conditions:

$(F1.)$ F is strictly increasing, i.e., for all $\alpha, \beta \in [0, \infty)$ such that $\alpha < \beta$, $F(\alpha) < F(\beta)$.

$(F2.)$ For each sequence $\{\alpha_n\}_{n\in\mathbb{N}}$ of positive numbers, $\lim_{n\to\infty} \alpha_n = 0$ if and only if $\lim_{n\to\infty} F(\alpha_n) = -\infty$.

$(F3.)$ There exists $k \in (0, 1)$ such that $\lim_{\alpha\to 0^+} \alpha^k F(\alpha) = 0$.

Let $F_1(\alpha) = ln\,(\alpha)$, $F_2(\alpha) = -\frac{1}{\sqrt{\alpha}}$ and $F_3(\alpha) = \alpha + ln\,(\alpha)$ for $\alpha > 0$, then $F_1, F_2, F_3 \in \mathcal{F}$.

Definition 2.4. ([22]) A mapping $\mathcal{T}: \mathcal{U} \to \mathcal{U}$ is said to be an F-contraction if there exists $\tau > 0$ such that

$$\forall\, y, z \in \mathcal{U},\ \big\{(d(\mathcal{T}y, \mathcal{T}z)) > 0 \Rightarrow \tau + F(d(\mathcal{T}y, \mathcal{T}z)) \leq F(d(y, z))\big\}. \tag{2.1}$$

Example 2.3. ([22]) Let $F: [0, \infty) \to \mathbb{R}$ be given by $F(\alpha) = ln\,\alpha$. Then F satisfies $(F1)$-$(F3)$. Each mapping satisfying (2.1) is an F-contraction such that

$$d(\mathcal{T}y, \mathcal{T}z) \leq e^{-\tau}\, d(y, z), \tag{2.2}$$

for all $y, z \in \mathcal{U}$ and $\mathcal{T}y \neq \mathcal{T}z$.

It is clear that for $y, z \in \mathcal{U}$ such that $\mathcal{T}y = \mathcal{T}z$ the inequality $d(\mathcal{T}y, \mathcal{T}z) \leq e^{-\tau}\, d(y, z)$ also holds, i.e., $\mathcal{T}$ is a Banach contraction [7].

Example 2.4. ([22]) Let $F: [0, \infty) \to \mathbb{R}$ be given by $F(\alpha) = \alpha + ln\,\alpha$ for $\alpha > 0$. From (2.1) we get

$$\frac{d(\mathcal{T}y, \mathcal{T}z)}{d(y, z)} e^{d(\mathcal{T}y, \mathcal{T}z) - d(y, z)} \leq e^{-\tau},$$

for all $y, z \in \mathcal{U}$ and $\mathcal{T}y \neq \mathcal{T}z$, i.e., $d(\mathcal{T}y, \mathcal{T}z) > 0$.

Wardowski [22] proved the following fixed point theorem.

Theorem 2.1. *([22]) Let $(\mathcal{U}, d)$ be a complete metric space and let $\mathcal{T}: \mathcal{U} \to \mathcal{U}$ be an F-contraction. Then $\mathcal{T}$ has a unique fixed point in $\mathcal{U}$.*

3. Main Results

In this section, we shall prove some unique common fixed point theorems in the setting of complete partial metric spaces through generalized integral type F-contractions.

Theorem 3.1. *Let $(\mathcal{U}, p)$ be a complete partial metric space and let $\mathcal{S}_1, \mathcal{S}_2: \mathcal{U} \to \mathcal{U}$ be two self-mappings. Suppose that there exist $F \in \mathcal{F}$ and $\tau > 0$ such that for all $y, z \in \mathcal{U}$ satisfying $p(\mathcal{S}_1 y, \mathcal{S}_2 z) > 0$, the following holds:*

$$\tau + F\Big(\int_0^{p(\mathcal{S}_1 y, \mathcal{S}_2 z)} \psi(t)dt\Big) \leq F\Big(\int_0^{\mu(y,z)} \psi(t)dt\Big), \tag{3.1}$$

where

$$\begin{aligned} \mu(y,z) &= \max\Big\{p(y,z), \frac{1}{3}[p(y,z) + p(z, \mathcal{S}_1 y) + p(y, \mathcal{S}_2 z)], \\ &\qquad \frac{1}{3}[p(y,z) + p(y, \mathcal{S}_1 y) + p(z, \mathcal{S}_2 z)]\Big\} \end{aligned} \tag{3.2}$$

and $\psi: [0, \infty) \to [0, \infty)$ is a Lebesgue-integrable mapping which is summable on each compact subset of $[0, \infty)$, nonnegative and for each $\varepsilon > 0$

$$\int_0^{\varepsilon} \psi(t)dt > 0, \tag{3.3}$$

and if F is continuous. Then $\mathcal{S}_1$ and $\mathcal{S}_2$ have a unique common fixed point in $\mathcal{U}$.

Proof. Let $z_0 \in \mathcal{U}$ be an arbitrary point. Define a sequence $\{z_n\}$ for $n \geq 0$ by

$$z_{2n+1} = \mathcal{S}_1 z_{2n} \text{ and } z_{2n+2} = \mathcal{S}_2 z_{2n+1}. \tag{3.4}$$

Step I. Now, we have to prove that $p(z_{n+1}, z_n) \to 0$ as $n \to \infty$. By equation (3.1), we have

$$\begin{aligned} \tau + F\Big(\int_0^{p(z_{2n+1}, z_{2n})} \psi(t)dt\Big) &= \tau + F\Big(\int_0^{p(\mathcal{S}_1 z_{2n}, \mathcal{S}_2 z_{2n-1})} \psi(t)dt\Big) \\ &\leq F\Big(\int_0^{\mu(z_{2n}, z_{2n-1})} \psi(t)dt\Big), \end{aligned} \tag{3.5}$$

where

$$\begin{aligned}\mu(z_{2n}, z_{2n-1}) &= \max\Big\{p(z_{2n}, z_{2n-1}), \frac{1}{3}[p(z_{2n}, z_{2n-1}) + p(z_{2n}, \mathcal{S}_2 z_{2n-1}) + p(z_{2n-1}, \mathcal{S}_1 z_{2n})], \\ &\quad \frac{1}{3}[p(z_{2n}, z_{2n-1}) + p(z_{2n}, \mathcal{S}_1 z_{2n}) + p(z_{2n-1}, \mathcal{S}_2 z_{2n-1})]\Big\} \\ &= \max\Big\{p(z_{2n}, z_{2n-1}), \frac{1}{3}[p(z_{2n}, z_{2n-1}) + p(z_{2n}, z_{2n}) + p(z_{2n-1}, z_{2n+1})], \\ &\quad \frac{1}{3}[p(z_{2n}, z_{2n-1}) + p(z_{2n}, z_{2n+1}) + p(z_{2n-1}, z_{2n})]\Big\} \\ &\le \max\Big\{p(z_{2n-1}, z_{2n}), \frac{1}{3}[p(z_{2n-1}, z_{2n}) + p(z_{2n-1}, z_{2n}) + p(z_{2n+1}, z_{2n})], \\ &\quad \frac{1}{3}[p(z_{2n-1}, z_{2n}) + p(z_{2n+1}, z_{2n}) + p(z_{2n-1}, z_{2n})]\Big\} \ (by (P3)\ and\ (P4)) \\ &= \max\Big\{p(z_{2n-1}, z_{2n}), p(z_{2n+1}, z_{2n})\Big\}. \qquad (3.6)\end{aligned}$$

If $\max\Big\{p(z_{2n-1}, z_{2n}), p(z_{2n+1}, z_{2n})\Big\} = p(z_{2n+1}, z_{2n})$, then it follows from (3.5)

$$\tau + F\Big(\int_0^{p(z_{2n+1}, z_{2n})} \psi(t)dt\Big) \le F\Big(\int_0^{p(z_{2n+1}, z_{2n})} \psi(t)dt\Big), \qquad (3.7)$$

which is a contradiction (as $\tau > 0$). Thus,

$$\max\Big\{p(z_{2n-1}, z_{2n}), p(z_{2n+1}, z_{2n})\Big\} = p(z_{2n-1}, z_{2n}). \qquad (3.8)$$

From equation (3.5), we have

$$F\Big(\int_0^{p(z_{2n+1}, z_{2n})} \psi(t)dt\Big) \le F\Big(\int_0^{p(z_{2n-1}, z_{2n})} \psi(t)dt\Big) - \tau. \qquad (3.9)$$

Continuing in the same fashion, we obtain

$$F\Big(\int_0^{p(z_{2n-1}, z_{2n})} \psi(t)dt\Big) \le F\Big(\int_0^{p(z_{2n-2}, z_{2n-1})} \psi(t)dt\Big) - \tau. \qquad (3.10)$$

Using (3.9) and (3.10), we get

$$\begin{aligned}F\Big(\int_0^{p(z_{2n+1}, z_{2n})} \psi(t)dt\Big) &\le F\Big(\int_0^{p(z_{2n}, z_{2n-1})} \psi(t)dt\Big) - \tau \\ &\le F\Big(\int_0^{p(z_{2n-1}, z_{2n-2})} \psi(t)dt\Big) - 2\tau \\ &\le \cdots \\ &\le F\Big(\int_0^{p(z_1, z_0)} \psi(t)dt\Big) - (2n)\tau. \qquad (3.11)\end{aligned}$$

Then, it follows $\lim_{n\to\infty} F\Big(\int_0^{p(z_{n+1},z_n)} \psi(t)dt\Big) = -\infty$. By $F \in \mathcal{F}$ and $(F2)$, we have

$$\lim_{n\to\infty} p(z_{n+1}, z_n) = 0. \tag{3.12}$$

Step II. Now, we show that $\{z_n\}$ is a p-Cauchy sequence. Put $b_n = p(z_{n+1}, z_n)$, $n = 0, 1, 2, \ldots$. By $F \in \mathcal{F}$ and $(F3)$, there exists $k \in (0, 1)$ such that

$$\lim_{n\to\infty} (b_n)^k F(b_n) = 0. \tag{3.13}$$

By (3.11), we have

$$\begin{aligned} &\Big(p(z_{2n+1}, z_{2n})\Big)^k F\Big(\int_0^{p(z_{2n+1},z_{2n})} \psi(t)dt\Big) - F\Big(\int_0^{p(z_1,z_0)} \psi(t)dt\Big) \\ &\leq \quad -(2n)\Big(p(z_{2n+1}, z_{2n})\Big)^k \tau \leq 0. \end{aligned} \tag{3.14}$$

Using the above inequality and (3.13), we get

$$\lim_{n\to\infty} n\Big(p(z_{n+1}, z_n)\Big)^k = 0. \tag{3.15}$$

Therefore, there exists a positive integer $N_1 \in \mathbb{N}$ such that $n\Big(p(z_{n+1}, z_n)\Big)^k < 1$ for all $n > N_1$, or

$$p(z_{n+1}, z_n) < \frac{1}{n^{1/k}}. \tag{3.16}$$

Let $m, n \in \mathbb{N}$ with $m > n > N_1$, using $(P4)$ (triangular inequality), we have

$$\begin{aligned} p(z_n, z_m) &\leq p(z_n, z_{n+1}) + p(z_{n+1}, z_{n+2}) + \ldots + p(z_{m-1}, z_m) \\ &\quad -[p(z_{n+1}, z_{n+1}) + p(z_{n+2}, z_{n+2}) + \ldots + p(z_{m-1}, z_{m-1})] \\ &\leq p(z_n, z_{n+1}) + p(z_{n+1}, z_{n+2}) + \ldots + p(z_{m-1}, z_m) \\ &= \sum_{r=n}^{m-1} p(z_{r+1}, z_r) \leq \sum_{r=n}^{\infty} p(z_{r+1}, z_r) \\ &\leq \sum_{r=n}^{\infty} \frac{1}{r^{1/k}}. \end{aligned} \tag{3.17}$$

As $k \in (0, 1)$, the series $\sum_{r=n}^{\infty} \left(\frac{1}{r^{1/k}}\right)$ is convergent, so

$$\lim_{n,m\to\infty} p(z_n, z_m) = 0. \tag{3.18}$$

Thus $\{z_n\}$ is a Cauchy sequence in $(\mathcal{U}, p)$. Therefore, $\{z_n\}$ is a Cauchy sequence in $(\mathcal{U}, d_w)$. Since $(\mathcal{U}, p)$ is a complete partial metric space, then by Lemma 2.1,

$(\mathcal{U}, d_w)$ is also complete. Thus, there exists a $v \in \mathcal{U}$ such that $\lim_{n\to\infty} z_n = v$ and $\lim_{n\to\infty} d_w(z_n, v) = 0$. Moreover, by Definition 2.2 (3') and equation (3.18), we have

$$p(v, v) = \lim_{n\to\infty} p(z_n, v) = \lim_{n,m\to\infty} p(z_n, z_m) = 0. \tag{3.19}$$

Step III. Now, we shall show that v is a common fixed point of $\mathcal{S}_1$ and $\mathcal{S}_2$. Using given contractive condition (3.1) for $y = z_{2n}$ and $z = v$, we have

$$\begin{aligned} \tau + F\Big(\int_0^{p(z_{2n+1}, \mathcal{S}_2 v)} \psi(t)dt\Big) &= \tau + F\Big(\int_0^{p(\mathcal{S}_1 z_{2n}, \mathcal{S}_2 v)} \psi(t)dt\Big) \\ &\leq F\Big(\int_0^{\mu(z_{2n}, v)} \psi(t)dt\Big), \end{aligned} \tag{3.20}$$

where

$$\begin{aligned} \mu(z_{2n}, v) &= \max\Big\{p(z_{2n}, v), \frac{1}{3}[p(z_{2n}, v) + p(z_{2n}, \mathcal{S}_2 v) + p(v, \mathcal{S}_1 z_{2n})], \\ &\qquad \frac{1}{3}[p(z_{2n}, v) + p(z_{2n}, \mathcal{S}_1 z_{2n}) + p(v, \mathcal{S}_2 v)]\Big\} \\ &= \max\Big\{p(z_{2n}, v), \frac{1}{3}[p(z_{2n}, v) + p(z_{2n}, \mathcal{S}_2 v) + p(v, z_{2n+1})], \\ &\qquad \frac{1}{3}[p(z_{2n}, v) + p(z_{2n}, z_{2n+1}) + p(v, \mathcal{S}_2 v)]\Big\}. \end{aligned} \tag{3.21}$$

Passing to limit as $n \to \infty$ in (3.21) and using (3.19), we obtain

$$\mu(z_{2n}, v) \to \max\Big\{0, \frac{p(v, \mathcal{S}_2 v)}{3}, \frac{p(v, \mathcal{S}_2 v)}{3}\Big\} = \frac{p(v, \mathcal{S}_2 v)}{3} < p(v, \mathcal{S}_2 v). \tag{3.22}$$

Now, using (3.20) and (3.22), we get

$$\tau + F\Big(\int_0^{p(z_{2n+1}, \mathcal{S}_2 v)} \psi(t)dt\Big) \leq F\Big(\int_0^{p(v, \mathcal{S}_2 v)} \psi(t)dt\Big). \tag{3.23}$$

Passing to limit as $n \to \infty$ in (3.23) and using continuity of F, we obtain

$$\tau + F\Big(\int_0^{p(v, \mathcal{S}_2 v)} \psi(t)dt\Big) \leq F\Big(\int_0^{p(v, \mathcal{S}_2 v)} \psi(t)dt\Big),$$

which is a contradiction since $\tau > 0$. Thus, we have $\mathcal{S}_2 v = v$. This shows that v is a fixed point of $\mathcal{S}_2$. Similar, we can show that $\mathcal{S}_1 v = v$. Hence v is a common fixed point of $\mathcal{S}_1$ and $\mathcal{S}_2$.

Step IV. Now, we shall show the uniqueness of the common fixed point. Assume that v' is another common fixed point of $\mathcal{S}_1$ and $\mathcal{S}_2$, that is, $\mathcal{S}_1 v' = v' = \mathcal{S}_2 v'$ with

$v \neq v'$. From the given contractive condition (3.1), we have

$$\begin{aligned} \tau + F\Big(\int_0^{p(v,v')} \psi(t)dt\Big) &= F\Big(\int_0^{p(\mathcal{S}_1 v,\mathcal{S}_2 v')} \psi(t)dt\Big) \\ &\leq F\Big(\int_0^{\mu(v,v')} \psi(t)dt\Big), \end{aligned} \tag{3.24}$$

where

$$\begin{aligned} \mu(v,v') &= \max\Big\{p(v,v'), \frac{1}{3}[p(v,v') + p(v',\mathcal{S}_1 v) + p(v,\mathcal{S}_2 v')], \\ &\qquad \frac{1}{3}[p(v,v') + p(v,\mathcal{S}_1 v) + p(v',\mathcal{S}_2 v')]\Big\} \\ &= \max\Big\{p(v,v'), \frac{1}{3}[p(v,v') + p(v',v) + p(v,v')], \\ &\qquad \frac{1}{3}[p(v,v') + p(v,v) + p(v',v')]\Big\}. \end{aligned} \tag{3.25}$$

Using condition $(P3)$ and (3.19), we get

$$\mu(v,v') \to \max\Big\{p(v,v'), p(v,v'), \frac{p(v,v')}{3}\Big\} = p(v,v'). \tag{3.26}$$

From (3.24) and (3.26), we obtain

$$\tau + F\Big(\int_0^{p(v,v')} \psi(t)dt\Big) \leq F\Big(\int_0^{p(v,v')} \psi(t)dt\Big),$$

which is a contradiction since $\tau > 0$. Thus, we have $v = v'$. This shows that the common fixed point of $\mathcal{S}_1$ and $\mathcal{S}_2$ is unique. This completes the proof. □

Theorem 3.2. *Let $(\mathcal{U}, p)$ be a complete partial metric space and let $\mathcal{R}_1, \mathcal{R}_2 : \mathcal{U} \to \mathcal{U}$ be two self-mappings. Suppose that there exist $F \in \mathcal{F}$ and $\tau > 0$ such that for all $y, z \in \mathcal{U}$ satisfying $p(\mathcal{R}_1 y, \mathcal{R}_2 z) > 0$, the following holds:*

$$\tau + F\Big(\int_0^{p(\mathcal{R}_1 y,\mathcal{R}_2 z)} \psi(t)dt\Big) \leq F\Big(\int_0^{\nu(y,z)} \psi(t)dt\Big), \tag{3.27}$$

where

$$\begin{aligned} \nu(y,z) &= \max\Big\{p(y,z), \frac{1}{2}[p(z,\mathcal{R}_1 y) + (y,\mathcal{R}_2 z)], \\ &\qquad \frac{p(y,\mathcal{R}_1 y)p(z,\mathcal{R}_2 z)}{1 + p(y,z)}, \frac{p(y,\mathcal{R}_1 y)p(z,\mathcal{R}_2 z)}{1 + p(\mathcal{R}_1 y,\mathcal{R}_2 z)}\Big\}, \end{aligned} \tag{3.28}$$

and F, ψ are as in Theorem 3.1. Then $\mathcal{R}_1$ and $\mathcal{R}_2$ have a unique common fixed point in $\mathcal{U}$.

Proof. Let $r_0 \in \mathcal{U}$ be an arbitrary point. Define a sequence $\{r_n\}$ for $n \geq 0$ by

$$r_{2n+1} = \mathcal{R}_1 r_{2n} \ \ and \ \ r_{2n+2} = \mathcal{R}_2 r_{2n+1}. \tag{3.29}$$

Step I. Now, we have to prove that $p(r_{n+1}, r_n) \to 0$ as $n \to \infty$. By equation (3.27), we have

$$\begin{aligned} \tau + F\Big(\int_0^{p(r_{2n+1}, r_{2n})} \psi(t)dt\Big) &= \tau + F\Big(\int_0^{p(\mathcal{R}_1 r_{2n}, \mathcal{R}_2 r_{2n-1})} \psi(t)dt\Big) \\ &\leq F\Big(\int_0^{\nu(r_{2n}, r_{2n-1})} \psi(t)dt\Big), \end{aligned} \tag{3.30}$$

where

$$\begin{aligned} \nu(r_{2n}, r_{2n-1}) &= \max\Big\{p(r_{2n}, r_{2n-1}), \frac{1}{2}[p(r_{2n}, \mathcal{R}_2 r_{2n-1}) + p(r_{2n-1}, \mathcal{R}_1 r_{2n})], \\ &\quad \frac{p(r_{2n}, \mathcal{R}_1 r_{2n})p(r_{2n-1}, \mathcal{R}_2 r_{2n-1})}{1 + p(r_{2n}, r_{2n-1})}, \frac{p(r_{2n}, \mathcal{R}_1 r_{2n})p(r_{2n-1}, \mathcal{R}_2 r_{2n-1})}{1 + p(\mathcal{R}_1 r_{2n}, \mathcal{R}_2 r_{2n-1})}\Big\} \\ &= \max\Big\{p(r_{2n}, r_{2n-1}), \frac{1}{2}[p(r_{2n}, r_{2n}) + p(r_{2n-1}, r_{2n+1})], \\ &\quad \frac{p(r_{2n}, r_{2n+1})p(r_{2n-1}, r_{2n})}{1 + p(r_{2n}, r_{2n-1})}, \frac{p(r_{2n}, r_{2n+1})p(r_{2n-1}, r_{2n})}{1 + p(r_{2n+1}, r_{2n})}\Big\} \\ &\leq \max\Big\{p(r_{2n}, r_{2n-1}), \frac{1}{2}[p(r_{2n-1}, r_{2n}) + p(r_{2n+1}, r_{2n})], \\ &\quad \frac{p(r_{2n+1}, r_{2n})p(r_{2n-1}, r_{2n})}{1 + p(r_{2n-1}, r_{2n})}, \frac{p(r_{2n-1}, r_{2n})p(r_{2n-1}, r_{2n})}{1 + p(r_{2n+1}, r_{2n})}\Big\} \\ &\quad (by\, (P3)\, and\, (P4)) \\ &= \max\Big\{p(r_{2n-1}, r_{2n}), p(r_{2n+1}, r_{2n})\Big\}. \end{aligned} \tag{3.31}$$

If $\max\Big\{p(r_{2n-1}, r_{2n}), p(r_{2n+1}, r_{2n})\Big\} = p(r_{2n+1}, r_{2n})$, then it follows from (3.30)

$$\tau + F\Big(\int_0^{p(r_{2n+1}, r_{2n})} \psi(t)dt\Big) \leq F\Big(\int_0^{p(r_{2n+1}, r_{2n})} \psi(t)dt\Big), \tag{3.32}$$

which is a contradiction (as $\tau > 0$). Thus,

$$\max\Big\{p(r_{2n-1}, r_{2n}), p(r_{2n+1}, r_{2n})\Big\} = p(r_{2n-1}, r_{2n}). \tag{3.33}$$

From equation (3.30), we have

$$F\Big(\int_0^{p(r_{2n+1}, r_{2n})} \psi(t)dt\Big) \leq F\Big(\int_0^{p(r_{2n-1}, r_{2n})} \psi(t)dt\Big) - \tau. \tag{3.34}$$

Continuing in the same manner, we obtain

$$F\Big(\int_0^{p(r_{2n-1}, r_{2n})} \psi(t)dt\Big) \leq F\Big(\int_0^{p(r_{2n-2}, r_{2n-1})} \psi(t)dt\Big) - \tau. \tag{3.35}$$

Using (3.34) and (3.35), we get

$$
\begin{aligned}
F\Big(\int_0^{p(r_{2n+1},r_{2n})}\psi(t)dt\Big) &\le F\Big(\int_0^{p(r_{2n},r_{2n-1})}\psi(t)dt\Big)-\tau\\
&\le F\Big(\int_0^{p(r_{2n-1},r_{2n-2})}\psi(t)dt\Big)-2\tau\\
&\le \cdots\\
&\le F\Big(\int_0^{p(r_1,r_0)}\psi(t)dt\Big)-(2n)\tau. \qquad (3.36)
\end{aligned}
$$

Then, it follows $\lim_{n\to\infty} F\Big(\int_0^{p(r_{n+1},r_n)}\psi(t)dt\Big) = -\infty$. By $F\in\mathcal{F}$ and $(F2)$, we have

$$\lim_{n\to\infty} p(r_{n+1},r_n)=0. \qquad (3.37)$$

Step II. Now, we show that $\{r_n\}$ is a p-Cauchy sequence. Put $q_n=p(r_{n+1},r_n)$, $n=0,1,2,\ldots$. By $F\in\mathcal{F}$ and $(F3)$, there exists $k\in(0,1)$ such that

$$\lim_{n\to\infty}(q_n)^kF(q_n)=0. \qquad (3.38)$$

By (3.36), we have

$$
\begin{aligned}
&\Big(p(r_{2n+1},r_{2n})\Big)^kF\Big(\int_0^{p(r_{2n+1},r_{2n})}\psi(t)dt\Big)-F\Big(\int_0^{p(r_1,r_0)}\psi(t)dt\Big)\\
&\le -(2n)\Big(p(r_{2n+1},r_{2n})\Big)^k\tau\le 0. \qquad (3.39)
\end{aligned}
$$

Using the above inequality and (3.38), we get

$$\lim_{n\to\infty} n\Big(p(r_{n+1},r_n)\Big)^k=0. \qquad (3.40)$$

Therefore, there exists a positive integer $N_p\in\mathbb{N}$ such that $n\Big(p(r_{n+1},r_n)\Big)^k<1$ for all $n>N_p$, or

$$p(r_{n+1},r_n)<\frac{1}{n^{1/k}}. \qquad (3.41)$$

Let $m,n\in\mathbb{N}$ with $m>n>N_p$, using $(P4)$ (triangular inequality), we have

$$
\begin{aligned}
p(r_n,r_m) &\le p(r_n,r_{n+1})+p(r_{n+1},r_{n+2})+\ldots+p(r_{m-1},r_m)\\
&\quad -[p(r_{n+1},r_{n+1})+p(r_{n+2},r_{n+2})+\ldots+p(r_{m-1},r_{m-1})]\\
&\le p(r_n,r_{n+1})+p(r_{n+1},r_{n+2})+\ldots+p(r_{m-1},r_m)\\
&= \sum_{x=n}^{m-1}p(r_{x+1},r_x)\le\sum_{x=n}^{\infty}p(r_{x+1},r_x)\\
&\le \sum_{x=n}^{\infty}\frac{1}{x^{1/k}}. \qquad (3.42)
\end{aligned}
$$

As $k \in (0,1)$, the series $\sum_{x=n}^{\infty}\left(\frac{1}{x^{1/k}}\right)$ is convergent, so

$$\lim_{n,m\to\infty} p(r_n, r_m) = 0. \tag{3.43}$$

Thus $\{r_n\}$ is a Cauchy sequence in $(\mathcal{U}, p)$. Therefore, $\{r_n\}$ is a Cauchy sequence in $(\mathcal{U}, d_w)$. Since $(\mathcal{U}, p)$ is a complete partial metric space, then by Lemma 2.1, $(\mathcal{U}, d_w)$ is also complete. Thus, there exists an $s \in \mathcal{U}$ such that $\lim_{n\to\infty} r_n = s$ and $\lim_{n\to\infty} d_w(r_n, s) = 0$. Moreover, by Definition 2.2 (3') and equation (3.43), we have

$$p(s,s) = \lim_{n\to\infty} p(r_n, s) = \lim_{n,m\to\infty} p(r_n, r_m) = 0. \tag{3.44}$$

Step III. Now, we shall show that s is a common fixed point of $\mathcal{R}_1$ and $\mathcal{R}_2$. Using given contractive condition (3.27) for $y = r_{2n}$ and $z = s$, we have

$$\begin{aligned} \tau + F\Big(\int_0^{p(r_{2n+1},\mathcal{R}_2 s)} \psi(t)dt\Big) &= \tau + F\Big(\int_0^{p(\mathcal{R}_1 r_{2n},\mathcal{R}_2 s)} \psi(t)dt\Big) \\ &\le F\Big(\int_0^{\nu(r_{2n},s)} \phi(t)dt\Big), \end{aligned} \tag{3.45}$$

where

$$\begin{aligned} \nu(r_{2n}, s) &= \max\Big\{p(r_{2n}, s), \frac{1}{2}[p(r_{2n}, \mathcal{R}_2 s) + p(s, \mathcal{R}_1 r_{2n})], \\ &\qquad \frac{p(r_{2n}, \mathcal{R}_1 r_{2n})p(s,\mathcal{R}_2 s)}{1 + p(r_{2n}, s)}, \frac{p(r_{2n}, \mathcal{R}_1 r_{2n})p(s,\mathcal{R}_2 s)}{1 + p(\mathcal{R}_1 r_{2n}, \mathcal{R}_2 s)}\Big\} \\ &= \max\Big\{p(r_{2n}, s), \frac{1}{2}[p(r_{2n}, \mathcal{R}_2 s) + p(s, r_{2n+1})], \\ &\qquad \frac{p(r_{2n}, r_{2n+1})p(s,\mathcal{R}_2 s)}{1 + p(r_{2n}, s)}, \frac{p(r_{2n}, r_{2n+1})p(s,\mathcal{R}_2 s)}{1 + p(r_{2n+1}, \mathcal{R}_2 s)}\Big\}. \end{aligned} \tag{3.46}$$

Passing to limit as $n \to \infty$ in (3.46) and using (3.44), we obtain

$$\nu(r_{2n}, s) \to \max\Big\{0, \frac{p(s,\mathcal{R}_2 s)}{2}, 0, 0\Big\} = \frac{p(s,\mathcal{R}_2 s)}{2} < p(s, \mathcal{R}_2 s). \tag{3.47}$$

Now, using (3.45) and (3.47), we get

$$\tau + F\Big(\int_0^{p(r_{2n+1},\mathcal{R}_2 s))} \psi(t)dt\Big) \le F\Big(\int_0^{p(s,\mathcal{R}_2 s)} \psi(t)dt\Big). \tag{3.48}$$

Passing to limit as $n \to \infty$ in (3.48) and using continuity of F, we obtain

$$\tau + F\Big(\int_0^{p(s,\mathcal{R}_2 s)} \psi(t)dt\Big) \le F\Big(\int_0^{p(s,\mathcal{R}_2 s)} \psi(t)dt\Big),$$

which is a contradiction since $\tau > 0$. Thus, we have $\mathcal{R}_2 s = s$. This shows that s is a fixed point of $\mathcal{R}_2$. By similar fashion we can show that $\mathcal{R}_1 s = s$. Hence s is a common fixed point of $\mathcal{R}_1$ and $\mathcal{R}_2$.

Step IV. Now, we show the uniqueness of the common fixed point. Assume that s' is another common fixed point of $\mathcal{R}_1$ and $\mathcal{R}_2$, that is, $\mathcal{R}_1 s' = s' = \mathcal{R}_2 s'$ with $s \neq s'$. From the given contractive condition (3.27), we have

$$\tau + F\Big(\int_0^{p(s,s')} \psi(t)dt\Big) = F\Big(\int_0^{p(\mathcal{R}_1 s,\mathcal{R}_2 s')} \psi(t)dt\Big) \leq F\Big(\int_0^{\nu(s,s')} \psi(t)dt\Big),$$

where

$$\begin{aligned} \nu(s,s') &= \max\Big\{p(s,s'), \frac{1}{2}[p(s,\mathcal{R}_2 s') + p(s',\mathcal{R}_1 s)], \\ &\qquad \frac{p(s,\mathcal{R}_1 s)p(s',\mathcal{R}_2 s')}{1+p(s,s')}, \frac{p(s,\mathcal{R}_1 s)p(s',\mathcal{R}_2 s')}{1+p(\mathcal{R}_1 s,\mathcal{R}_2 s')}\Big\} \\ &= \max\Big\{p(s,s'), \frac{1}{2}[p(s,s') + p(s',s)], \\ &\qquad \frac{p(s,s)p(s',s')}{1+p(s,s')}, \frac{p(s,s)p(s',s')}{1+p(s,s')}\Big\}. \end{aligned} \tag{3.49}$$

Using condition $(P3)$ and (3.44) in (3.49), we get

$$\nu(s,s') \to \max\Big\{p(s,s'), p(s,s'), 0, 0\Big\} = p(s,s'). \tag{3.50}$$

From (3.49) and (3.50), we obtain

$$\tau + F\Big(\int_0^{p(s,s')} \psi(t)dt\Big) \leq F\Big(\int_0^{p(s,s')} \psi(t)dt\Big),$$

which is a contradiction since $\tau > 0$. Thus, we have $s = s'$. This shows that the common fixed point of $\mathcal{R}_1$ and $\mathcal{R}_2$ is unique. This completes the proof. □

4. Consequences of Theorem 3.1

Corollary 4.1. *Let $(\mathcal{U}, p)$ be a complete partial metric space and let $\mathcal{S}:\mathcal{U} \to \mathcal{U}$ be a self-mapping. Suppose that there exist $F \in \mathcal{F}$ and $\tau > 0$ such that for all $y, z \in \mathcal{U}$ satisfying $p(\mathcal{S}y, \mathcal{S}z) > 0$, the following holds:*

$$\tau + F\Big(\int_0^{p(\mathcal{S}y,\mathcal{S}z)} \psi(t)dt\Big) \leq F\Big(\int_0^{\mu(y,z)} \psi(t)dt\Big),$$

where

$$\begin{aligned} \mu(y,z) &= \max\Big\{p(y,z), \frac{1}{3}[p(y,z) + p(z,\mathcal{S}y) + (y,\mathcal{S}z)], \\ &\qquad \frac{1}{3}[p(y,z) + p(y,\mathcal{S}y) + p(z,\mathcal{S}z)]\Big\}, \end{aligned}$$

and $\psi\colon [0,\infty) \to [0,\infty)$ is a Lebesque-integrable mapping which is summable on each compact subset of $[0,\infty)$ nonnegative and for each $\varepsilon > 0$

$$\int_0^{\varepsilon} \psi(t)dt > 0,$$

and if F is continuous. Then $\mathcal{S}$ has a unique fixed point in $\mathcal{U}$.

Corollary 4.2. *Let $(\mathcal{U}, p)$ be a complete partial metric space and let $\mathcal{S}\colon\mathcal{U} \to \mathcal{U}$ be a self-mapping. Suppose that there exist $F \in \mathcal{F}$ and $\tau > 0$ such that for all $y, z \in \mathcal{U}$ satisfying $p(\mathcal{S}y, \mathcal{S}z) > 0$, the following holds:*

$$\tau + F\Big(\int_0^{p(\mathcal{S}y,\mathcal{S}z)} \psi(t)dt\Big) \le F\Big(\int_0^{p(y,z)} \psi(t)dt\Big),$$

where F and ψ are as in Corollary 4.1. Then $\mathcal{S}$ has a unique fixed point in $\mathcal{U}$.

Let $(\mathcal{U}, p)$ be a complete partial metric space and let $\mathcal{S}\colon\mathcal{U} \to \mathcal{U}$ be a self-mapping. Suppose that there exist $F \in \mathcal{F}$ and $\tau > 0$ such that for all $y, z \in \mathcal{U}$ satisfying $p(\mathcal{S}y, \mathcal{S}z) > 0$, the following holds:

$$\tau + F\Big(\int_0^{p(\mathcal{S}y,\mathcal{S}z)} \psi(t)dt\Big) \le F\Big(\int_0^{\frac{1}{3}[p(y,z)+p(z,\mathcal{S}y)+(y,\mathcal{S}z)]} \psi(t)dt\Big),$$

where F and ψ are as in Corollary 4.1. Then $\mathcal{S}$ has a unique fixed point in $\mathcal{U}$.

Corollary 4.3. *Let $(\mathcal{U}, p)$ be a complete partial metric space and let $\mathcal{S}\colon\mathcal{U} \to \mathcal{U}$ be a self-mapping. Suppose that there exist $F \in \mathcal{F}$ and $\tau > 0$ such that for all $y, z \in \mathcal{U}$ satisfying $p(\mathcal{S}y, \mathcal{S}z) > 0$, the following holds:*

$$\tau + F\Big(\int_0^{p(\mathcal{S}y,\mathcal{S}z)} \psi(t)dt\Big) \le F\Big(\int_0^{\frac{1}{3}[p(y,z)+p(y,\mathcal{S}y)+(z,\mathcal{S}z)]} \psi(t)dt\Big),$$

where F and ψ are as in Corollary 4.1. Then $\mathcal{S}$ has a unique fixed point in $\mathcal{U}$.

We give an example to validate the result.

Example 4.1. Let $\mathcal{U} = [0,1]$ and $p(y,z) = \max\{y,z\}$ for all $y, z \in \mathcal{U}$. Then $(\mathcal{U}, p)$ is a complete partial metric space. Let $\mathcal{R}_1, \mathcal{R}_2\colon\mathcal{U} \to \mathcal{U}$ and $\psi\colon (0,\infty) \to (0,\infty)$ be defined by $\mathcal{R}_1(y) = \frac{y}{8}$, $\mathcal{R}_2(y) = 0$ and $\psi(t) = 2t$ for all $t \ge 0$. If $F\colon [0,\infty) \to \mathbb{R}$ be given by $F(\beta) = ln\,\beta$. Then all conditions of Theorem 3.1 and the contractive condition (3.1) are satisfied for some $\tau > 0$ and for $p(y,z) > 0$.

If $y > z$, then we have

$$\begin{aligned}\tau + F\left(\int_0^{p(\mathcal{R}_1(y),\mathcal{R}_2(z))} \psi(t)dt\right) &= \tau + ln\left(\frac{y^2}{64}\right) \le ln\,(y^2) \\ &= F\Big(\int_0^{\nu(y,z)} \psi(t)dt\Big).\end{aligned}$$

If $y < z$, then we have

$$\begin{aligned}\tau + F\bigg(\int_0^{p(\mathcal{R}_1(y),\mathcal{R}_2(z))}\psi(t)dt\bigg) &= \tau + ln\left(\frac{y^2}{64}\right)\\ &< \tau + ln\left(\frac{z^2}{64}\right) \leq ln\,(z^2)\\ &= F\bigg(\int_0^{\nu(y,z)}\psi(t)dt\bigg).\end{aligned}$$

Hence $0 \in \mathcal{U}$ is a common fixed point of $\mathcal{R}_1$ and $\mathcal{R}_2$.

5. Conclusion

In this paper, we prove some unique common fixed point theorems for generalized integral type F-contraction in the set up of complete partial metric spaces and give some consequences as corollaries of the main results. Also, an illustrated example is provided to validate the result. The results presented in this paper generalize and extend several results from the existing literature.

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