FACTA UNIVERSITATIS (NIŠ)
Ser. Math. Inform. Vol. 31, No 2 (2016), 315–333

ON LOWER AND UPPER WEAKLY α-CONTINUOUS INTUITIONISTIC FUZZY MULTIFUNCTIONS

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Abstract. The aim of this paper is to introduce the concepts of upper and lower weakly α-continuous intuitionistic fuzzy multifunctions and obtain some of their properties.
Keywords:Intuitionistic fuzzy sets, Intuitionistic fuzzy topology, Intuitionistic fuzzy multifunctions, lower weakly α-continuous Intuitionistic fuzzy multifunctions and upper weakly α-continuous Intuitionistic fuzzy multifunctions

1. Introduction

After the introduction of fuzzy sets by Zadeh [40] in 1965 and fuzzy topology by Chang [10] in 1967, several research studies were conducted on the generalization of the notions of fuzzy sets and fuzzy topology. The concept of intuitionistic fuzzy sets was introduced by Atanassov [2, 3, 4] as a generalization of fuzzy sets. In the last 32 years various concepts of fuzzy mathematics have been extended for intuitionistic fuzzy sets. In 1997 Coker [11] introduced the concept of intuitionistic fuzzy topological spaces as a generalization of fuzzy topological spaces. In 1999, Ozbakir and Coker [28] introduced the concept intuitionistic fuzzy multifunctions and studied their lower and upper intuitionistic fuzzy semi continuity from a topological space to an intuitionistic fuzzy topological space. In the recent past some weak and strong forms of lower and upper semi-continuity of intuitionistic fuzzy multifunctions have been studied in [6] [7, 8][33, 34, 35, 36, 37, 38, 39]. In the present paper we extend the concepts of lower and upper weakly α-continuous multifunctions due to Popa and Noiri [30] to intuitionistic fuzzy multifunctions and obtain some of their characterizations and properties.

2. Preliminaries

Throughout this paper (X,τ) and (Y,Γ) represent a topological space and an intuitionistic fuzzy topological space, respectively.

Received July 11, 2015; accepted February 03, 2016
2010 *Mathematics Subject Classification.* Primary 54A99; Secondary 03E99

Definition 2.1. [17, 24] *A subset A of a topological space (X, τ) is called:*

(a) Semi-open if $A \subset Cl(Int(A))$.

(b) Semi-closed if its complement is semi-open.

(c) α-open if $A \subset Int(Cl(Int(A)))$.

(d) α-closed if its complement is α-open.

(e) pre-open if $A \subset Int(Cl(A))$.

(f) pre-closed if its complement is pre-open.

Remark 2.1. *[25] Every open set is α-open and every α-open set is semi-open (resp. pre-open) but the converses may not be true.*

The family of all α-open (resp. semi-open, pre-open) subsets of a topological space (X, τ) is denoted by $\alpha O(X)$ (resp. SO(X), PO(X))similarly for the family of all α-closed (resp. semi-closed, pre-closed) subsets of topological space (X, τ) is denoted by $\alpha C(X)$ (resp. SC(X), PC(X)). The intersection of all α-closed (resp. semi-closed) sets of X containing a set A of X is called the α-closure [19] (resp. semi-closure) of A. It is denoted by $\alpha Cl(A)$ (resp. sCl(A)). The union of all α-open (resp. semi-open) subsets of A of X is called the α-interior [19] (resp. semi-interior) of A. It is denoted by $\alpha Int(A)$ (resp. $sInt(A)$). A subset A of X is α-closed (resp. semi-closed) if and only if $A \supset Cl(Int(Cl(A)))$ (resp.$A \supset Int(Cl(A))$. A subset N of a topological space (X, τ) is called a α-neighborhood [18] of a point x of X if there exists an α-open set O of X such that $x \in O \subset N$. A is an α-open in X if and only if it is a α-neighborhood of each of its points. A subset A of a topological space X is said to be regular-open (resp. regular-closed) if $A = Int(Cl(A))$(resp. $A = Cl(Int(A))$). The family of regular open (resp. regular-closed) sets of X is denoted by $RO(X)$ (resp. $RC(X)$). The θ-closure of A is defined to be the collection of all $x \in X$ such that $A \cap Cl(U) \neq \phi$ for every open-neighborhood U of x, is denoted by $Cl_\theta(A)$. The $Cl_\theta(A)$ is closed in X and $Cl(V) = Cl_\theta(V)$ for all open set U of X. A subset V of X is called an α-neighborhood of a subset A of X if there exists $U \in \alpha O(X)$ such that $A \subset U \subset V$. A mapping f from a topological space (X, τ) to another topological space (X^*, τ^*) is said to be α-continuous [20, 21] if the inverse image of every open set of X^* is α-open in X.

Lemma 2.1. [30] *The following properties hold for a subset A of a topological space (X, τ):*

(a) A is α-closed in $X \Leftrightarrow sInt(Cl(A) \subset A$;

(b) $sInt(Cl(A)) = Cl(Int(Cl(A)))$;

(c) $\alpha Cl(A) = A \cup Cl(Int(Cl(A)))$.

Lemma 2.2. [30] *The following are equivalent for a subset A of a topological space (X, τ):*

(a) $A \in \alpha O(X)$,

(b) $U \subset A \subset Int(Cl(U))$ *for some open set* U *of* X.

(c) $U \subset A \subset sCl(U)$ *for some open set* U *of* X.

(d) $A \subset sCl(Int(A))$.

Definition 2.2. [2, 3, 4] *Let Y be a nonempty fixed set. An intuitionistic fuzzy set $\tilde{A}$ in Y is an object having the form*

$$\tilde{A} = \{< x, \mu_{\tilde{A}}(y), \nu_{\tilde{A}}(y) >: y \in Y\}$$

where the functions $\mu_{\tilde{A}}(y) : Y \to I$ and $\nu_{\tilde{A}}(y) : Y \to I$ denotes the degree of membership (namely $\mu_{\tilde{A}}(y)$) and the degree of non membership (namely $\nu_{\tilde{A}}(y)$) of each element $y \in Y$ to the set $\tilde{A}$ respectively, and $0 \leq \mu_{\tilde{A}}(y) + \mu_{\tilde{A}}(y) \leq 1$ for each $y \in Y$.

Definition 2.3. [2, 3, 4] *Let Y be a nonempty set and the intuitionistic fuzzy sets $\tilde{A}$ and $\tilde{B}$ be in the form $\tilde{A} = \{< x, \mu_{\tilde{A}}(y), \nu_{\tilde{A}}(y) >: y \in Y\}$, $\tilde{B} = \{< x, \mu_{\tilde{B}}(y), \nu_{\tilde{B}}(y) >: y \in Y\}$ and let $\tilde{B}_\alpha : \alpha \in \Lambda$ be an arbitrary family of intuitionistic fuzzy sets in Y. Then:*

(a). $\tilde{A} \subseteq \tilde{B}$ *if* $\forall y \in Y$ $[\mu_{\tilde{A}}(y) \leq \mu_{\tilde{B}}(y)$ *and* $\nu_{\tilde{A}}(y) \geq \nu_{\tilde{B}}(y)]$

(b). $\tilde{A} = \tilde{B}$ *if* $\tilde{A} \subseteq \tilde{B}$ *and* $\tilde{B} \subseteq \tilde{A}$;

(c). $\tilde{A}^c = \{< x, \nu_{\tilde{A}}(y), \mu_{\tilde{A}}(y) >: y \in Y\}$;

(d). $\tilde{0} = \{< y, 0, 1 >: y \in Y\}$ *and* $\tilde{1} = \{< y, 1, 0 >: y \in Y\}$

(e). $\cap \tilde{A}_\alpha = \{< x, \wedge \mu_{\tilde{A}}(y), \vee \nu_{\tilde{A}}(y) >: y \in Y\}$

(f). $\cup \tilde{A}_\alpha = \{< x, \vee \mu_{\tilde{A}}(y), \wedge \nu_{\tilde{A}}(y) >: y \in Y\}$.

Definition 2.4. [12] *Let Y be a nonempty set and $c \in Y$ a fixed element in Y. If $\alpha \in (0, 1]$ and $\beta \in [0, 1)$ are two real numbers such that $\alpha + \beta < 1$ then,*

(a) $c(\alpha, \beta) =< y, c_\alpha, c_{1-\beta} >$ *is called an intuitionistic fuzzy point (IFP in short) in Y, where α denotes the degree of membership of $c(\alpha, \beta)$, and β denotes the degree of non-membership of $c(\alpha, \beta)$.*

(b) $c(\beta) =< y, 0, 1 - c_{1-\beta} >$ *is called a vanishing intuitionistic fuzzy point (VIFP in short) in Y, where β denotes the degree of non membership of $c(\beta)$.*

Definition 2.5. [12] *Two Intuitionistic Fuzzy Sets* $\tilde{A}$ *and* $\tilde{B}$ *of* Y *are said to be quasi-coincident* ($\tilde{A}q\tilde{B}$ *for short) if* $\exists y \in Y$ *such that*

$\mu_{\tilde{A}}(y) > \nu_{\tilde{B}}(y)$ *or* $\nu_{\tilde{A}}(y) < \mu_{\tilde{B}}(y)$

Definition 2.6. [12] *An intuitionistic fuzzy point* $c(\alpha, \beta)$ *is said to be quasi-coincidence with the intuitionistic fuzzy set* $\tilde{A} =< (\mu_{\tilde{A}}(y), \nu_{\tilde{A}}(y)) >$ *denoted by* $c(\alpha, \beta)q\tilde{A}$ *if* $\alpha > \nu_{\tilde{A}}(c)$ *or* $\beta < \mu_{\tilde{A}}(c)$

Definition 2.7. [12] *An intuitionistic fuzzy* $\tilde{A}$ *in an intuitionistic fuzzy topological space* (Y, Γ) *is said to be q-neighborhood of* $c(\alpha, \beta)$ *if there exists an intuitionistic fuzzy open set* $\tilde{B}$ *in* Y *such that* $c(\alpha, \beta)q\tilde{B} \leq \tilde{A}$.

Lemma 2.3. [12] *For any two intuitionistic fuzzy sets* $\tilde{A}$ *and* $\tilde{B}$ *of* Y, $\sim (\tilde{A}q\tilde{B}) \Leftrightarrow \tilde{A} \subset \tilde{B}^c$.

Definition 2.8. [12] *An intuitionistic fuzzy topology on a non empty set* Y *is a family* Γ *of intuitionistic fuzzy sets in* Y *which satisfy the following axioms:*

O_1. $\tilde{0}, \tilde{1} \in \Gamma$,

O_2. $\tilde{A}_1 \cap \tilde{A}_2 \in \Gamma$ *for any* $\tilde{A}_1, \tilde{A}_2 \in \Gamma$,

O_3. $\cup\tilde{A}_\alpha \in \Gamma$ *for arbitrary family* $\{\tilde{A}_\alpha : \alpha \in \Lambda\} \in \Gamma$.

In this case the pair (Y, Γ) is called an intuitionistic fuzzy topological space and each intuitionistic fuzzy set in Γ, is known as an intuitionistic fuzzy open set in Y. The complement $\tilde{B}^c$ of an intuitionistic fuzzy open set $\tilde{B}$ is called an intuitionistic fuzzy closed set in Y.

Definition 2.9. [11] *Let* (Y, Γ) *be an intuitionistic fuzzy topological space and* $\tilde{A}$ *be an intuitionistic fuzzy set in* Y. *Then the closure and the interior of* $\tilde{A}$ *are defined, respectively, by:*
$Cl(\tilde{A}) = \cap\{\tilde{K} : \tilde{K}$ *is an intuitionistic fuzzy closed set in* Y *and* $\tilde{A} \subseteq \tilde{K}\}$,
$Int(\tilde{A}) = \cup\{\tilde{G} : \tilde{G}$ *is an intuitionistic fuzzy open set in* Y *and* $\tilde{G} \subseteq \tilde{A}\}$.

Lemma 2.4. [11] *For any intuitionistic fuzzy set* $\tilde{A}$ *in* (Y, Γ) *we have:*

(a) $\tilde{A}$ *is an intuitionistic fuzzy closed set in* $Y \Leftrightarrow Cl(\tilde{A}) = \tilde{A}$

(b) $\tilde{A}$ *is an intuitionistic fuzzy open set in* $Y \Leftrightarrow Int(\tilde{A}) = \tilde{A}$

(c) $Cl(\tilde{A}^c) = (Int(\tilde{A})^c)$

(d) $Int(\tilde{A}^c) = (Cl(\tilde{A})^c)$

Definition 2.10. [14] *An intuitionistic fuzzy point $c(\alpha,\beta)$ is said to be a θ-cluster point of an intuitionistic fuzzy set $\tilde{A}$ if for each q-neighborhood $\tilde{B}$ of $c(\alpha,\beta)$, $\tilde{A}qCl(\tilde{B})$. The set of all θ-cluster points of $\tilde{A}$ is called θ-closure of $\tilde{A}$ and is denoted by $Cl_\theta(\tilde{A})$. An intuitionistic fuzzy set $\tilde{A}$ is called intuitionistic fuzzy θ-closed if $\tilde{A} = Cl_\theta(\tilde{A})$. The compliment of intuitionistic fuzzy θ-closed is called intuitionistic fuzzy θ-open set. The θ-interior of $\tilde{A}$ denoted by $Int_\theta(\tilde{A})$ is defined by $Int_\theta(\tilde{A}) = (Cl_\theta(\tilde{A}^c))^c$.*

Definition 2.11. [28] *Let X and Y are two nonempty sets. A function $F : (X,\tau) \to (Y,\Gamma)$ is called intuitionistic fuzzy multifunction if $F(x)$ is an intuitionistic fuzzy set in Y, $\forall x \in X$.*

Definition 2.12. [33] *Let F :$(X,\tau) \to (Y,\Gamma)$ is an intuitionistic fuzzy multifunction and A be a subset of X. Then $F(A) = \cup_{x\in A} F(x)$.*

Lemma 2.5. [33] *Let $F : (X,\tau) \to (Y,\Gamma)$ be an intuitionistic fuzzy multifunction. Then*

(a) $A \subseteq B \Rightarrow F(A) \subseteq F(B)$ for any subsets A and B of X.

(b) $F(A \cap B) \subseteq F(A) \cap F(B)$ for any subsets A and B of X.

(c) $F(\cup_{\alpha\in\Lambda} A_\alpha) = \cup\{F(A_\alpha) : \alpha \in \Lambda\}$ for any family of subsets $\{A_\alpha : \alpha \in \Lambda\}$ in X.

Definition 2.13. [28] *Let $F : (X,\tau) \to (Y,\Gamma)$ is an intuitionistic fuzzy multifunction. Then the upper inverse $F^+(\tilde{A})$ and lower inverse $F^-(\tilde{A})$ of an intuitionistic fuzzy set $\tilde{A}$ in Y are defined as follows:*

$F^+(\tilde{A}) = \{x \in X : F(x) \subseteq \tilde{A}\}$

$F^-(\tilde{A}) = \{x \in X : F(x)q\tilde{A}\}$

Lemma 2.6. [33] *Let $F : (X,\tau) \to (Y,\Gamma)$ be an intuitionistic fuzzy multifunction and $\tilde{A}, \tilde{B}$ be intuitionistic fuzzy sets in Y. Then:*

(a) $F^+(\tilde{1}) = F^-(\tilde{1}) = X$

(b) $F^+(\tilde{A}) \subseteq F^-(\tilde{A})$

(c) $[F^-(\tilde{A})]^c = [F^+(\tilde{A})^c]$

(d) $[F^+(\tilde{A})]^c = [F^-(\tilde{A})^c]$

(e) If $\tilde{A} \subseteq \tilde{B}$, then $F^+(\tilde{A}) \subseteq F^+(\tilde{B})$

(f) If $\tilde{A} \subseteq \tilde{B}$, then $F^-(\tilde{A}) \subseteq F^-(\tilde{B})$

Definition 2.14. [16] *A subset $\tilde{A}$ of an intuitionistic fuzzy topological space (Y,Γ) is called :*

(a) intuitionistic fuzzy Semi open if $\tilde{A} \subset Cl(Int(\tilde{A}))$.

(b) intuitionistic fuzzy Semi closed if its complement is semi open.

Definition 2.15. [28] *An Intuitionistic fuzzy multifunction* $F : (X, \tau) \to (Y, \Gamma)$ *is said to be:*

(a) Intuitionistic fuzzy upper semi-continuous at a point $x_0 \in X$ *, if for any intuitionistic fuzzy open set* $\tilde{W} \subset Y$ *such that* $F(x_0) \subset \tilde{W}$ *there exists an open set* $U \subset X$ *containing* x_0 *such that* $F(U) \subset \tilde{W}$ *.*

(b) Intuitionistic fuzzy lower semi-continuous at a point $x_0 \in X$ *, if for any intuitionistic fuzzy open set* $\tilde{W} \subset Y$ *such that* $F(x_0)q\tilde{W}$ *there exists an open set* $U \subset X$ *containing* x_0 *such that* $F(x)q\tilde{W}, \forall x \in U$ *.*

(c) Intuitionistic fuzzy upper semi-continuous (intuitionistic fuzzy lower semi-continuous) if it is intuitionistic fuzzy upper semi-continuous (Intuitionistic fuzzy lower semi-continuous) at each point of X*.*

Definition 2.16. [7] An Intuitionistic fuzzy multifunction $F : (X, \tau) \to (Y, \Gamma)$ is said to be:

(a) Intuitionistic fuzzy lower α-continuous at a point $x_0 \in X$, if for any intuitionistic fuzzy open set $\tilde{W} \subset Y$ such that $F(x_0)q\tilde{W}$ there exists $U \in \alpha O(X)$ containing x_0 such that $F(x)q\tilde{W}, \forall x \in U$.

(b) Intuitionistic fuzzy upper α-continuous at a point $x_0 \in X$, if for any intuitionistic fuzzy open set $\tilde{W} \subset Y$ such that $F(x_0) \subseteq \tilde{W}$ there exists $U \in \alpha O(X)$ containing x_0 such that $F(U) \subseteq \tilde{W}$.

(c) Intuitionistic fuzzy upper α-continuous (resp. Intuitionistic fuzzy lower α-continuous) if it is intuitionistic fuzzy upper α-continuous (resp. intuitionistic fuzzy lower α-continuous) at every point of X.

3. Lower Weakly α-continuous Intuitionistic Fuzzy Multifunctions

Definition 3.1. *An Intuitionistic fuzzy multifunction* $F : (X, \tau) \to (Y, \Gamma)$ *is said to be:*

(a) Intuitionistic fuzzy lower weakly α*-continuous at a point* $x_0 \in X$*, if for each* $U \in SO(X)$ *containing* x_0 *and each intuitionistic fuzzy open set* $\tilde{W} \subset Y$ *such that* $F(x_0)q\tilde{W}$ *there exists a nonempty open set* $V \subset U$ *such that* $F(x)qCl(\tilde{W})$*,* $\forall x \in V$*.*

(b) Intuitionistic fuzzy lower weakly α*-continuous if it is intuitionistic fuzzy lower weakly* α*-continuous at each point of* X*.*

Definition 3.2. Let $\tilde{A}$ be an intuitionistic fuzzy set of an intuitionistic fuzzy topological space (Y,Γ). Then $\tilde{V}$ is said to be a neighbourhood of $\tilde{A}$ in Y if there exists an intuitionistic fuzzy open set $\tilde{U}$ of Y such that $\tilde{A} \subset \tilde{U} \subset \tilde{V}$.

Theorem 3.1. *Let F :$(X,\tau) \to (Y,\Gamma)$ be an intuitionistic fuzzy multifunction and let $x \in X$. Then the following statements are equivalent:*

(a) F is intuitionistic fuzzy lower weakly α-continuous at x.

(b) For each intuitionistic fuzzy open set $\tilde{B}$ of Y with $F(x)q\tilde{B}$, there exists $U \in \alpha O(X)$ containing x such that $F(x)qCl(\tilde{B})$, $\forall x \in U$.

(c) $x \in \alpha Int(F^-(Cl(\tilde{B})))$ for every intuitionistic fuzzy open set $\tilde{B}$ of Y such that $F(x)q\tilde{B}$.

(d) $x \in Int(Cl(Int(F^-(Cl(\tilde{B})))))$ for every intuitionistic fuzzy open set $\tilde{B}$ of Y such that $F(x)q\tilde{B}$.

Proof. **(a)⇒(b)**. Let $x \in X$ and $\tilde{B}$ be any intuitionistic fuzzy open set of Y such that $F(x)q\tilde{B}$. For each $U \in SO(X)$ such that $x \in U$ there exists a nonempty open set G_U such that $G_U \subset U$ and $F(x)qCl(\tilde{B})\forall x \in G_U$. Let $N = \cup\{G_U : U \in SO(X)\}$. Put $M = N \cup \{x\}$, then N is open in $X, x \in sCl(N)$ and $F(x)qCl(\tilde{B})\forall x \in N$. Thus we have by Lemma 2.1 $M \in SO(X)$. Hence $F(x)qCl(\tilde{B})\forall x \in M$.

(b)⇒(c). Let $\tilde{B}$ be any intuitionistic fuzzy open set of Y such that $F(x)q\tilde{B}$, then there exists $M \in \alpha O(X)$ such that $F(x)qCl(\tilde{B}))\forall x \in M$. Then $x \in M \subset (F^-(Cl(\tilde{B})))$ and hence $x \in \alpha Int(F^-(Cl(\tilde{B})))$.

(c)⇒(d). Let $\tilde{B}$ be any intuitionistic fuzzy open set of Y such that $F(x)q\tilde{B}$, Let $x \in \alpha Int(F^-(Cl(\tilde{B})))$, then there exists $M \in \alpha O(X)$ such that $F(M)qCl(\tilde{B}))$. Then $x \in M \subset (F^-(Cl(\tilde{B})))$ and hence $x \in U \subset Int(Cl(Int(F^-(Cl(\tilde{B})))))$.

(d)⇒(a). Let $\tilde{B}$ be any intuitionistic fuzzy open set of Y such that $F(x)q\tilde{B}$ and $U \in SO(X)$ containig x. Then we have $x \in Int(Cl(Int(F^-(Cl(\tilde{B}))))) = sCl(Int(F^-(Cl(\tilde{B}))))$, therefore $\phi \neq U \cap Int(F^-(Cl(\tilde{B}))) \in SO(X)$. Put $G = U \cap Int(F^-(Cl(\tilde{B})))$, then G is a nonempty open set of X and $G \subset U$ and $F(G)qCl(\tilde{B})$, by Lemma 2.6. Hence, F is intuitionistic fuzzy lower weakly α-continuous at x. □

Definition 3.3. [29] *Let X and Y are two non empty sets. A multifunction $F : X \to Y$ is called fuzzy multifunction if $F(x)$ is a fuzzy set in Y, $\forall x \in X$.*

Corollary 3.1. *Let F be a fuzzy multifunction from a topological space $(X, \urcorner)$ into a fuzzy topological space (Y,σ) and let $x \in X$. Then the following statements are equivalent:*

(a) F is fuzzy lower weakly α-continuous at x.

(b) For each fuzzy open set B of Y with $F(x)qB$, there exists $U \in \alpha O(X)$ containing x such that $F(x)qCl(B)$, $\forall x \in U$. .

(c) $x \in \alpha Int(F^{-}(Cl(B)))$ *for every fuzzy open set* B *of* Y *such that* $F(x)qB$.

(d) $x \in Int(Cl(Int(F^{-}(Cl(B)))))$ *for every fuzzy open set* B *of* Y *such that* $F(x)qB$.

Corollary 3.2. [30] *For a multifunction* $F : X \to Y$ *and a point* $x \in X$. *Then the following statements are equivalent:*

(a) F *is lower weakly* α*-continuous at* x.

(b) For each open set B *of* Y *with* $F(x)qB$, *there exists* $U \in \alpha O(X)$ *containing* x *such that* $F(x)qCl(B)$, $\forall x \in U$. .

(c) $x \in \alpha Int(F^{-}(Cl(B)))$ *for every open set* B *of* Y *such that* $F(x)qB$.

(d) $x \in Int(Cl(Int(F^{-}(Cl(B)))))$ *for every open set* B *of* Y *such that* $F(x)qB$.

Theorem 3.2. *Let* $F : (X, \tau) \to (Y, \Gamma)$ *be an intuitionistic fuzzy multifunction. Then the following statements are equivalent:*

(a) F *is intuitionistic fuzzy lower weakly* α*-continuous;*

(b) For each intuitionistic fuzzy open set $\tilde{B}$ *of* Y *with* $F(x)q\tilde{B}$, *there exists* $U \in \alpha O(X)$ *containing* x *such that* $U \subset F^{-}(Cl(\tilde{B}))$;

(c) $F^{-}(\tilde{G}) \subset Int(Cl(Int(F^{-}Cl(\tilde{G}))))$ *for every intuitionistic fuzzy open set* $\tilde{G}$ *of* Y;

(d) $Cl(Int(Cl(F^{+}(Int(\tilde{V})) \subset F^{+}(Int(\tilde{V}))$ *for every intuitionistic fuzzy closed set* $\tilde{V}$ *of* Y;

(e) $\alpha Cl(F^{+}Int(\tilde{V}) \subset F^{+}(\tilde{V})$ *for every intuitionistic fuzzy closed set* $\tilde{V}$ *of* Y;

(f) $\alpha Cl(F^{+}Int(Cl(\tilde{B})) \subset F^{+}(Cl(\tilde{B}))$ *for every intuitionistic fuzzy closed set* $\tilde{B}$ *of* Y;

(g) $F^{-}(Int(\tilde{B})) \subset \alpha Int(F^{-}(Cl(Int(\tilde{B}))))$, *for each intuitionistic fuzzy subset* $\tilde{B}$ *of* Y;

(h) $F^{-}(\tilde{V}) \subset \alpha Int(F^{-}(Cl(\tilde{V})))$, *for each intuitionistic fuzzy open set* $\tilde{B}$ *of* Y;

(i) $\alpha Cl(F^{+}(Int(\tilde{A})) \subset F^{+}(\tilde{A})$ *for every intuitionistic fuzzy regular set* $\tilde{A}$ *of* Y;

(j) $\alpha Cl(F^{+}(\tilde{B})) \subset F^{+}(Cl(\tilde{B}))$, *for each Intuitionistic fuzzy open set* $\tilde{B}$ *of* Y.

(k) $\alpha Cl(F^{+}(Int(Cl_{\theta}(\tilde{B})) \subset F^{+}(Cl_{\theta}(\tilde{B}))$, *for each Intuitionistic fuzzy subset* $\tilde{B}$ *of* Y.

Proof. **(a)⇒(b)**. Similar to Theorem 3.1

(b)⇒(c). Let x be arbitrarily chosen in X and $\tilde{B}$ be any intuitionistic fuzzy open set of Y such that $F(x)q\tilde{B}$, so $x \in F^-(\tilde{B})$. By hypothesis there exists $U \in \alpha O(X)$ containing x such that $F(x)qCl(\tilde{B}); \forall x \in U$ implies $x \in U \subset F^-(Cl(\tilde{B}))$ since $U \in \alpha O(X)$, we have $x \in F^-(\tilde{B}) \subset Int(Cl(Int(F^-(Cl(\tilde{B})))))$.

(c)⇒(d). Let $\tilde{G}$ be any arbitrary intuitionistic fuzzy closed set of Y. Then $\tilde{G}^c$ is intuitionistic fuzzy open set of Y. By (c) $F^-(\tilde{G}^c) \subset Int(Cl(Int(F^-(Cl(\tilde{G}^c)))))$ by lemma 2.6 (c) we have $Cl(Int(Cl(F^+(Int(\tilde{G}))))) \subset F^+(Int(\tilde{G}))$.

(d)⇒(e). Suppose that (d) holds, let $\tilde{V}$ be any arbitrary intuitionistic fuzzy closed set of Y thus we have $Cl(Int(Cl(F^+(Int(\tilde{V}))))) \subset F^+(Int(\tilde{V}))$ and hence $\alpha Cl(F^+(Int(\tilde{V}))) \subset F^+(\tilde{V})$ by lemma 2.1 and 2.2.

(e)⇒(f). Suppose that (e) holds and let $\tilde{V}$ be any intuitionistic fuzzy set of Y. Then $Cl(\tilde{V})$ is intuitionistic fuzzy closed set of Y therefore $\alpha Cl(F^+(Int(Cl(\tilde{V})))) \subset F^+(Cl(\tilde{V}))$.

(e)⇒(i). Obvious.

(f)⇒(g). Let $\tilde{B}$ be any intuitionistic fuzzy subset of Y, then $[F^-(Int(\tilde{B}))]^c = F^+(Cl(\tilde{B}^c)) \supset \alpha Cl(F^+(Int(Cl(\tilde{B}^c)))) = \alpha Cl(F^+(Cl(Int(\tilde{B}))^c) = \alpha Cl(F^-(Cl(Int(\tilde{B}))))^c = [\alpha Int(F^-(Cl(Int(\tilde{B}))))]^c$. Thus we obtained $F^-(Int(\tilde{B})) \subset \alpha Int(F^-(Cl(Int(\tilde{B}))))$

(g)⇒(h). Obvious.

(i)⇒(j). Let $\tilde{A}$ be any intuitionistic fuzzy open set of Y then $Cl(\tilde{A})$ is intuitionistic fuzzy regular closed in Y and hance we have $\alpha Cl(F^+(\tilde{A})) \subset \alpha Cl(F^+(Int(Cl(\tilde{A})))) \subset \alpha Cl(F^+(Cl(\tilde{A}))) \subset F^+(Cl(\tilde{B}))$.

(j)⇒(h). Let $\tilde{B}$ be any intuitionistic fuzzy open set of Y, then $[\alpha Int(F^-(Cl(\tilde{B})))]^c = \alpha Cl(F^-(Cl(\tilde{B})))^c = \alpha Cl(F^+(Int(Cl(\tilde{B})^c)) \subset F^+(Cl(Cl(\tilde{B}))^c)) = F^+(Cl(Int(\tilde{B}))) = [F^-(Int(Cl(\tilde{B})))]^c$. Thus we obtained $F^-(\tilde{B}) = F^-(Int(Cl(\tilde{B}))) \subset \alpha Int(F^-(Cl(\tilde{B})))$.

(h)⇒(a). Let x be any point of X and $\tilde{B}$ be any intuitionistic fuzzy open set of Y such that $F(x)q\tilde{B}$. Then it follows that $x \in F^-(B) \subset \alpha Int(F^-(Cl((B)))) \subset Int(Cl(Int(F^-Cl(B))))$. Thus F is intuitionistic fuzzy lower weakly α continuous multifunction.

(j)⇒(k). Let $\tilde{B}$ be any intuitionistic fuzzy subset of Y, put $\tilde{B} = Int(Cl_\theta(\tilde{V})$ in (j). Then because $Cl_\theta(\tilde{V})$ is intuitionistic fuzzy closed in Y, we have $\alpha Cl(F^+(Int(Cl_\theta(\tilde{V})) \subset F^+(Cl_\theta(\tilde{V}))$.

(k)⇒(j). Let $\tilde{V}$ be any intuitionistic fuzzy regular closed set of Y. Therefore, we have $Cl(\tilde{V}) = Cl_\theta(\tilde{V})$ for every intuitionistic fuzzy open set $\tilde{V}$ of Y, thus $\alpha Cl(F^+(Int(\tilde{V}))) = \alpha Cl(F^+(Int(Cl(\tilde{V})))) = \alpha Cl(F^+(Int(Cl_\theta(\tilde{V})))) \subset F^+(Cl_\theta(\tilde{V})) = F^+(Cl(\tilde{V})) = F^+(\tilde{V})$. □

Corollary 3.3. *Let F be a fuzzy multifunction from a topological space $(X, \urcorner)$ into a fuzzy topological space (Y, σ). Then the following statements are equivalent:*

(a) F is fuzzy lower weakly α-continuous;

(b) *For each fuzzy open set* B *of* Y *with* $F(x)qB$*, there exists* $U \in \alpha O(X)$ *containing* x *such that* $U \subset F^{-}(Cl(B))$*;*

(c) $F^{-}(G) \subset Int(Cl(Int(F^{-}Cl(G))))$ *for every fuzzy open set* G *of* Y*;*

(d) $Cl(Int(Cl(F^{+}(Int(V)) \subset F^{+}(Int(V))$ *for every fuzzy closed set* V *of* Y*;*

(e) $\alpha Cl(F^{+}Int(V) \subset F^{+}(V)$ *for every fuzzy closed set* V *of* Y*;*

(f) $\alpha Cl(F^{+}Int(Cl(B)) \subset F^{+}(Cl(B))$ *for every fuzzy closed set* B *of* Y*;*

(g) $F^{-}(Int(B)) \subset \alpha Int(F^{-}(Cl(Int(B))))$*, for each fuzzy subset* B *of* Y*;*

(h) $F^{-}(V) \subset \alpha Int(F^{-}(Cl(V)))$*, for each fuzzy open set* B *of* Y*;*

(i) $\alpha Cl(F^{+}(Int(A)) \subset F^{+}(A)$ *for every fuzzy regular set* A *of* Y*;*

(j) $\alpha Cl(F^{+}(B)) \subset F^{+}(Cl(B))$*, for each fuzzy open set* B *of* Y*.*

Corollary 3.4. [30] *Let* F *be a multifunction from a topological space* $(X, \urcorner)$ *into another topological space* (Y, ζ)*. Then the following statements are equivalent:*

(a) F *is lower weakly* α*-continuous;*

(b) *For each open set* B *of* Y *with* $F(x)qB$*, there exists* $U \in \alpha O(X)$ *containing* x *such that* $U \subset F^{-}(Cl(B))$*;*

(c) $F^{-}(G) \subset Int(Cl(Int(F^{-}Cl(G))))$ *for every open set* G *of* Y*;*

(d) $Cl(Int(Cl(F^{+}(Int(V)) \subset F^{+}(Int(V))$ *for every closed set* V *of* Y*;*

(e) $\alpha Cl(F^{+}Int(V) \subset F^{+}(V)$ *for every closed set* V *of* Y*;*

(f) $\alpha Cl(F^{+}Int(Cl(B)) \subset F^{+}(Cl(B))$ *for every closed set* B *of* Y*;*

(g) $F^{-}(Int(B)) \subset \alpha Int(F^{-}(Cl(Int(B))))$*, for each subset* B *of* Y*;*

(h) $F^{-}(V) \subset \alpha Int(F^{-}(Cl(V)))$*, for each open set* B *of* Y*;*

(i) $\alpha Cl(F^{+}(Int(A)) \subset F^{+}(A)$ *for every regular set* A *of* Y*;*

(j) $\alpha Cl(F^{+}(B)) \subset F^{+}(Cl(B))$*, for each open set* B *of* Y*.*

Lemma 3.1. $F : (X, \tau) \to (Y, \Gamma)$ *is an intuitionistic fuzzy lower weakly* α*-continuous multifunction, then for each* $x \in X$ *and each* $\tilde{B} \subset Y$ *with* $F(x)q(Int_{\theta}(\tilde{B}))$ *there exists* $U \in \alpha O(X)$ *such that* $U \subset F^{-}(\tilde{B})$*.*

Proof. Since $F(x)q(Int_{\theta}(\tilde{B}))$ there exists a nonempty intuitionistic fuzzy set $\tilde{A}$ of Y such that $\tilde{A} \subset Cl(\tilde{A}) \subset \tilde{B}$ and $F(x)q\tilde{A}$. Since F is lower weakly α-continuous there exists $U \in \alpha O(X)$ such that $F(u)qCl(\tilde{A}) : \forall u \in U$ and hence $U \subset F^{-}(B)$. □

Theorem 3.3. *For an intuitionistic fuzzy multifunction $F : (X, \tau) \to (Y, \Gamma)$, followings are equivalent:*

(a) F is intuitionistic fuzzy lower weakly α-continuous.

(b) $\alpha Cl(F^+(\tilde{B})) \subset F^+(Cl_\theta(\tilde{B}))$ for every intuitionistic fuzzy subset $\tilde{B}$ of Y.

(c) $F(\alpha Cl(A)) \subset Cl_\theta(F(A))$ for every subset A of X.

Proof. **(a)⇒(b)**. Let $\tilde{B}$ be any intuitionistic fuzzy open set of Y. Suppose $x \in F^-(Cl_\theta(\tilde{B}))^c = F^-(Int_\theta(\tilde{B}^c))$, by Lemma 3.1, there exists $U \in \alpha O(X)$ such that $U \subset F^-(\tilde{B}^c) = [F^+(\tilde{B})]^c$. Thus $U \cap [F^+(\tilde{B})]^c \neq \phi$ therefore $x \in \alpha Cl(F^+(\tilde{B}))^c$.

(b)⇒(a). Let $\tilde{A}$ be any intuitionistic fuzzy open set of Y. Since $Cl(\tilde{A}) = Cl_\theta(\tilde{A})$ for every intuitionistic fuzzy open subset of Y and we have $\alpha Cl(F^+(\tilde{B})) \subset F^+(Cl_\theta(\tilde{B}))$, by theorem 3.2 F is intuitionistic fuzzy lower weakly α-continuous.

(b)⇒(c). Let A be any nonempty subset of X, by (b) we have

$$\alpha Cl(A) \subset \alpha Cl(F^+(F(A))) \subset F^+(Cl_\theta(F(A)))$$

therefore we obtain $F(\alpha Cl(A)) \subset Cl_\theta(F(A))$.

(c)⇒(b). Let $\tilde{B}$ be any intuitionistic fuzzy open set of Y. By (c) we have

$$F(\alpha Cl(F^+(\tilde{B}))) \subset Cl_\theta(F(F^+(A))) \subset Cl_\theta(\tilde{B})$$

Therefore $\alpha Cl(F^+(\tilde{B})) \subset F^+(Cl_\theta(\tilde{B}))$ □

4. Upper Weakly α-Continuous Intuitionistic Fuzzy Multifunctions

Definition 4.1. *An intuitionistic fuzzy multifunction $F : (X, \tau) \to (Y, \Gamma)$ is said to be:*

(a) Intuitionistic fuzzy upper weakly α-continuous at a point $x_0 \in X$, if for each $U \in SO(X)$ containing x_0 and each intuitionistic fuzzy open set $\tilde{W} \subset Y$ such that $F(x_0) \subset \tilde{W}$ there exists a nonempty open set $V \subset U$ such that $F(V) \subset Cl(\tilde{W})$.

(b) Intuitionistic fuzzy upper weakly α-continuous if it is intuitionistic fuzzy upper weakly α-continuous at each point of X.

Theorem 4.1. *Let $F : (X, \tau) \to (Y, \Gamma)$ be an intuitionistic fuzzy multifunction and let $x \in X$. Then the following statements are equivalent:*

(a) F is intuitionistic fuzzy upper weakly α-continuous at x.

(b) For each intuitionistic fuzzy open set $\tilde{B}$ of Y with $F(x) \subset \tilde{B}$, there exists $U \in \alpha O(X)$ containing x such that $F(U) \subset Cl(\tilde{B})$.

(c) $x \in \alpha Int(F^+(Cl(\tilde{B})))$ *for every intuitionistic fuzzy open set* $\tilde{B}$ *of* Y *such that* $F(x) \subset \tilde{B}$.

(d) $x \in Int(Cl(Int(F^+(Cl(\tilde{B})))))$ *for every intuitionistic fuzzy open set* $\tilde{B}$ *of* Y *such that* $F(x) \subset \tilde{B}$.

Proof. **(a)⇒(b)**. Let $x \in X$ and $\tilde{B}$ be any intuitionistic fuzzy open set of Y such that $F(x) \subset \tilde{B}$. For each $U \in SO(X)$ such that $x \in U$ there exists a nonempty open set G_U such that $G_U \subset U$ and $F(G_U) \subset Cl(\tilde{B})$. Let $A = \cup\{G_U : U \in SO(X)\}$. Put $S = A \cup \{x\}$, then A is open in $X, x \in sCl(A)$ and $F(A) \subset Cl(\tilde{B})$. Thus we have by Lemma 2.1 $S \in SO(X)$. Hence $F(S) \subset Cl(\tilde{B})$.

(b)⇒(c). Let $\tilde{B}$ be any intuitionistic fuzzy open set of Y such that $F(x) \subset \tilde{B}$,then there exists $S \in \alpha O(X)$ such that $F(S) \subset Cl(\tilde{B}))$. Then $x \in S \subset (F^+(Cl(\tilde{B})))$ and hence $x \in \alpha Int(F^+(Cl(\tilde{B})))$.

(c)⇒(d). Let $\tilde{B}$ be any intuitionistic fuzzy open set of Y such that $F(x) \subset \tilde{B}$, Let $x \in \alpha Int(F^+(Cl(\tilde{B})))$, then there exists $S \in \alpha O(X)$ such that $F(S) \subset Cl(\tilde{B}))$. Then $x \in S \subset (F^+(Cl(\tilde{B})))$ and hence $x \in U \subset Int(Cl(Int(F^+(Cl(\tilde{B})))))$.

(d)⇒(a). Let $\tilde{B}$ be any intuitionistic fuzzy open set of Y such that $F(x) \subset \tilde{B}$ and $U \in SO(X)$ containing x. Then we have $x \in Int(Cl(Int(F^+(Cl(\tilde{B}))))) = sCl(Int(F^+(Cl(\tilde{B}))))$, therefore $\phi \neq U \cap Int(F^+(Cl(\tilde{B}))) \in SO(X)$. Put $G = U \cap Int(F^+(Cl(\tilde{B})))$, then G is a nonempty open set of X and $G \subset U$ and $F(G) \subset Cl(\tilde{B})$, by Lemma 2.6, hence F is intuitionistic fuzzy upper weakly α-continuous at x. □

Corollary 4.1. *Let* F *be a fuzzy multifunction from a topological space* $(X, \urcorner)$ *into a fuzzy topological space* (Y, σ) *and let* $x \in X$. *Then the following statements are equivalent:*

(a) F *is fuzzy upper weakly* α*-continuous at* x.

(b) *For each fuzzy open set* B *of* Y *with* $F(x) \subset B$, *there exists* $U \in \alpha O(X)$ *containing* x *such that* $F(U) \subset Cl(B)$.

(c) $x \in \alpha Int(F^+(Cl(B)))$ *for every fuzzy open set* B *of* Y *such that* $F(x) \subset B$.

(d) $x \in Int(Cl(Int(F^+(Cl(B)))))$ *for every fuzzy open set* B *of* Y *such that* $F(x) \subset B$.

Corollary 4.2. [27] *Let* F *be a multifunction from a topological space* $(X, \urcorner)$ *into another topological space* (Y, ζ) *and let* $x \in X$. *Then the following statements are equivalent:*

(a) F *is upper weakly* α*-continuous at* x.

(b) *For each open set* B *of* Y *with* $F(x) \subset B$, *there exists* $U \in \alpha O(X)$ *containing* x *such that* $F(U) \subset Cl(B)$.

(c) $x \in \alpha Int(F^+(Cl(B)))$ *for every open set* B *of* Y *such that* $F(x) \subset B$.

(d) $x \in Int(Cl(Int(F^+(Cl(B)))))$ *for every open set* B *of* Y *such that* $F(x) \subset B$.

Definition 4.2. *Let* $\tilde{A}$ *be an intuitionistic fuzzy set of an intuitionistic fuzzy topological space* (Y, Γ)*. Then* $\tilde{V}$ *is said to be an* α*-neighbourhood of* $\tilde{A}$ *in* Y *if there exists an intuitionistic fuzzy* α*-open set* $\tilde{U} \subset Y$ *such that* $\tilde{A} \subset \tilde{U} \subset \tilde{V}$.

Theorem 4.2. *For an intuitionistic fuzzy multifunction* $F : (X, \tau) \to (Y, \Gamma)$ *the following statements are equivalent:*

(a) F *is intuitionistic fuzzy upper weakly* α*-continuous;*

(b) For each intuitionistic fuzzy open set $\tilde{B}$ *of* Y *with* $F(x) \subset \tilde{B}$*, there exists* $U \in \alpha O(X)$ *containing* x *such that* $F(U) \subset (Cl(\tilde{B}))$*;*

(c) $F^+(\tilde{G}) \subset Int(Cl(Int(F^+Cl(\tilde{G}))))$ *for every intuitionistic fuzzy open set* $\tilde{G}$ *of* Y*;*

(d) $Cl(Int(Cl(F^-(Int(\tilde{V})) \subset F^-(\tilde{V})$ *for every intuitionistic fuzzy closed set* $\tilde{V}$ *of* Y*;*

(e) $\alpha Cl(F^- Int(\tilde{V}) \subset F^-(\tilde{V})$ *for every intuitionistic fuzzy closed set* $\tilde{V}$ *of* Y*;*

(f) $\alpha Cl(F^-(Int(Cl(\tilde{B}))) \subset F^-(Cl(\tilde{B}))$ *for every intuitionistic fuzzy set* $\tilde{B}$ *of* Y*;*

(g) $F^+(Int(\tilde{B})) \subset \alpha Int(F^+(Cl(Int(\tilde{B}))))$*, for each intuitionistic fuzzy subset* $\tilde{B}$ *of* Y*;*

(h) $F^+(\tilde{V}) \subset \alpha Int(F^+(Cl(\tilde{V})))$*, for each intuitionistic fuzzy open set* $\tilde{B}$ *of* Y*;*

(i) $\alpha Cl(F^-(Int(\tilde{A})) \subset F^-(\tilde{A})$ *for every intuitionistic fuzzy regular closed set* $\tilde{A}$ *of* Y*;*

(j) $\alpha Cl(F^-(\tilde{B})) \subset F^-(Cl(\tilde{B}))$*, for each Intuitionistic fuzzy open set* $\tilde{B}$ *of* Y.

(k) $\alpha Cl(F^-(Cl_\theta(\tilde{B})) \subset F^-(Cl_\theta(\tilde{B}))$*, for each Intuitionistic fuzzy subset* $\tilde{B}$ *of* Y.

Proof. **(a)⇒(b)**. Similar to Theorem 4.1.

(b)⇒(c). Let x be arbitrarily chosen in X and $\tilde{B}$ be any intuitionistic fuzzy open set of Y such that $F(x) \subset \tilde{B}$, so $x \in F^+(\tilde{B})$. By hypothesis there exists $U \in \alpha O(X)$ containing x such that $F(U) \subset Cl(\tilde{B})$ implies $x \in U \subset F^+(Cl(\tilde{B}))$ since $U \in \alpha O(X)$, we have $x \in F^+(\tilde{B}) \subset Int(Cl(Int(F^+(Cl(\tilde{B})))))$.

(c)⇒(d). Let $\tilde{G}$ be any arbitrary intuitionistic fuzzy closed set of Y. Then $\tilde{G}^c$ is intuitionistic fuzzy open set of Y. By (c) $F^+(\tilde{G}^c) \subset Int(Cl(Int(F^+(Cl(\tilde{G}^c)))))$ by lemma 2.6 (d) we have $Cl(Int(Cl(F^-(Int(\tilde{G}))))) \subset F^-(\tilde{G})$.

(d)⇒(e). Suppose that (d) holds, let $\tilde{V}$ be any arbitrary intuitionistic fuzzy closed set of Y thus we have $Cl(Int(Cl(F^-(Int(\tilde{V}))))) \subset F^-(\tilde{V})$ and hence $\alpha Cl(F^-(Int(\tilde{V}))) \subset F^-(\tilde{V})$ by lemma 2.1 and 2.2.

(e)⇒(f). Suppose that (e) holds and let $\tilde{V}$ be any intuitionistic fuzzy set of Y. Then $Cl(\tilde{V})$ is intuitionistic fuzzy closed set of Y therefore $\alpha Cl(F^-(Int(Cl(\tilde{V})))) \subset F^-(Cl(\tilde{V}))$.

(e)⇒(i). Obvious.

(f)⇒(g). Let $\tilde{B}$ be any intuitionistic fuzzy subset of Y, then $[F^+(Int(\tilde{B}))]^c = F^-(Cl(\tilde{B}^c)) \supset \alpha Cl(F^-(Int(Cl(\tilde{B}^c)))) = \alpha Cl(F^-(Cl(Int(\tilde{B}))^c) = \alpha Cl(F^+(Cl(Int(\tilde{B}))))^c = [\alpha Int(F^+(Cl(Int(\tilde{B}))))]^c$. Thus we obtained $F^+(Int(\tilde{B})) \subset \alpha Int(F^+(Cl(Int(\tilde{B}))))$

(g)⇒(h). Obvious.

(i)⇒(j). Let $\tilde{A}$ be any intuitionistic fuzzy open set of Y then $Cl(\tilde{A})$ is regular closed in Y and hance we have $\alpha Cl(F^-(\tilde{A})) \subset \alpha Cl(F^-(Int(Cl(\tilde{A})))) \subset \alpha Cl(F^-(Cl(\tilde{A}))) \subset F^-(Cl(\tilde{B}))$.

(j)⇒(h). Let $\tilde{B}$ be any intuitionistic fuzzy open set of Y, then $[\alpha Int(F^+(Cl(\tilde{B})))]^c = \alpha Cl(F^+(Cl(\tilde{B})))^c = \alpha Cl(F^-(Int(Cl(\tilde{B}))^c)) \subset F^-(Cl(Cl(\tilde{B}))^c)) = F^-(Cl(Int(\tilde{B})))) = [F^+(Int(Cl(\tilde{B})))]^c$. Thus we obtained $F^+(\tilde{B}) = F^+(Int(Cl(\tilde{B}))) \subset \alpha Int(F^+(Cl(\tilde{B})))$.

(h)⇒(a). Let x be any point of X and $\tilde{B}$ be any intuitionistic fuzzy open set of Y such that $F(x) \subset \tilde{B}$. Then it follows that $x \in F^+(B) \subset \alpha Int(F^+(Cl((B)))) \subset Int(Cl(Int(F^+Cl(B))))$. Thus F is intuitionistic fuzzy upper weakly α continuous multifunction.

(j)⇒(k). Let $\tilde{V}$ be any intuitionistic fuzzy subset of Y, put $\tilde{B} = Int(Cl_\theta(\tilde{V})$ in (j). Then because $(Cl_\theta(\tilde{V})$ is closed in Y, we have $\alpha Cl(F^-(Int(Cl_\theta(\tilde{V})) \subset F^-(Cl_\theta(\tilde{V}))$.

(k)⇒(j). Let $\tilde{V}$ be any intuitionistic fuzzy regular closed set of Y. Therefore, we have $Cl(\tilde{V}) = Cl_\theta(\tilde{V})$ for every intuitionistic fuzzy open set $\tilde{V}$ of Y, thus $\alpha Cl(F^-(Int(\tilde{V}))) = \alpha Cl(F^-(Int(Cl(\tilde{V})))) = \alpha Cl(F^-(Int(Cl_\theta(Int(\tilde{V}))))) \subset F^-(Cl_\theta(Int(\tilde{V}))) = F^-(Cl(Int(\tilde{V}))) = F^-(\tilde{V})$ □

Corollary 4.3. *Let F be a fuzzy multifunction from a topological space $(X, \urcorner)$ into a fuzzy topological space (Y, σ) and let $x \in X$. Then the following statements are equivalent:*

(a) F is fuzzy upper weakly α-continuous;

(b) For each fuzzy open set B of Y with $F(x) \subset B$, there exists $U \in \alpha O(X)$ containing x such that $F(U) \subset Cl(B)$;

(c) $F^+(G) \subset Int(Cl(Int(F^+Cl(G))))$ for every fuzzy open set G of Y;

(d) $Cl(Int(Cl(F^-(Int(V)) \subset F^-(V)$ for every fuzzy closed set V of Y;

(e) $\alpha Cl(F^-Int(V) \subset F^-(V)$ for every fuzzy closed set V of Y;

(f) $\alpha Cl(F^-(Int(Cl(B))) \subset F^-(Cl(B))$ for every fuzzy set B of Y;

(g) $F^+(Int(B)) \subset \alpha Int(F^+(Cl(Int(B))))$, for each fuzzy subset B of Y;

(h) $F^+(V) \subset \alpha Int(F^+(Cl(V)))$, for each fuzzy open set B of Y;

(i) $\alpha Cl(F^-(Int(A)) \subset F^-(A)$ *for every fuzzy regular closed set* A *of* Y*;*

(j) $\alpha Cl(F^-(B)) \subset F^-(Cl(B))$*, for each fuzzy open set* B *of* Y*.*

Corollary 4.4. [30] *Let* F *be a multifunction from a topological space* $(X, \urcorner)$ *into another topological space* (Y, ζ) *and let* $x \in X$*. Then the following statements are equivalent:*

(a) F *is upper weakly* α*-continuous;*

(b) For each open set B *of* Y *with* $F(x) \subset B$*, there exists* $U \in \alpha O(X)$ *containing* x *such that* $F(U) \subset Cl(B)$*;*

(c) $F^+(G) \subset Int(Cl(Int(F^+Cl(G))))$ *for every open set* G *of* Y*;*

(d) $Cl(Int(Cl(F^-(Int(V)) \subset F^-(V)$ *for every closed set* V *of* Y*;*

(e) $\alpha Cl(F^- Int(V) \subset F^-(V)$ *for every closed set* V *of* Y*;*

(f) $\alpha Cl(F^-(Int(Cl(B))) \subset F^-(Cl(B))$ *for every set* B *of* Y*;*

(g) $F^+(Int(B)) \subset \alpha Int(F^+(Cl(Int(B))))$*, for each subset* B *of* Y*;*

(h) $F^+(V) \subset \alpha Int(F^+(Cl(V)))$*, for each open set* B *of* Y*;*

(i) $\alpha Cl(F^-(Int(A)) \subset F^-(A)$ *for every regular closed set* A *of* Y*;*

(j) $\alpha Cl(F^-(B)) \subset F^-(Cl(B))$*, for each open set* B *of* Y*.*

5. Properties of Upper(lower) Weakly α-Continuous Intuitionistic Fuzzy Multifunctions

Lemma 5.1. [21] *Let* U *and* X_0 *be a subset of topological space* X*. The following properties hold:*

(a) If $A \in SO(X) \cup PO(X)$ *and* $B \in \alpha(X)$*,then* $A \cap B \in \alpha(A)$*.*

(b) If $A \subset B \subset X, A \in \alpha(B)$ *and* $B \in \alpha(X)$*, then* $A \in \alpha(X)$*.*

Theorem 5.1. *If an intuitionistic fuzzy multifunction* $F : (X, \tau) \to (Y, \Gamma)$ *is intuitionistic fuzzy upper weakly* α*-continuous (resp. intuitionistic fuzzy lower weakly* α*-continuous) and* $A \in PO(X) \cup SO(X)$*, then the restriction* $F|A : A \to Y$ *is intuitionistic fuzzy upper weakly* α*-continuous (resp. intuitionistic fuzzy lower weakly* α*-continuous).*

Proof. We prove only the assertion for F intuitionistic fuzzy upper weakly α-continuous, the proof for F intuitionistic fuzzy lower weakly α-continuous being analogous. Let $x \in X$ and $\tilde{V}$ be any intuitionistic fuzzy open set of Y such that $(F|A)(x) \subset \tilde{V}$. Since F intuitionistic fuzzy upper weakly α-continuous and

$(F|A)(x) = F(x)$, there exists $U \in \alpha(X)$ containing x such that $F(U) \subset Cl(\tilde{V})$. Set $U_0 = U \cap A$. Then by Lemma 5.1 we have $x \in U_0 \in (A)$ and $(F|A)(U_0) = F(U_0) \subset Cl(\tilde{V})$. This shows that $F|A : A \to Y$ is intuitionistic fuzzy upper weakly α-continuous. □

Theorem 5.2. *An intuitionistic fuzzy multifunction $F : (X, \tau) \to (Y, \Gamma)$ is intuitionistic fuzzy upper weakly α -continuous (resp. intuitionistic fuzzy lower weakly α-continuous) if for each $x \in X$ there exists $X_0 \in \alpha(X)$ containing x such that the restriction $F|X_0 : X_0 \to Y$ is intuitionistic fuzzy upper weakly α-continuous (resp. intuitionistic fuzzy lower weakly α-continuous).*

Proof. We prove only the assertion for F intuitionistic fuzzy upper weakly α-continuous, the proof for F intuitionistic fuzzy lower weakly α-continuous being analogous. Let $x \in X$ and $\tilde{V}$ be any intuitionistic fuzzy open set of Y such that $F(x) \subset \tilde{V}$. There exists $X_0 \in \alpha(X)$ containing x such that $(F|X_0)(x) = F(x)$ intuitionistic fuzzy upper weakly α-continuous, there exists $U_0 \in \alpha(X_0)$ containing x such that $(F|X_0)(U_0) \subset Cl(\tilde{V})$. Then By Lemma 5.1 we have $x \in U_0 \in \alpha(X)$ and $F(u) = (F|X_0)(u), \forall u \in U_0$. This shows that $F : X \to Y$ is intuitionistic fuzzy upper weakly α-continuous. □

Theorem 5.3. *If $F : (X, \tau) \to (Y, \Gamma)$ is intuitionistic fuzzy multifunction, such that $F(x)$ is closed in Y for each $x \in X$ and Y is a normal space, then the following are equivalent:*

(a) F is intuitionistic fuzzy upper α-continuous.

(b) F is intuitionistic fuzzy upper weakly α-continuous.

Proof. $(a) \to (b)$. Obvious.

$(b) \to (a)$. Suppose that F is intuitionistic fuzzy upper weakly α-continuous. Let $x \in X$ and $\tilde{V}$ if intuitionistic fuzzy open set of Y such that $F(x) \subset \tilde{V}$. Since $F(x)$ is intuitionistic fuzzy closed in Y, by normality of Y there exists an intuitionistic fuzzy open set $\tilde{W}$ of Y such that $F(x) \subset \tilde{W} \subset Cl(\tilde{W}) \subset \tilde{V}$. Since F is intuitionistic fuzzy upper weakly α-continuous, there exists $U \in \alpha(X)$ containing x such that $F(U) \subset Cl(\tilde{W})$; hence $F(U) \subset \tilde{V}$. This shows that F is intuitionistic fuzzy upper α-continuous. □

Theorem 5.4. *If $F : (X, \tau) \to (Y, \Gamma)$ is intuitionistic fuzzy multifunction, such that $F(x)$ is open in Y for each $x \in X$, then the following are equivalent:*

(a) F is intuitionistic fuzzy lower α-continuous.

(b) F is intuitionistic fuzzy lower weakly α-continuous.

Proof. $(a) \to (b)$. Obvious.

$(b) \to (a)$. Suppose that F is intuitionistic fuzzy lower weakly α-continuous. Let $x \in X$ and $\tilde{V}$ if intuitionistic fuzzy open set of Y such that $F(x) \cap \tilde{V}$. There exists an open set $U \in \alpha(X)$ containing x such that $F(u)qCl(\tilde{W}), \forall u \in U$. Since $F(u)$ is intuitionistic fuzzy open in Y,hence $F(u)q\tilde{W}, \forall u \in U$. This shows that F is intuitionistic fuzzy lower α-continuous. □

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