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E-BOCHNER CURVATURE TENSOR ON ALMOST $C(\lambda)$ MANIFOLDS

Brijesh Kumar Gupta[1] **and Braj Bhushan Chaturvedi**[1]

[1]**Department of Pure and Applied Mathematics, Guru Ghasidas Vishwavidyalaya, Bilaspur (Chhattisgarh), India**

Abstract. The present paper deals with the study of E-Bochner curvature tensor on an almost $C(\lambda)$ manifolds with the conditions $B^e(\xi, X).S = 0$, $B^e(\xi, X).R = 0$, $R.B^e(\xi, X) = 0$ and $B^e(\xi, X).B^e = 0$, where R, S and B^e denote Riemannian curvature tensor, Ricci tensor and E-Bochner curvature tensor, respectively. Also, we study ξ-E-Bochner flat $C(\lambda)$ manifolds.
Keywords: Almost contact manifold, E-Bochner curvature tensor, $C(\lambda)$ manifolds, Ricci tensor, Einstien manifold and Pseudosymmetric manifold.

1. Introduction

In 1981, D. Janssens and L. Vanhecke [4] first introduced the idea of the $C(\lambda)$ manifold. An almost contact metric manifold $M^{2n+1}(\phi, \xi, \eta, g)$ is said to be an almost $C(\lambda)$ manifold if the curvature tensor R of the manifold has the form [13]

(1.1)
$$R(X, Y)Z = R(\phi X, \phi Y)Z - \lambda[g(Y, Z)X - g(X, Z)Y - \phi Xg(\phi Y, Z) + g(\phi X, Z)\phi Y],$$

for any vector fields $X, Y, Z \in TM$ and λ is a real number.
D. Janssens and L. Vanhecke [4] also proved that if $\lambda = 0$, $\lambda = 1$ and $\lambda = -1$ then $C(\lambda)$ manifold becomes cosymplectic, Sasakian and Kenmotsu manifolds respectively. In 2013, Ali Akbar and Avijit Sarkar[1] studied conharmonic and concircular curvature tensors in an almost $C(\lambda)$ manifold. They proved that the concircular and

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Corresponding Author: Department of Pure and Applied Mathematics, Guru Ghasidas Vishwavidyalaya, Bilaspur (Chhattisgarh), India | E-mail: brijeshggv75@gmail.com
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conharmonic curvature tensors in $C(\lambda)$ manifold vanish if either $\lambda = 0$ or the manifold is a special type of η-Einstein manifold. In 1949, S. Bochner [14] gave the idea of the Bochner curvature tensor. D. E. Blair[5] explain the Bochner curvature tensor geometrically in 1975, Matsumoto and Chuman [10] constructed a curvature tensor from the Bochner curvature tensor with the help of Boothby-Wangs fibrations[18] and called it C-Bochner curvature tensor. J. S. Kim, M. M. Tripathi and J.Choi [9] studied the C-Bochner curvature tensor of a contact metric manifold in 2005. C-Bochner curvature tensor was also studied by several authors, viz., [4, 7, 12, 17] in different approaches. As an extension of C-Bochner curvature tensor, in 1991 Endo [8] defined the E-Bochner curvature tensor B^e.
The E-Bochner curvature tensor B^e is defined by [8]

(1.2) $$B^e(X,Y)Z = B(X,Y)Z - \eta(X)B(\xi,Y)Z - \eta(Y)B(X,\xi)Z - \eta(Z)B(X,Y)\xi.$$

where B is the C-Bochner curvature tensor defined by [10]

(1.3)
$$\begin{aligned} B(X,Y)Z = R(X,Y)Z &+ \frac{1}{2(n+2)}\Big\{S(X,Z)Y - S(Y,Z)X \\ &+ g(X,Z)QY - g(Y,Z)QX + S(\phi X,Z)\phi Y \\ &- S(\phi Y,Z)\phi X + g(\phi X,Z)Q\phi Y - g(\phi Y,Z)Q\phi X \\ &+ 2S(\phi X,Y)\phi Z + 2g(\phi X,Y)Q\phi Z - S(X,Z)\eta(Y)\xi \\ &+ S(Y,Z)\eta(X)\xi - \eta(X)\eta(Z)QY + \eta(Y)\eta(Z)QX\Big\} \\ &- \frac{\tau+2n}{2(n+2)}\Big\{g(\phi X,Z)\phi Y - g(\phi Y,Z)\phi X \\ &+ 2g(\phi X,Y)\phi Z\Big\} - \frac{\tau-4}{2(n+2)}\big\{g(X,Z)Y - g(Y,Z)X\big\} \\ &+ \frac{\tau}{2(n+2)}\Big\{g(X,Z)\eta(Y)\xi - g(Y,Z)\eta(X)\xi \\ &+ \eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X\Big\}, \end{aligned}$$

where $\tau = \frac{r+2n}{2(n+2)}$, Q is Ricci operator i.e. $g(QX,Y) = S(X,Y)$ for all X and Y and r is a scalar curvature of the manifold.
We have gone through the developments in $C(\lambda)$ manifold and then plan to study the E-Bochner curvature tensor in almost $C(\lambda)$ manifold. This paper is organized as follows:
The first section of the paper is introductory, and we provided the basic definition; the second part of the paper is the preliminaries and we have written some basic formula required for the calculation. In section 3 we studied E-Bochner pseudosymmetric in almost $C(\lambda)$ manifold and proved that the $C(\lambda)$ manifold will be E-Bochner pseudosymmetric if in $C(\lambda)$ manifold either $L_{B^e} = -\lambda$ or $C(\lambda)$ manifold is Kenmotsu manifold. In section 4, we have studied E-Bochner semi-symmetric and proved that the $C(\lambda)$ manifold is E-Bochner semi-symmetric if either $C(\lambda)$ manifold

is cosymplectic manifold or a Kenmotsu manifold. Besides this, in this section we have proved that the E-Bochner curvature tensor satisfies $B^e(\xi, X).S = 0$ if and only if the $C(\lambda)$ manifold is either cosymplectic or Ricci curvature tensor satisfies $S(X, U) = -2n\lambda\eta(X)\eta(U)$. Also, we have proved the relation $B^e(\xi, X).B^e = 0$ hold if and only if the manifold is Kenmotsu manifold. Finally, in section 5 we have discussed the ξ-E-Bochner flat curvature tensor on $C(\lambda)$ manifolds.

2. Preliminaries

A Riemannian manifold (M^{2n+1}, g) of dimension $(2n + 1)$ is said to be an almost contact metric manifold [3] if there exists a tensor field ϕ of type (1, 1), a vector field ξ (called the structure vector field) and a 1-form η on M such that

$$\phi^2(X) = -X + \eta(X)\xi, \tag{2.1}$$

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \tag{2.2}$$

and

$$\eta(\xi) = 1, \tag{2.3}$$

for any vector fields X, Y on M . In an almost contact metric manifold, we have

$$\phi\xi = 0, \quad \eta o\phi = 0. \tag{2.4}$$

Then such type of manifold is a called contact metric manifold if $d\eta = \Phi$, where $\Phi(X, Y) = g(X, \phi Y)$, is called the fundamental 2-form of $M^{(2n+1)}$.
A contact metric manifold is said to be K-contact manifold if and only if the covarient derivative of ξ satisfies

$$\nabla_X \xi = -\phi X, \tag{2.5}$$

for any vector field X on M.
The almost contact metric structure of M is said to be normal if

$$[\phi, \phi](X, Y) = -2d\eta(X, Y)\xi, \tag{2.6}$$

for any vector fields X and Y, where $[\phi, \phi]$ denotes the Nijenhuis torsion of ϕ.
A normal contact metric manifold is called a Sasakian manifold. An almost contact metric manifold is Sasakian if and only if

$$(\nabla_X \phi)Y = g(X, Y)\xi - \eta(Y)X, \tag{2.7}$$

for any vector fields X, Y.
An almost $C(\lambda)$ manifold satisfies the following relations [13]

$$R(X, Y)\xi = R(\phi X, \phi Y)\xi - \lambda\{\eta(Y)X - \eta(X)Y\}, \tag{2.8}$$

$$R(X,\xi)\,Y = \lambda\,\{g(X,Y)\xi - \eta(Y)X\}\,, \tag{2.9}$$

$$R(\xi,Y)\,Z = \lambda\,\{\eta(Z)Y - g(Y,Z)\xi\}\,, \tag{2.10}$$

$$R(X,\xi)\,\xi = \lambda\,\{\eta(X)\xi - X\}\,, \tag{2.11}$$

$$R(\xi,Y)\xi = \lambda\,\{Y - \eta(Y)\xi\}\,, \tag{2.12}$$

$$S(X,Y) = Ag(X,Y) + B\eta(X)\eta(Y), \tag{2.13}$$

where $A = -\lambda(2n-1)$ and $B = -\lambda$, since $g(QX,Y) = S(X,Y)$, where Q is the Ricci-operator.
From straight forward calculation of (2.13) we can write the following

$$QX = AX + B\eta(X)\xi, \tag{2.14}$$

$$S(X,\xi) = (A+B)\eta(X), \tag{2.15}$$

$$S(\xi,\xi) = (A+B), \tag{2.16}$$

and

$$r = -4n^2\lambda. \tag{2.17}$$

With the help of equations (1.2)-(1.3) and (2.8)-(2.16), we have

$$B^e(\xi,Y)Z = \eta(Z)\frac{2(\lambda+1)}{(n+2)}[\eta(Y)\xi - Y], \tag{2.18}$$

$$B^e(X,Y)\xi = \frac{2(\lambda+1)}{(n+2)}[\eta(Y)\,X - \eta(X)\,Y], \tag{2.19}$$

$$B^e(X,\xi)Z = \eta(Z)\frac{2(\lambda+1)}{(n+2)}[X - \eta(X)\xi], \tag{2.20}$$

and

$$B^e(\xi,\xi)\xi = 0. \tag{2.21}$$

This is required E-Bochner curvature tensor in $C(\lambda)$ manifolds.

3. E-Bochner Pseudosymmetric $C(\lambda)$ manifolds

Let (M, g) be a Riemannian manifold and let ∇ be the Levi-Civita connection of (M, g). A Riemannian manifold is called locally symmetric if $\nabla R = 0$, where R is the Riemannian curvature tensor of (M, g). The locally symmetric manifolds have been studied by different differential geometers through different approaches and they extent it e.x. semi-symmetric manifolds by Szabo [19], recurrent manifolds by Walker [2], conformally recurrent manifolds by Adati and Miyazawa [15].
According to Z. I. $Szab^{'}o$[19], if the manifold M satisfies the condition

$$(R(X,Y).R)(U,V)W = 0, \quad X,Y,U,V,W \in \chi(M) \tag{3.1}$$

then the manifold is called semi-symmetric manifold for all vector fields X and Y.
For a (0, k)- tensor field T on M, $k \geq 1$ and a symmetric (0, 2)-tensor field A on M the (0, k+2)-tensor fields R.T and Q(A, T) are defined by

$$\begin{aligned}(R.T)(X_1, ...X_k; X, Y) = &-T(R(X,Y)X_1, X_2, ...X_k)\\ &- .... - T(X_1, ...X_{k-1}, R(X,Y)X_k),\end{aligned} \tag{3.2}$$

and

$$\begin{aligned}Q(A,T)(X_1, ...X_k; X, Y) = &-T((X \wedge_A Y)X_1, X_2, ...X_k)\\ &- .... - T(X_1, ...X_{k-1}, (X \wedge_A Y)X_k),\end{aligned} \tag{3.3}$$

where $X \wedge_A Y$ is the endomorphism given by

$$(X \wedge_A Y)Z = A(Y,Z)X - A(X,Z)Y. \tag{3.4}$$

According to R. Deszcz [11] a Riemannian manifold is said to be pseudosymmetric if

$$R.R = L_R \, Q(g,R), \tag{3.5}$$

holds on $U_r = \left\{x \in M | R - \frac{r}{n(n-1)}G \neq 0 \ \ at \ x\right\}$, where G is (0, 4)-tensor defined by $G(X_1, X_2, X_3, X_4) = g((X_1 \wedge X_2)X_3, X_4)$ and L_R is some smooth function on U_R.
A Riemannian manifold M is said to be E-Bochner pseudosymmetric if

$$R.B^e = L_{B^e} \, Q(g, B^e), \tag{3.6}$$

holds on the set $U_{B^e} = \{x \in M : B^e \neq 0 \ \ at \ x\}$, where L_{B^e} is some function on U_{B^e} and B^e is the E-Bochner curvature tensor.
Let M^{2n+1} be E-Bochner pseudosymmetric $C(\lambda)$ manifold and then from equation(3.6), we have

$$(R(X,\xi).B^e)(U,V)W = L_{B^e}[((X \wedge_g \xi).B^e)(U,V)W]. \tag{3.7}$$

Using equations (3.2) and (3.3) in equation (3.7), we get

$$\begin{aligned} &R(X,\xi)B^e(U,V)W - B^e(R(X,\xi)U,V)W \\ &- B^e(U,R(X,\xi)V)W - B^e(U,V)R(X,\xi)W \\ &= L_{B^e}\Big\{(X\wedge_g \xi)B^e(U,V)W - B^e((X\wedge_g \xi)U,V)W \\ &- B^e(U,(X\wedge_g \xi)V)W - B^e(U,V)(X\wedge_g \xi)W\Big\}. \end{aligned} \tag{3.8}$$

Again, using equations (2.9) and (3.4) in (3.8), we infer

$$\begin{aligned} &(\lambda)\Big\{g(X,B^e(U,V)W)\xi - g(\xi,B^e(U,V)W)X + \eta(U)B^e(X,V)W \\ &- g(X,U)B^e(\xi,V)W + \eta(V)B^e(U,X)W - g(X,V)B^e(U,\xi)W \\ &+ \eta(W)B^e(U,V)X - g(X,W)B^e(U,V)\xi\Big\} \\ &= L_{B^e}\Big\{g(\xi,B^e(U,V)W)X - g(X,B^e(U,V)W)\xi - \eta(U)B^e(X,V)W \\ &+ g(X,U)B^e(\xi,V)W - \eta(V)B^e(U,X)W + g(X,V)B^e(U,\xi)W \\ &- \eta(W)B^e(U,V)X + g(X,W)B^e(U,V)\xi\Big\}. \end{aligned} \tag{3.9}$$

The above expression can be written as

$$\begin{aligned} &(L_{B^e}+\lambda)\Big\{g(\xi,B^e(U,V)W)X - g(X,B^e(U,V)W)\xi - \eta(U)B^e(X,V)W \\ &+ g(X,U)B^e(\xi,V)W - \eta(V)B^e(U,X)W + g(X,V)B^e(U,\xi)W \\ &- \eta(W)B^e(U,V)X + g(X,W)B^e(U,V)\xi\Big\} = 0, \end{aligned} \tag{3.10}$$

which implies that either

$$\begin{aligned} &(a)\ L_{B^e} = -\lambda \\ &or \\ &(b)\ \Big\{g(\xi,B^e(U,V)W)X \\ &- g(X,B^e(U,V)W)\xi - \eta(U)B^e(X,V)W \\ &+ g(X,U)B^e(\xi,V)W - \eta(V)B^e(U,X)W + g(X,V)B^e(U,\xi)W \\ &- \eta(W)B^e(U,V)X + g(X,W)B^e(U,V)\xi\Big\} = 0. \end{aligned} \tag{3.11}$$

Putting $W=\xi$ and using equations (1.3) and (2.18) in equation (3.11(b)), we have

$$B^e(X,V)W = \frac{2(\lambda+1)}{(n+2)}[g(X,V)\,U - g(X,U)\,V]. \tag{3.12}$$

Now, contracting V in above equation, we get

$$\frac{2(\lambda+1)}{(n+2)}2n\,g(X,U) = 0. \tag{3.13}$$

This implies that

$$\lambda = -1. \tag{3.14}$$

using equations (3.14) in (2.18) in (3.12), we have

$$B^e(X,V)W = 0, \quad B^e(\xi,V)W = 0. \tag{3.15}$$

Therefore with the help of equations (3.11(b)) and (3.15) we conclude that:

Proposition 3.1. *A $C(\lambda)$ manifold M^{2n+1} $(n > 1)$ is E-Bochner pseudosymmetric if either $L_{B^e} = -\lambda$ or $C(\lambda)$ manifold is a Kenmotsu manifold.*

Now, since λ is a real number and if $C(\lambda)$ manifold be E-Bochner pseudosymmetric then we have $\lambda = -1$ or $L_{B^e} = -\lambda$ holds on M^{2n+1} which implies that $L_{B^e} = -\lambda$ will be a real number in both cases therefore we can state the following corollary.

Corollary 3.1. *Every $C(\lambda)$ manifold is E-Bochner pseudosymmetric and has the form $R.B^e = -\lambda\, Q(g, B^e)$.*

Corollary 3.2. *Every $C(\lambda)$ manifold is E-Bochner pseudosymmetric and has the form $R.B^e = Q(g, B^e)$.*

4. E-Bochner semi-symmetric $C(\lambda)$ manifolds

In an (2n+1)-dimensional alomost $C(\lambda)$ the E-Bochner semi-symmetric $C(\lambda)$ manifold is defined by

$$(R(X,Y).B^e)(U,V)W = 0. \tag{4.1}$$

The above equation can be written as

$$\begin{aligned} &R(X,Y)B^e(U,V)W - B^e(R(X,Y)U,V)W \\ &- B^e(U,R(X,Y)V)W - B^e(U,V)R(X,Y)W = 0. \end{aligned} \tag{4.2}$$

Putting $Y = \xi$ in above equation we get

$$\begin{aligned} &\lambda\Big[g(X,B^e(U,V)W)\xi - X\eta(B^e(U,V)W) \\ &- g(X,U)B^e(\xi,V)W + \eta(U)B^e(X,V)W \\ &- g(X,V)B^e(U,\xi)W + \eta(V)B^e(U,X)W \\ &+ \eta(W)B^e(U,V)X - g(X,W)B^e(U,V)\xi\Big] = 0. \end{aligned} \tag{4.3}$$

From (4.3), we have either $\lambda = 0$ or

$$\begin{aligned} &\Big[g(X,B^e(U,V)W)\xi - X\eta(B^e(U,V)W) \\ &- g(X,U)B^e(\xi,V)W + \eta(U)B^e(X,V)W \\ &- g(X,V)B^e(U,\xi)W + \eta(V)B^e(U,X)W \\ &+ \eta(W)B^e(U,V)X - g(X,W)B^e(U,V)\xi\Big] = 0, \end{aligned} \tag{4.4}$$

for $\lambda = 0$ the manifold is a cosymplectic manifold.
Now putting $W = U = \xi$ and using equation (2.18) in above equation, we have

$$\frac{2(\lambda+1)}{(n+2)}\eta(X)V - g(X,V)\xi = 0. \tag{4.5}$$

again putting $X = \phi X$, $V = \phi V$ and using equation (2.4), we have

$$\frac{2(\lambda+1)}{(n+2)}g(\phi X,\phi V)\xi = 0. \tag{4.6}$$

Since $g(\phi X,\phi V)\xi \neq 0$, in general therefore we obtain from (4.5) $\lambda = -1$. Therefore in this case manifold is a Kenmotsu manifold.
Thus we conclude

Proposition 4.1. *If $C(\lambda)$ manifold M^{2n+1} $(n > 1)$ is an E-Bochner semi-symmetric $C(\lambda)$ manifold then either $C(\lambda)$ manifold is a cosymplectic manifold or a Kenmotsu manifold.*

Now we propose

Theorem 4.1. *In a $C(\lambda)$ manifold M^{2n+1} $(n > 1)$, $B^e(\xi, X).S = 0$ if and only if either $C(\lambda)$ manifold is a Kenmotsu manifold or in $C(\lambda)$ manifold the Ricci tensor satisfies $S(X,U) = -2n\lambda\eta(X)\eta(U)$.*

Proof If $C(\lambda)$ manifold satisfying $B^e(\xi, X).S = 0$.
Then from equation (3.2), we have

$$S(B^e(\xi,X)U,\xi) + S(U,B^e(\xi,X)\xi) = 0, \tag{4.7}$$

From equation (2.12), we have

$$S(B^e(\xi,X)U,\xi) = -2n\lambda\eta(B^e(\xi,X)U). \tag{4.8}$$

Now with the help of equations (2.18) and (4.8), we have

$$S(B^e(\xi,X)U,\xi) = 0. \tag{4.9}$$

Again in view of the equation (2.18), we have

$$S(B^e(\xi,X)\xi,U) = -\frac{2(\lambda+1)}{(n+2)}(S(X,U) + 2n\lambda\eta(X)\eta(U)). \tag{4.10}$$

By using expressions (4.10) and (4.9) in (4.7), we infer

$$\frac{2(\lambda+1)}{(n+2)}(S(X,U) + 2n\lambda\eta(X)\eta(U)) = 0, \tag{4.11}$$

which implies that $\lambda = -1$ or

$$S(X,U) = -2n\lambda\eta(X)\eta(U). \tag{4.12}$$

Conversely if the manifold satisfies the relation (4.12), then in view of equation (2.18), we have

$$\begin{aligned} B^e(\xi, X).S &= -S(B^e(\xi, X)U, \xi) - S(U, B^e(\xi, X)\xi) \\ &= -\frac{2(\lambda+1)}{(n+2)}(S(X,U) + 2n\lambda\eta(X)\eta(U)) \\ &= 0. \end{aligned} \tag{4.13}$$

Again, if the manifold is Kenmotsu then we easily obtain from (2.18) that $B^e(\xi, X).S = 0$.
As a particular case of theorem 4.1 we can state the following corollary :

Corollary 4.1. *A $C(\lambda)$ manifold M^{2n+1} $(n > 1)$ satisfies $B^e(\xi, X).S = 0$ is a special type of η-Einstein manifold.*

Now we take $B^e(\xi, U).R = 0$.
Then from equation (3.2), we have

$$\begin{aligned} &B^e(\xi, U)R(X,Y)Z - R(B^e(\xi, U)X, Y)Z \\ &- R(X, B^e(\xi, U)Y)Z - R(X,Y)B^e(\xi, U)Z = 0, \end{aligned} \tag{4.14}$$

which in view of the equation (2.18), we have

$$\begin{aligned} &\frac{2(\lambda+1)}{(n+2)}\Big\{\eta(U)\eta(R(X,Y)Z)\xi - \eta(R(X,Y)Z)U) \\ &- \eta(X)\eta(U)R(\xi, Y)Z + \eta(X)R(U,Y)Z \\ &- \eta(U)\eta(Y)R(X,\xi)Z + \eta(Y)R(X,U)Z \\ &- \eta(U)\eta(Z)R(X,Y)\xi + \eta(Z)R(X,Y)U\Big\} = 0, \end{aligned} \tag{4.15}$$

From (4.15) we have either $\lambda = -1$, or

$$\begin{aligned} &\Big\{\eta(U)\eta(R(X,Y)Z)\xi - \eta(R(X,Y)Z)U) \\ &- \eta(X)\eta(U)R(\xi, Y)Z + \eta(X)R(U,Y)Z \\ &- \eta(U)\eta(Y)R(X,\xi)Z + \eta(Y)R(X,U)Z \\ &- \eta(U)\eta(Z)R(X,Y)\xi + \eta(Z)R(X,Y)U\Big\} = 0. \end{aligned} \tag{4.16}$$

For $\lambda = -1$, the manifold is Kenmotsu ˙
Putting $X = Z = \xi$ in (4.16) and using (2.10) in the above equation, we infer

$$R(\phi U, \phi Y)\xi = \lambda[g(Y,U)\xi - \eta(U)\eta(Y)]. \tag{4.17}$$

Thus, we conclude

Proposition 4.2. *In $C(\lambda)$ manifold M^{2n+1} $(n > 1)$ if $B^e(\xi, U).R = 0$ then the manifold is either a Kenmotsu manifold or $R(\phi U, \phi Y)\xi = \lambda[g(Y,U)\xi - \eta(U)\eta(Y)]$.*

Now we propose

Theorem 4.2. *In $C(\lambda)$ manifold M^{2n+1} $(n > 1)$, $B^e(\xi, X).B^e = 0$, if and only if the manifold is Kenmotsu manifold.*

Proof If $C(\lambda)$ manifold satisfying $B^e(\xi, X).B^e = 0$. Then from equation (3.2), we have

$$\begin{aligned} &B^e(\xi, X)B^e(U, V)W - B^e(B^e(\xi, X)U, V)W \\ &- B^e(U, B^e(\xi, X)V)W - B^e(U, V)B^e(\xi, X)W = 0, \end{aligned} \tag{4.18}$$

which in view of the equation (2.18), we get

$$\begin{aligned} &\frac{2(\lambda+1)}{(n+2)}\Big\{\eta(B^e(U, V)W)\eta(X)\xi - \eta(B^e(U, V)W)X \\ &- \eta(U)\eta(X)B^e(\xi, V)W + \eta(U)B^e(X, V)W \\ &- \eta(X)\eta(V)B^e(U, \xi)W + \eta(V)B^e(U, X)W \\ &- \eta(W)\eta(X)B^e(U, V)\xi + \eta(W)B^e(U, V)X\Big\} = 0. \end{aligned} \tag{4.19}$$

By using $U = \xi$ in above equation, we infer

$$\frac{2(\lambda+1)}{(n+2)}\Big\{(B^e(X, V)W + \eta(W)\frac{2(\lambda+1)}{(n+2)}(\eta(V)X + \eta(X)V)\Big\} = 0, \tag{4.20}$$

which implies that either $\lambda = -1$ or

$$B^e(X, V)W = \frac{2(\lambda+1)}{(n+2)}\eta(W)[\eta(V)\,X - \eta(X)\,V], \tag{4.21}$$

contracting V in above equation, we have

$$\frac{2(\lambda+1)}{(n+2)}2n\eta(W)\eta(X) = 0, \tag{4.22}$$

This implies that $\lambda = -1$, for $\lambda = -1$, the manifold is Kenmotsu. Conversely, in the case if the manifold is Kenmotsu then from (2.18) we obtain $B^e(\xi, X).B^e = 0$ holds if and only if the manifold is Kenmotsu.

5. ξ-E- Bochner flat curvature tensor on $C(\lambda)$ manifolds

A contact metric manifold is said to be ξ-conformally flat contact metric manifold if the conformal curvature tensor of the manifold satisfies

$$C(X, Y)\xi = 0, \tag{5.1}$$

for any vector fields X and Y.
This idea was introduced by Zhen, Cabrerizo, M. Fernandez and Fernandez [6] in

1997. In 2012 U.C.De , Ahmet Yildiz, Mine Turan and Bilal E. Acet [16] defined ξ-concircularly flat manifold if the concircular curvature tensor $\tilde{C}(X,Y)\xi = 0$ holds on M.
Now, we define ξ- E-Bochner flat $C(\lambda)$ manifold.

Definition 5.1. A $C(\lambda)$ manifolds is said to be ξ- E-Bochner flat $C(\lambda)$ manifold if the E-Bochner curvature tensor B^e of type (1, 3) of $C(\lambda)$ manifold satisfies

$$B^e(X,Y)\xi = 0, \tag{5.2}$$

for any vector fields X and Y.
Putting $Z = \xi$ in equation (1.2), we have

$$B^e(X,Y)\xi = -\eta(X)B(\xi,Y)\xi - \eta(Y)B(X,\xi)\xi. \tag{5.3}$$

Now from equations (1.3), (2.18) and (5.3), we get

$$\frac{2(\lambda+1)}{(n+2)}[\eta(Y)X - \eta(X)Y] = 0 \tag{5.4}$$

putting $Y = \xi$ in above equation we have

$$\frac{2(\lambda+1)}{n+2}(X - \eta(X)\xi) = 0. \tag{5.5}$$

Now taking inner product with a vector field V, we have

$$\frac{2(\lambda+1)}{n+2}(g(X,V) - \eta(X)\eta(V)) = 0. \tag{5.6}$$

Replacing X by QX in above equation, we get

$$\frac{2(\lambda+1)}{n+2}(g(QX,V) - \eta(QX)\eta(V)) = 0, \tag{5.7}$$

since S(X,Y)=g(QX, Y), then from above equation we have

$$\frac{2(\lambda+1)}{n+2}(S(X,V) - \eta(QX)\eta(V)) = 0. \tag{5.8}$$

Now with the help of equation (2.11) and (4.8), we have

$$\frac{2(\lambda+1)}{n+2}(S(X,V) + 2n\lambda\eta(X)\eta(V))) = 0. \tag{5.9}$$

this implies that either

$$\lambda = -1, \tag{5.10}$$

or

$$S(X,V) = -2n\lambda\eta(X)\eta(V). \tag{5.11}$$

Theorem 5.1. *In a ξ-E-Bochner flat $C(\lambda)$ manifold either $\lambda = -1$ or $C(\lambda)$ manifold is a special type of η-Einstein manifold.*

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