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CONVOLUTION PROPERTIES FOR CERTAIN SUBCLASSES OF MEROMORPHIC P-VALENT FUNCTIONS BY MEANS OF CASSINIAN OVALS

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Abstract. In the present paper we introduce two new sub-categories $\mathscr{MS}^*_{q,\eta}(p,s;d)$ and $\mathscr{MK}_{q,\eta}(p,s;d)$ for a variety of meromorphic operations using a q -derivative operator defined on a perforated unit disk. We use Cassinian Oval $\sqrt{1+dz}$ with $d \in (0,1]$ as a subordinant function. We also find the necessary and sufficient conditions for the activities of these clauses.
Key words: Meromorphic p-Valent functions, Hadamard product (or convolution), Subordination between analytic functions, Q-derivative operator, Cassinian ovals.

1. Introduction

In the present scenario, the concept of q-calculus has spawned a surprising effort by researchers for its use in many branches of mathematics and physics. Q-calculus is standard Calculus without limit point view. Jackson [5, 6, 7] introduced and studied the q-derivative and q-integral. Using q-calculus classes for various tasks in Geometric Function Theory is presented and investigated with different views and opinions (see [1], [8], [13], [15], [16], [19] and references therein). The purpose of this paper is to introduce and read two new sections of p-valent meromorphic activities using q derivative operators in accordance with the principle of subordinations.

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Let Σ_p denote the class of meromorphic functions of the form

$$h(z) = z^{-p} + \sum_{n\geq 1} b_n z^{n-p} \qquad (p \in \mathbb{N}), \tag{1.1}$$

which are analytic and p-valent in the punctured unit disc $\mathbb{U}^* = \mathbb{U}\backslash\{0\}$, where $\mathbb{U} = \{z : z \in \mathbb{C}, |z| < 1\}$. Given two functions f and $g \in \Sigma_p$, we say that f is subordinated to g in $\mathbb{U}$ and write $f(z) \prec g(z)$, if there exists a Schwarz function ω, analytic in $\mathbb{U}$, with $\omega(0) = 0, |\omega(z)| < |z|$, $z \in \mathbb{U}$, such that $f(z) = g(\omega(z))$ in $\mathbb{U}$. In particular, if $g(z)$ is univalent in $\mathbb{U}$, we have the following equivalence: $f(z) \prec g(z),\ z \in \mathbb{U} \Longleftrightarrow f(0) = g(0) \quad \text{and} f(\mathbb{U}) \subset g(\mathbb{U})$.

For $0 < q < 1$, the q-derivative of a function h is defined by (see [4, 5, 6, 7])

$$\mathscr{D}_q h(z) = \frac{h(qz) - h(z)}{(q-1)z} \qquad (z \in \mathbb{U}), \tag{1.2}$$

provided that $h^{'}(0)$ exists.
From (1.2), it can be easily obtain that

$$\mathscr{D}_q h(z) = \frac{-[p]_q}{q^p z^{p+1}} + \sum_{n=1}^{\infty} [n-p]_q b_n z^{n-p-1},$$

where

$$[n]_q = \frac{1-q^n}{1-q}.$$

As q$\rightarrow 1^-$, $[n]_q \rightarrow n$ and $\lim_{q\rightarrow 1^-} \mathscr{D}_q h(z) = h^{'}(z)$. Also, we have

$$[n+p]_q = [n]_q + q^n[p]_q = q^p[n]_q + [p]_q,$$
$$[n-p]_q = q^{-p}[n]_q - q^{-p}[p]_q,$$
$$[0]_q = 0, [1]_q = 1.$$

For $h \in \Sigma_p$ given by (1.1) and $g \in \Sigma_p$ given by

$$g(z) = z^{-p} + \sum_{n\geq 1} a_n z^{n-p} \qquad (p \in \mathbb{N}),$$

the Hadamard product (or convolution) of h and g is defined by

$$(h * g)(z) = z^{-p} + \sum_{n\geq 1} b_n a_n z^{n-p} = (g * h)(z).$$

Highly encouraged by the work of Aouf [11], Seoudy et *al.* [12], Srivastava et *al.* [17], we define the following two subclasses for Σ_p using the q -derivative operator $\mathscr{D}_q$ and the principle of subordination between analytical functions:

Definition 1.1. *Let $0 < q < 1$, $d \in (0,1]$ and $s \in \mathbb{C}\backslash\{0\}$. A function h belonging to Σ_p is said to be in the class $\mathscr{MS}^*_{q,\eta}(p,s;d)$ if it satisfies*

$$1 - \frac{1}{s}\left[\frac{z\mathscr{D}_q h(z)}{(1-\frac{\eta}{q^p})h(z) - \frac{\eta}{[p]_q} z\mathscr{D}_q h(z)} + \frac{[p]_q}{q^p}\right] \prec \sqrt{1+dz}. \tag{1.3}$$

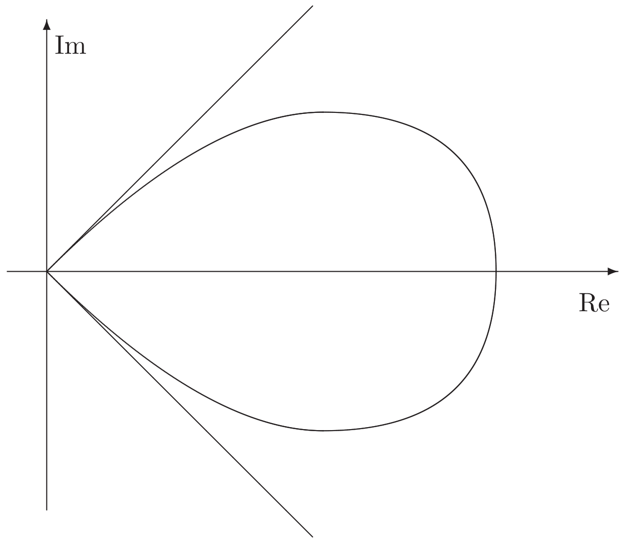
Definition 1.2. *Let $0 < q < 1$, $d \in (0,1]$ and $s \in \mathbb{C}\backslash\{0\}$. A function h belonging to Σ_p is said to be in the class $\mathscr{MK}_{q,\eta}(p,s;d)$ if it satisfies*

$$1 - \frac{1}{s}\left[\frac{z\mathscr{D}_q\big(z\mathscr{D}_q h(z)\big)}{(1-\frac{\eta}{q^p})\big(z\mathscr{D}_q h(z)\big) - \frac{\eta}{[p]_q} z\mathscr{D}_q\big(z\mathscr{D}_q h(z)\big)} + \frac{[p]_q}{q^p}\right] \prec \sqrt{1+dz}. \tag{1.4}$$

We also verify from both above definitions that

$$h \in \mathscr{MK}_{q,\eta}(p,s;d) \Leftrightarrow -\frac{q^p}{[p]_q} z\mathscr{D}_q h \in \mathscr{MS}^*_{q,\eta}(p,s;d). \tag{1.5}$$

Notice that for $d \in (0,1)$ the set $q_d(\mathbb{U}) = \{\omega \in \mathbb{C} : \mathrm{Re}(\omega) > 0 \cap |\omega^2 - 1| < d\}$ is the interior of the right half of the Cassini's curve $|\omega^2 - 1| = d$ and in the special case $d = 1$ this curve is the Bernoulli's lemniscate; see [10], pp. 231-235.



The image of the unit circle under the function $\sqrt{1+z}$

The function $\omega(z) = \sqrt{1+z}$ maps $\mathbb{U}$ onto a set bounded by Bernoulli lemniscate, and the class of functions $h \in \sum_p$ of holomorphic functions such that $zh'(z)/h(z) \prec$

$\sqrt{1+z}$ was considered in [14], while the class $zh'(z)/h(z) \prec \sqrt{1+dz}$ was considered in [2]. For detailed study about Cassinian ovals and related topics one can refer [2, 3, 9, 18] .

In the present investigations, we derive the necessary and sufficient condition for functions belonging to the subclasses $\mathscr{MS}^*_{q,\eta}(p,s;d)$ and $\mathscr{MK}_{q,\eta}(p,s;d)$.

2. Main Results

Unless otherwise mentioned, we assume throughout this section that $0 < q < 1$, $0 \leq \eta < 1$, $d \in (0,1)$, $s \in \mathbb{C}\backslash\{0\}$ and $\theta \in [0, 2\pi)$.

Theorem 2.1. *If $h \in \Sigma_p$, then $h \in \mathscr{MS}^*_{q,\eta}(p,s;d)$ if and only if*

$$z^p \left[h(z) * \frac{1 + \left\{ \left(1 - \frac{\eta}{q^p}\right) M(\theta) - \left(q + \frac{\eta}{q^p [p]_q}\right) \right\} z}{z^p (1-z)(1-qz)} \right] \neq 0 \qquad (z \in \mathbb{U}^*), \tag{2.1}$$

where

$$M(\theta) = \frac{e^{-i\theta}(1 + \sqrt{1 + de^{i\theta}})}{dsq^s}. \tag{2.2}$$

Proof. It is easy to verify that for any function $h \in \Sigma_p$

$$h(z) * \frac{1}{z^p(1-z)} = h(z) \tag{2.3}$$

and

$$h(z) * \frac{1 - \left(q + \frac{1}{[p]_q}\right) z}{z^p(1-z)(1-qz)} = -\frac{q^p}{[p]_q} z\mathscr{D}_q h(z). \tag{2.4}$$

First, if $h \in \mathscr{MS}^*_{q,\eta}(p,s;d)$, in order to prove that (2.1) holds we will write (1.3) by using the definition of the subordination, that is

$$-\frac{q^p}{[p]_q} \frac{z\mathscr{D}_q h(z)}{(1 - \frac{\eta}{q^p})h(z) - \frac{\eta}{[p]_q} z\mathscr{D}_q h(z)} = 1 - s\frac{q^p}{[p]_q}\left(1 - \sqrt{1 + d\omega(z)}\right) \qquad (z \in \mathbb{U}^*),$$

where ω is a schwarz function, hence

(2.5)

$$z^p \left[-q^p z\mathscr{D}_q h(z) - \left\{ [p]_q + sq^p(\sqrt{1 + de^{i\theta}} - 1) \right\} \left(\left(1 - \frac{\eta}{q^p}\right) h(z) - \frac{\eta}{[p]_q} z\mathscr{D}_q h(z) \right) \right] \neq 0.$$

Now from (2.3) and (2.4), we may write (2.5) as

$$z^p \Bigg[\left(h(z) * \frac{\{1 - (q + \frac{1}{[p]_q})z\}[p]_q}{z^p(1-z)(1-qz)} \right)$$

$$-\left\{[p]_q+sq^p(\sqrt{1+de^{i\theta}}-1)\right\}\left\{\left(1-\frac{\eta}{q^p}\right)\left(h(z)*\frac{1}{z^p(1-z)}\right)+\frac{\eta}{q^p}\left(h(z)*\frac{\{1-(q+\frac{1}{[p]_q})z\}}{z^p(1-z)(1-qz)}\right)\right\}\Bigg]\neq 0 \quad (z\in\mathbb{U}^*),$$

which is equivalent to

$$z^p\left[h(z)*\frac{1+\left\{\left(1-\frac{\eta}{q^p}\right)\frac{e^{-i\theta}\left(\sqrt{1+de^{i\theta}}+1\right)}{dsq^p}-\left(q+\frac{\eta e^{-i\theta}\left(\sqrt{1+de^{i\theta}}+1\right)}{dq^p[p]_q}\right)\right\}z}{z^p(1-z)(1-qz)}\left[-sq^p(\sqrt{1+de^{i\theta}}-1)\right]\right]\neq 0$$

or

$$z^p\left[h(z)*\frac{1+\left\{\left(1-\frac{\eta}{q^p}\right)\frac{e^{-i\theta}\left(\sqrt{1+de^{i\theta}}+1\right)}{dsq^p}-\left(q+\frac{\eta e^{-i\theta}\left(\sqrt{1+de^{i\theta}}+1\right)}{dq^p[p]_q}\right)\right\}z}{z^p(1-z)(1-qz)}\right]\neq 0\,(z\in\mathbb{U}^*),$$

which leads to (2.1), which proves the necessary part of Theorem 2.1.
Reversely, suppose that $h\in\Sigma_p$ satisfy the condition (2.1). Since it was shown in the first part of the proof that assumption (2.1) is equivalent to (2.5), we obtain that

(2.6)
$$-\frac{q^p}{[p]_q}\frac{z\mathscr{D}_qh(z)}{(1-\frac{\eta}{q^p})h(z)-\frac{\eta}{[p]_q}z\mathscr{D}_qh(z)}\neq 1+\frac{sq^p}{[p]_q}\left(\sqrt{1+de^{i\theta}}-1\right) \qquad (z\in\mathbb{U}^*),$$

and let us assume that

$$\varphi(z)=-\frac{q^p}{[p]_q}\frac{z\mathscr{D}_qh(z)}{(1-\frac{\eta}{q^p})h(z)-\frac{\eta}{[p]_q}z\mathscr{D}_qh(z)} \text{ and } \psi(z)=1+\frac{sq^p}{[p]_q}\left(\sqrt{1+de^{i\theta}}-1\right).$$

The relation (2.6) means that

$$\varphi(\mathbb{U}^*)\cap\psi(\partial\mathbb{U}^*)=\phi.$$

Thus, the simply connected domain is included in a connected component of $\mathbb{C}\backslash\psi(\partial\mathbb{U}^*)$. Therefore, using the fact that $\varphi(0)=\psi(0)$ and the univalence of the function ψ, it follows that $\varphi(z)\prec\psi(z)$, which implies that $f\in\mathscr{MS}^*_{q,\eta}(p,s;d)$. Thus, the proof of Theorem 2.1 is completed. □

Theorem 2.2. *If $f\in\Sigma_p$, then $f\in\mathscr{MK}_{q,\eta}(p,s;d)$ if and only if*

(2.7)
$$z^p\left[h(z)*\frac{1-\left[\left\{(q+\frac{\eta}{q^p[p]_q})-(1-\frac{\eta}{q^p})M(\theta)\right\}\left(1-\frac{1}{[p]_q}\right)+\frac{1+q}{[p]_q}+q^2\right]z-\left\{(1-\frac{\eta}{q^p})M(\theta)-(q+\frac{\eta}{q^p[p]_q})\right\}(q+\frac{1}{[p]_q})qz^2}{z^p(1-z)(1-qz)(1-q^2z)}\right]\neq 0$$

where $z\in\mathbb{U}^*$ and $M(\theta)$ is given by (2.2).

Proof. From (1.5) it follows that $h\in\mathscr{MK}_{q,\eta}(p,s;d)$ if and only if $-\frac{q^p}{[p]_q}z\mathscr{D}_qh\in\mathscr{MS}^*_{q,\eta}(p,s;d)$. Then from Theorem 2.1, the function $-\frac{q^p}{[p]_q}z\mathscr{D}_qh\in\mathscr{MS}^*_{q,\eta}(p,s;d)$ if and only if

$$z^p\left[-\frac{q^p}{[p]_q}z\mathscr{D}_qh(z)*g(z)\right]\neq 0, \qquad (z\in\mathbb{U}^*), \tag{2.8}$$

where

$$g(z) = \frac{1 + \left\{\left(1 - \frac{\eta}{q^p}\right)M(\theta) - \left(q + \frac{\eta}{q^p[p]_q}\right)\right\}z}{z^p(1-z)(1-qz)}.$$

On a basic computation we note that

$$\mathscr{D}_q g(z) = \frac{g(qz) - g(z)}{(q-1)z} =$$

$$\frac{-[p]_q + [1 + q + [p]_q q^2 + \{(q + \frac{\eta}{q^p[p]_q}) - (1 - \frac{\eta}{q^p})M(\theta)\}([p]_q - 1)]z + \{(1 - \frac{\eta}{q^p})M(\theta) - (q + \frac{\eta}{q^p[p]_q})\}(q + q^2[p]_q)z^2}{q^p z^{p+1}(1-z)(1-qz)(1-q^2z)}$$

and therefore

$$-\frac{q^p}{[p]_q} z\mathscr{D}_q g(z) =$$

$$= \frac{1 - \left[\{(q + \frac{\eta}{q^p[p]_q}) - (1 - \frac{\eta}{q^p})M(\theta)\}(1 - \frac{1}{[p]_q}) + \frac{1+q}{[p]_q} + q^2\right]z - \{(1 - \frac{\eta}{q^p})M(\theta) - (q + \frac{\eta}{q^p[p]_q})\}(q + \frac{1}{[p]_q})qz^2}{z^p(1-z)(1-qz)(1-q^2z)}.$$

Using the above relation and the identity

$$\left(-\frac{q^p}{[p]_q} z\mathscr{D}_q h(z)\right) * g(z) = h(z) * \left(-\frac{q^p}{[p]_q} z\mathscr{D}_q g(z)\right),$$

it is simple to check that (2.8) is identical to (2.7). Thus, the proof of Theorem 2.2 is completed. □

Theorem 2.3. *A necessary and sufficient condition for the function h defined by (1.1) to be in the class $\mathscr{MS}^*_{q,\eta}(p, s; d)$ is that*

(2.9)

$$1 + \sum_{n\geq 1} \frac{\left(1 - \frac{\eta}{q^p}\right)e^{-i\theta}\left(\sqrt{1 + de^{i\theta}} + 1\right)[n]_q + \left(1 - \frac{\eta[n]_q}{q^p[p]_q}\right)dsq^p}{dsq^p} b_n z^n \neq 0 \qquad (z \in \mathbb{U}^*).$$

Proof. From Theorem 2.1, we find that $f \in \mathscr{MS}^*_{q,\eta}(p, s; d)$ if and only if (2.1) holds.

Since

$$\frac{1}{z^p(1-z)(1-qz)} = \frac{1}{z^p} + (1+q)z^{1-p} + (1+q+q^2)z^{2-p} + (1+q+q^2+q^3)z^{3-p} + \cdots, \quad (z \in \mathbb{U}^*),$$

hence

$$\frac{1 + \left\{\left(1 - \frac{\eta}{q^p}\right)M(\theta) - \left(q + \frac{\eta}{q^p[p]_q}\right)\right\}z}{z^p(1-z)(1-qz)} = \frac{1}{z^p} + \sum_{n\geq 1}\left(1 + \left\{\left(1 - \frac{\eta}{q^p}\right)M(\theta) - \frac{\eta}{q^p[p]_q}\right\}[n]_q\right)z^{n-p},$$

where $M(\theta)$ is given by (2.2).

Now a simple computation shows that (2.1) is identical to (2.9). Thus, the proof of Theorem 2.3 is completed. □

Theorem 2.4. *A necessary and sufficient condition for the function h defined by (1.1) to be in the class $\mathscr{MK}_{q,\eta}(p, s; d)$ is that*

(2.10)

$$1 + \sum_{n\geq 1} \frac{\left(1 - \frac{\eta}{q^p}\right)e^{-i\theta}\left(\sqrt{1 + de^{i\theta}} + 1\right)[n]_q + \left(1 - \frac{\eta[n]_q}{q^p[p]_q}\right)dsq^p}{dsq^p}\left(1 - \frac{[n]_q}{[p]_q}\right) b_n z^n \neq 0 \qquad (z \in \mathbb{U}^*).$$

Proof. From Theorem 2.2, we find that $h \in \mathscr{MK}_{q,\eta}(p,s;d)$ if and only if (2.7) holds.

Since

$$\frac{1}{z^p(1-z)(1-qz)(1-q^2z)} = \frac{1}{z^p}+(1+q+q^2)z^{1-p}+(1+q+2q^2+q^3+q^4)z^{2-p}$$
$$+(1+q+2q^2+2q^3+2q^4+q^5+q^6)z^{3-p}+\cdots, \qquad (z \in \mathbb{U}^*),$$

hence

$$\frac{1-\left[\left\{\left(q+\frac{\eta}{q^p[p]_q}\right)-\left(1-\frac{\eta}{q^p}\right)M(\theta)\right\}\left(1-\frac{1}{[p]_q}\right)+\frac{1+q}{[p]_q}+q^2\right]z-\left\{\left(1-\frac{\eta}{q^p}\right)M(\theta)-\left(q+\frac{\eta}{q^p[p]_q}\right)\right\}\left(q+\frac{1}{[p]_q}\right)qz^2}{z^p(1-z)(1-qz)(1-q^2z)}$$
$$=\frac{1}{z^p}+\sum_{n=1}^{\infty}\left(1+\left\{(1-\frac{\eta}{q^p})M(\theta)-\frac{\eta}{q^p[p]_q}\right\}[n]_q\right)\left(1-\frac{[n]_q}{[p]_q}\right)z^{n-p} \qquad (z \in \mathbb{U}^*),$$

where $M(\theta)$ is given by (2.2).
Now a simple computation shows that (2.7) is identical to (2.10). Thus, the proof of Theorem 2.4 is completed. □

Theorem 2.5. *If $h \in \Sigma_p$ satisfies the inequality*

$$\text{(2.11)} \qquad \sum_{n=1}^{\infty}\left[\left|1-\frac{\eta}{q^p}\right|\left(\left|\sqrt{1+de^{i\theta}}+1\right|\right)[n]_q+\left|s(1-\frac{\eta[n]_q}{q^p[p]_q})\right|dq^p\right]|s_n| < d|s|q^p$$

then $h \in \mathscr{MS}^*_{q,\eta}(p,s;d)$.

Proof. Since

$$\left|1+\sum_{n\geq 1}\frac{(1-\frac{\eta}{q^p})e^{-i\theta}\left(\sqrt{1+de^{i\theta}}+1\right)[n]_q+(1-\frac{\eta[n]_q}{q^p[p]_q})dsq^p}{dsq^p}s_nz^n\right|$$
$$\geq 1-\left|\sum_{n\geq 1}\frac{(1-\frac{\eta}{q^p})e^{-i\theta}\left(\sqrt{1+de^{i\theta}}+1\right)[n]_q+(1-\frac{\eta[n]_q}{q^p[p]_q})dsq^p}{dsq^p}s_nz^n\right|$$
$$\geq 1-\sum_{n\geq 1}\frac{\left|1-\frac{\eta}{q^p}\right|\left(\left|\sqrt{1+de^{i\theta}}+1\right|\right)[n]_q+\left|s(1-\frac{\eta[n]_q}{q^p[p]_q})\right|dq^p}{d|s|q^p}|s_n| > 0.$$

Thus, the inequality (2.11) holds and our result follows from Theorem 2.3. □

Using similar arguments to those in the proof of Theorem 2.5, we may also prove the next result.

Theorem 2.6. *If $h \in \Sigma_p$ satisfies the inequality*

(2.12)

$$\sum_{n\geq 1}\left[\left|1-\frac{\eta}{q^p}\right|\left(\left|\sqrt{1+de^{i\theta}}+1\right|\right)[n]_q+\left|s(1-\frac{\eta[n]_q}{q^p[p]_q})\right|dq^p\right]\left(1-\frac{[n]_q}{[p]_q}\right)|s_n| < d|s|q^p$$

then $f \in \mathscr{MK}_{q,\eta}(p,s;d)$.

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